Tensor Analysis and Applications 2019

Exercise 10

Due 17/6/2019

1.) Let $\alpha \in \bigwedge^1(\mathbb{R}^3)$, $\beta \in \bigwedge^2(\mathbb{R}^3)$, and $\gamma \in \bigwedge^3(\mathbb{R}^3)$.

   i.) Compute $\alpha \wedge \beta$.

   ii.) Compute $\mathrm{d}\beta$ und $\mathrm{d}\gamma$.

   iii.) Compare $\mathrm{d}\alpha$ to the classical expression for the curl of a vector field.

   iv.) For $\xi \in \bigwedge^1(\mathbb{R}^3)$ compute $\alpha \wedge \xi$ and compare the resulting expression to the cross product.

2.) Compute $\mathrm{d}(f\,\alpha)$ for $f \in \bigwedge^0(\mathbb{R}^3)$ and $\alpha \in \bigwedge^1(\mathbb{R}^3)$ using that the exterior derivative satisfies the product rule. Find the vector calculus identity the resulting expression corresponds to.

3.) Let $\alpha = \alpha_1(x)\,\mathrm{d}x^1 + \alpha_2(x)\,\mathrm{d}x^2 \in \mathcal{T}_1^0(\mathbb{R}^2)$. Show that the symmetric part of the "naive" derivative $\partial\alpha_i/\partial x^j$ is not a tensor under coordinate transformations (it might be useful to use the decomposition of a matrix in its symmetric and anti-symmetric part).