Tensor Analysis and Applications 2019

Exercise 11

Due 20/6/2019

1.) Let $F = B_x\, dy \wedge dz + B_y\, dz \wedge dx + B_z\, dx \wedge dy + E_x\, dx \wedge dt + E_y\, dy \wedge dt + E_z\, dz \wedge dt \in \bigwedge^2(\mathbb{R}_t \times \mathbb{R}^3)$ be the Faraday 2-form, with E, B, corresponding to the electric and magnetic fields.

   i.) Compute $\mathrm{d}F$.

   ii.) Identify the resulting expression with the classical vector calculus form of Maxwell's equations.

2.) A simple transport equation for a scalar quantity is

$$\frac{\partial f}{\partial t} + \mathcal{L}_X f = 0. \tag{1}$$

Rewrite this equation using the exterior derivative.

i.)

3.) Let $v^\flat$ the 1-form associated with the velocity vector field $v \in \mathfrak{X}(\mathbb{R}^n)$ of an incompressible fluid. Then the vorticity of the fluid is given by $\zeta = \mathrm{d}v^\flat$. In fact, the vorticity ζ completely specifies the velocity vector field. What does this say about $v^\flat$.