Tensor Analysis and Applications 2019

Exercise 4

Due 29/4/2019

1.) Using a co-metric $g \in T^2_0(V)$ we can identify a co-vector $\alpha \in V^*$ with a vector $\bar{\alpha} \in V$ as

$$\bar{\alpha}^j = g^{ij} \alpha_i \tag{1a}$$

and one can then consider the pairing

$$\sum_{i=1}^{n} \alpha^i \, v^i . \tag{1b}$$

Let A be a coordinate transformation for V with components A^j_i. Show that the above pairing is not covariant.

2.) Let $g \in T^0_2(V)$ be a metric for V and $\bar{g} \in T^0_2(V^*)$ be one for V^*.

i.) Why does g provide an isomorphism between V and V^*?

ii.) Using the interior product we can associate vectors v_α and v_β with $\alpha, \beta \in V^*$ by

$$v_\alpha = \mathrm{i}_\alpha \bar{g} \tag{2a}$$

$$v_\beta = \mathrm{i}_\beta \bar{g} \tag{2b}$$

If we require that g and $\bar{g}$ satisfy the natural compatibility

$$\bar{g}(\alpha, \beta) = g(v_\alpha, v_\beta) \tag{2c}$$

then, in components in the usual sense of matrix calculus, $\bar{g}$ is the inverse of g. Show this.