

Tensor Analysis and Applications 2019

Exercise 2

Due 16/4/2019

- 1.) Let V be a vector space with basis $\{e_1, \dots, e_n\}$ and V^* its dual.
 - i.) Write the biorthogonality conditions for the dual basis $\{e^1, \dots, e^n\}$ in matrix notation.
 - ii.) Given an arbitrary basis $\{e_1, \dots, e_n\}$, how can you determine numerically if a given set of vectors $\{u_1, \dots, u_n\}$ forms a basis? How can the dual basis be computed numerically?
 - iii.) Write a program (e.g. using Numpy¹ or Matlab) that, for a randomly generated set of vectors in \mathbb{R}^2 , checks if the vectors form a basis, computes the dual basis if this is the case, and plots primary and dual basis vectors.
- 2.) From the definition of an inner product, derive its coordinate expression. Explain how the inner product is compatible with the linear structure of the vector space and how this carries over to the coordinate representation. Is this also true for the norm?

¹An introduction to python and Numpy can be found on the course page.