Tensor Analysis and Applications 2019

Exercise 5

Due 6/5/2019

1.) Let $\omega \in T^2_0(V)$ and non-degenerate. Show that $\omega(X, X) = 0$ if and only if ω is anti-symmetric.

2.) Let $V = \mathbb{R}^3$ and $\{e_i\}$ be the canonical basis. Consider a second basis

$$\bar{e}_1 = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \qquad \bar{e}_2 = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \qquad \bar{e}_3 = \begin{pmatrix} -1 \\ 5 \\ 3 \end{pmatrix} \tag{1}$$

specified with respect to $\{e_i\}$. Furthermore, let the metric $g \in T^0_2(V)$ be given by

$$g = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix}, \tag{2}$$

also specified with respect to $\{e_i\}$.

i.) Construct the dual basis $\{\bar{e}^i\}$. Plot the primary and dual vectors.

ii.) Transform g to $\{\bar{e}_i\}$. Verify for a few examples that $g(v, w)$ is independent of the coordinate system that is used.

iii.) For the co-vectors

$$\alpha = \begin{pmatrix} 1 & 0.5 & 3 \end{pmatrix} \qquad \beta = \begin{pmatrix} -1.5 & 2 & 3 \end{pmatrix} \tag{3}$$

given with respect to $\{\bar{e}_i\}$ compute the associated vectors $\alpha^\sharp$ and $\beta^\sharp$. Verify numerically that $g(\alpha, \beta) = g(\alpha^\sharp, \beta^\sharp)$.