Tensor Analysis and Applications 2019

Exercise 14

Due 2/7/2019

A (singular) inverse chart map for the 2-sphere S^2 is given by

$$\eta(\theta,\phi) = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \sin\theta\,\cos\phi \\ \sin\theta\,\sin\phi \\ \cos\theta \end{pmatrix} \tag{1}$$

The metric on S^2 is given by

$$g_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & \sin\theta \end{pmatrix}. \tag{2}$$

1.) Compute the tangent vectors for S^2 as the push-forward as the ones in the chart.

2.) Let $\mu \in \Omega^2(S^2)$. Compute the pullback of μ in the chart. Relate it to the classical formula for integral of a function on S^2

3.) Compute the Hodge dual for all differential form basis functions on S^2.

4.) Derive the classical formulas for gradient, curl, and divergence on S^2. (Note that most books use an orthonormal frame on S^2 whereas the natural one induced by the above inverse chart map is not unit length.)

5.) Derive a coordinate expression for $\mathcal{L}_u \alpha$ for $\alpha \in \Omega^1(S^2)$.