Tensor Analysis and Applications 2019

Exercise 6

Due 13/5/2019

1.) Let $t \in T^1_1(\mathbb{R}^2)$ be given by $t = e_1 \otimes e^2 - 2e_2 \otimes e^2$ and let $\phi : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation with matrix representation

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}. \tag{1}$$

i.) Compute the inverse ϕ^{-1} of ϕ.

ii.) Compute the push-forward $\phi_* t$ using linearity and the push-forward of the basis functions.

iii.) Compute the push-forward $\phi_* t$ using index (or matrix) notation.

2.) Let

$$f(t) = -2t^3 + 3t^2 + 2x - 5 \tag{2}$$

$$g(t) = -0.5t^3 + 2t^2 - 3 \tag{3}$$

be two particle trajectories as functions of time.

i.) Plot the functions on $t \in [-2, 1]$.

ii.) Compute the tangents to the functions at $t = -1.251$.

iii.) What happens to the tangents when the particles traverse their trajectories in half the time?

iv.) Assume the particles have equal mass and the collide at $t = -1.251$ elastically (i.e. energy is conserved) and "stick" together. What is the tangent of the joint particle at $t = -1.251$.

v.) Which mathematical concept captures the essential properties of tangents?