Mathematical Tools for Computer Graphics 2017

Tutorial 4

In this tutorial we will construct the dual basis for monomials up to degree 4. This provides an example for how to work with biorthogonal bases in the context of function spaces.

An orthonormal basis for the space of polynomials.

1.) Plot the first five Legendre polynomials $P_l(x)$.

2.) Show numerically that the functions are orthogonal but *not* orthonormal.

Construction of the dual basis.

1.) Construct the basis matrix B for the monomials x^m with respect to (normalized!) Legendre polynomials $P_l(x)$. The normalization factor for the $P_l(x)$ is

$$C_l = \sqrt{\frac{2l+1}{2}}.$$

How can B be constructed economically without recourse to numerical integration?

2.) Given B verify that the Legendre polynomials indeed span the same space as the monomials.

3.) Construct and plot the dual basis functions $\tilde{x}_n$ for the monomials.

4.) Verify that the constructed functions $\tilde{x}_n$ satisfy the biorthogonality condition using numerical integration.

5.) Verify for an example that the dual functions $\tilde{x}_n$ indeed provide the expansion coefficients of polynomials with respect to the monomials.