

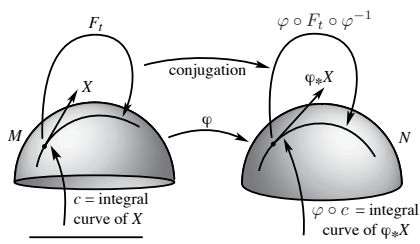
Exercise 9

Due 6/6/2019

- 1.) Let $F_t : [a, b] \times V \rightarrow V$ be a flow on the linear space V generated by a vector field $X \in \mathfrak{X}(V)$ and $\varphi : V \rightarrow W$ a diffeomorphism.

- i.) Define the push-forward $\varphi_* F_t$ of F_t by φ .
- ii.) What can be said about the relationship between the vector field X and $\varphi_* F_t$?

See the following figure (from [MRA, Ch. 4.2]) for inspiration.



- 2.) Let the vector field $X \in \mathfrak{X}(\mathbb{R}^2)$ be given by

$$X(x) = \cos \theta \frac{\partial}{\partial x} - \sin \theta \frac{\partial}{\partial y} \quad (1)$$

and let the covector fields $\alpha, \beta \in \mathfrak{X}^*(\mathbb{R}^2)$ be

$$\alpha = \sin \theta dx + \cos \theta dy \quad (2)$$

$$\beta = 0.2 dx \quad (3)$$

- i.) Compute the pairings $\alpha(X)$ and $\beta(X)$ on $[-1, 1] \times [-1, 1]$.
- ii.) Which properties would the vector / covector fields have to satisfy for the pairing to be well defined on all of \mathbb{R}^2 .