

Scientific Computing II

Assignment 1

Due: 21/11/2019, 23:59

In this assignment we solve the advection equation

$$u_t + v u_x = 0 \quad (1)$$

with constant velocity $v = \pm 1$ for $t \in [0, 5]$ and $x \in [-10, 10]$, see Fig. 1. The initial conditions are

$$u_1(x, 0) = e^{-x^2} \quad (2a)$$

$$u_2(x, 0) = H(x) = \begin{cases} 1 & x \in [-1/2, 1/2] \\ 0 & \text{otherwise} \end{cases} \quad (2b)$$

For the spatial and time discretization we use

$$h_x = 0.01 \quad (3a)$$

$$h_t = \{0.005, 0.01, 0.02\} \quad (3b)$$

with Courant number $C = v h_t / h_x$. Python skeleton code is available [here](#).

- 1.) Implement the analytic solution. (1 points)
- 2.) Implement the following time integrators: (5 points)
 - a) Explicit Euler / centered differences: $u_i^{n+1} = u_i^n + \frac{1}{2}C(u_{i+1}^n - u_{i-1}^n)$
 - b) Implicit Euler / centered differences: $u_i^{n+1} = u_i^n + \frac{1}{2}C(u_{i+1}^{n+1} - u_{i-1}^{n+1})$
 - c) Leap frog: $u_i^{n+1} = u_i^{n-1} + C(u_{i+1}^n - u_{i-1}^n)$
 - d) Crank-Nicholson: $u_i^{n+1} = u_i^n + \frac{1}{4}C((u_{i+1}^n - u_{i-1}^n) + (u_{i+1}^{n+1} - u_{i-1}^{n+1}))$
 - e) Lax-Wendroff: $u_i^{n+1} = u_i^n + \frac{1}{2}C((u_{i+1}^n - u_{i-1}^n) + C(u_{i+1}^n - 2u_i^n + u_{i-1}^n))$

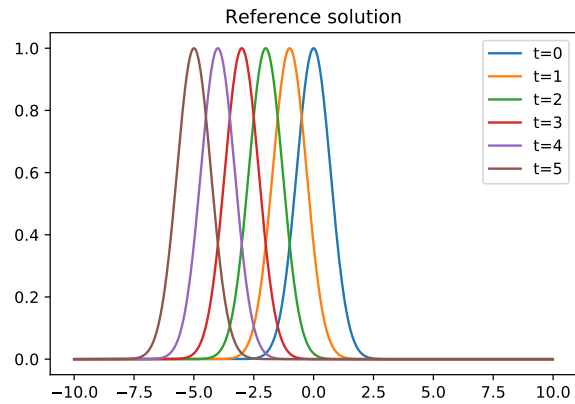


Figure 1: Reference solution for transport of u_1 .

- 3.) Perform experiments with the settings in Eq. 3a and generate plots at $t = \{0, 1, 2, 3, 4, 5\}$.
- 4.) Do your results agree with the CFL condition for the respective method? For this please either compute or look up the conditions. (2 points)

Please submit your implementation and the graphs before the deadline to wr@isg.cs.uni-magdeburg.de.