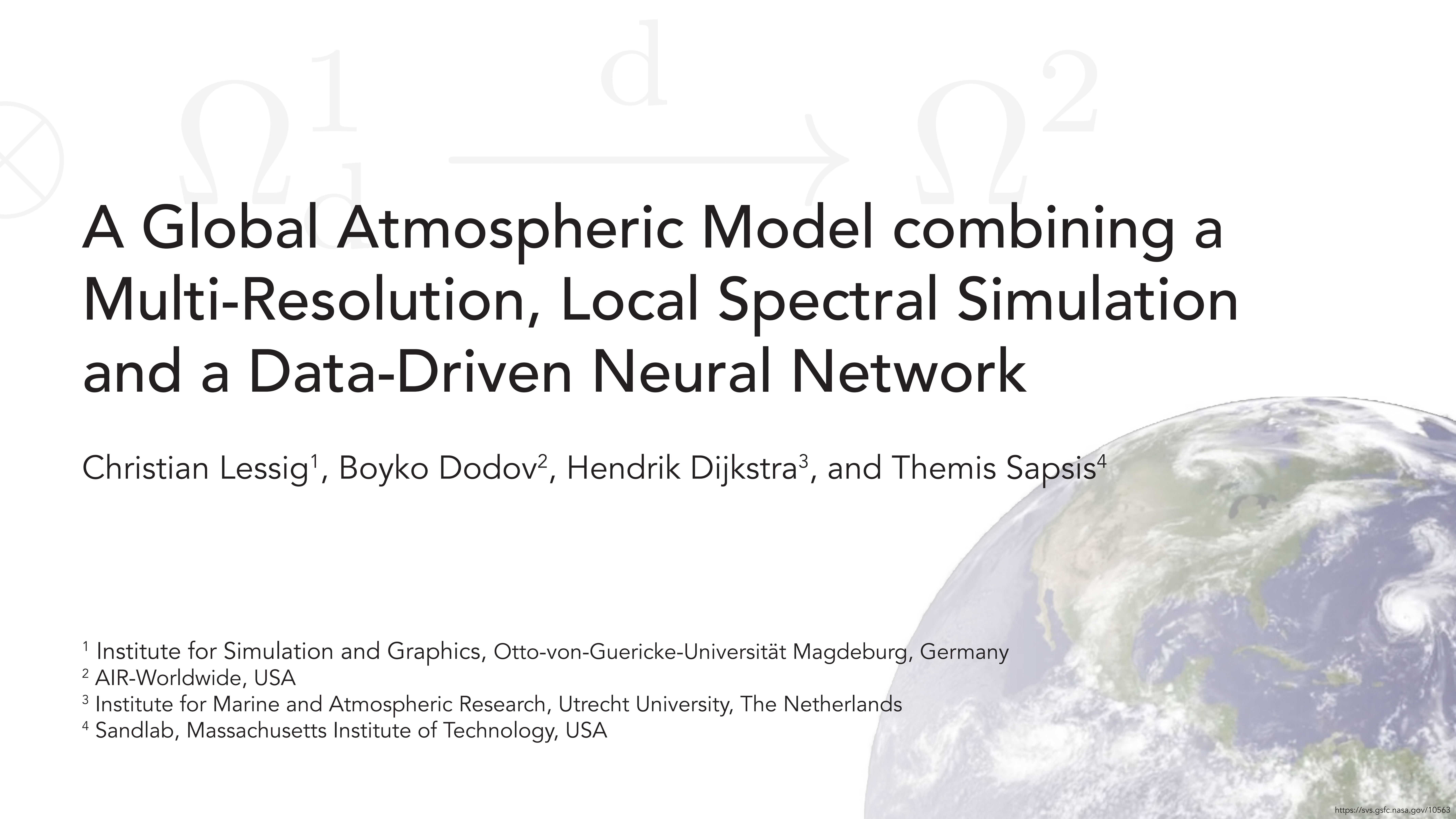




A Global Atmospheric Model combining a Multi-Resolution, Local Spectral Simulation and a Data-Driven Neural Network

Christian Lessig¹, Boyko Dodov², Hendrik Dijkstra³, and Themis Sapsis⁴

¹ Institute for Simulation and Graphics, Otto-von-Guericke-Universität Magdeburg, Germany

² AIR-Worldwide, USA

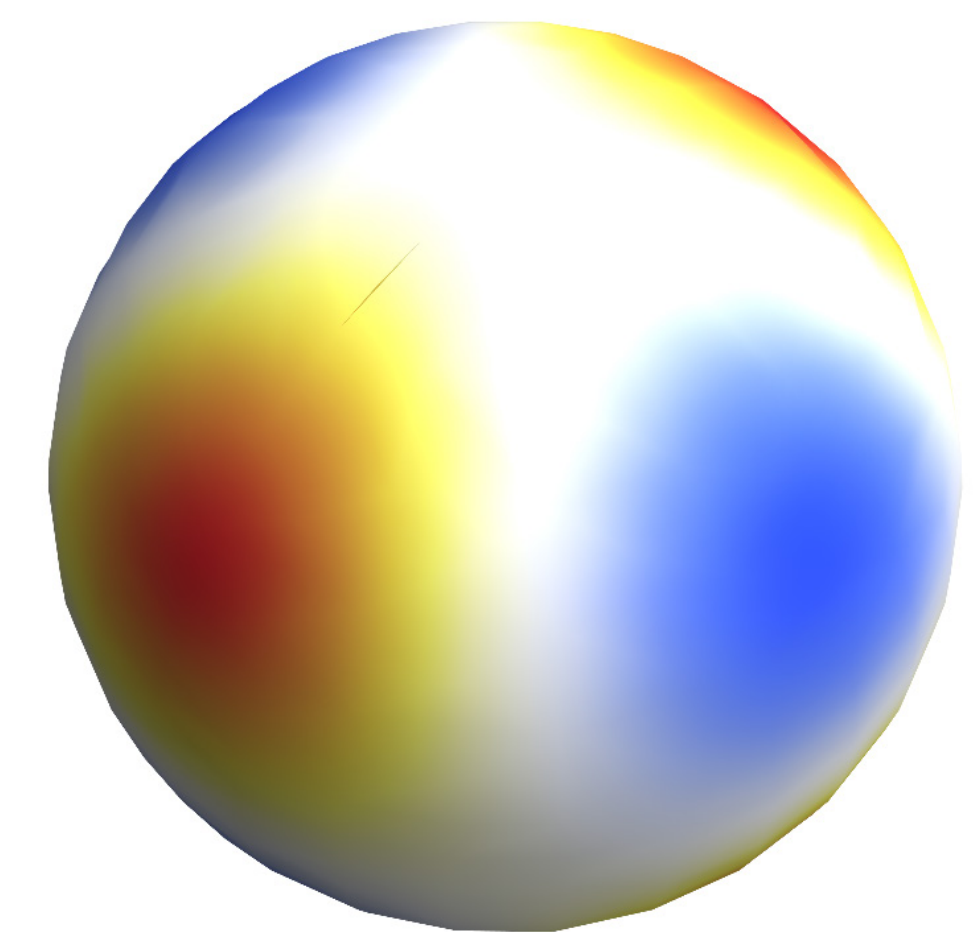
³ Institute for Marine and Atmospheric Research, Utrecht University, The Netherlands

⁴ Sandlab, Massachusetts Institute of Technology, USA

Motivation

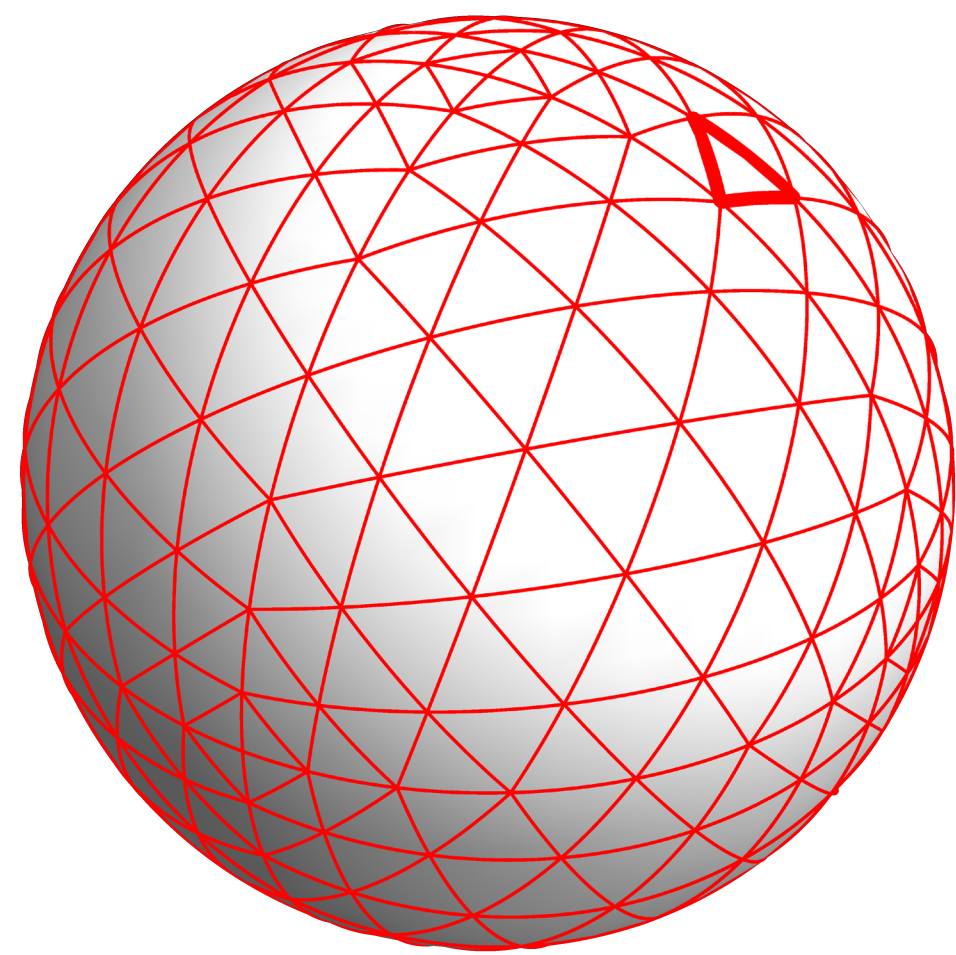
Motivation

Spectral
models

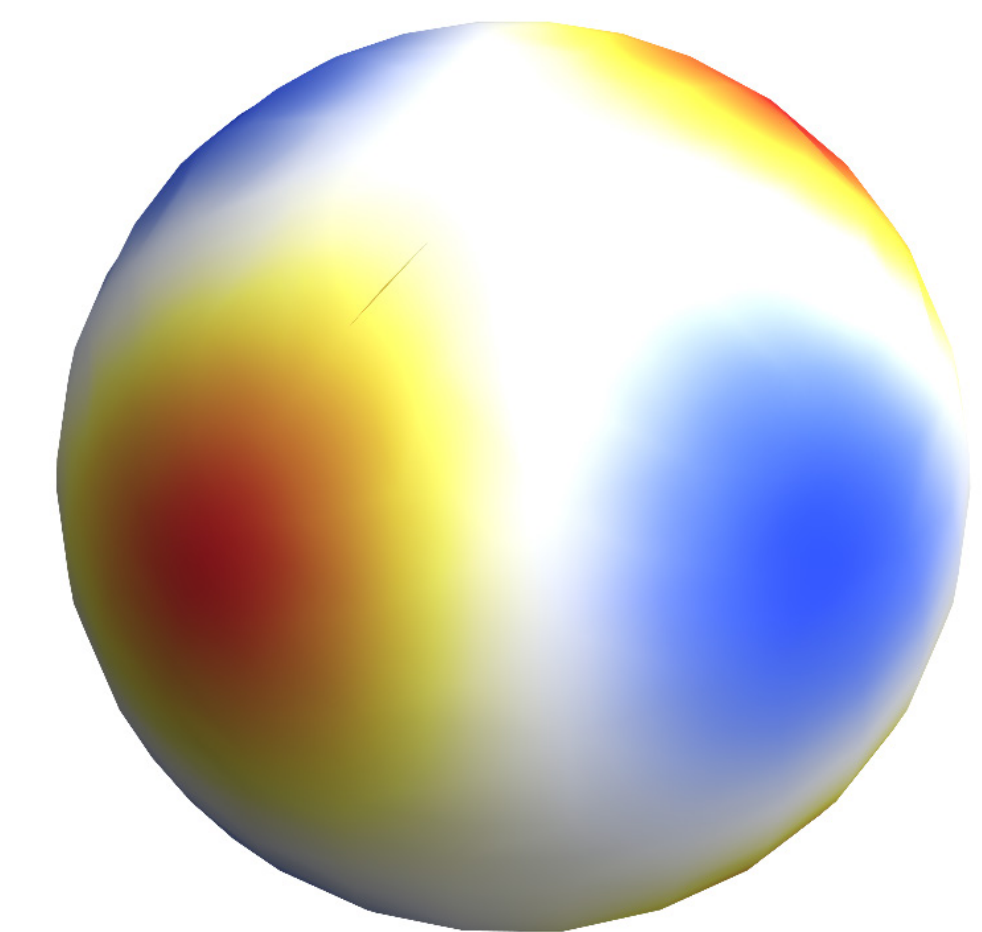


Motivation

Finite element
type models

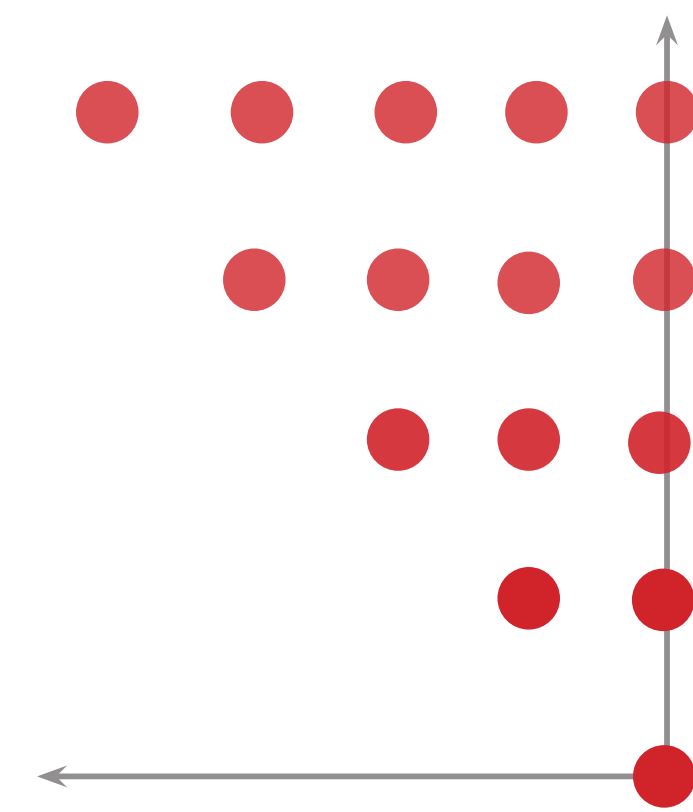
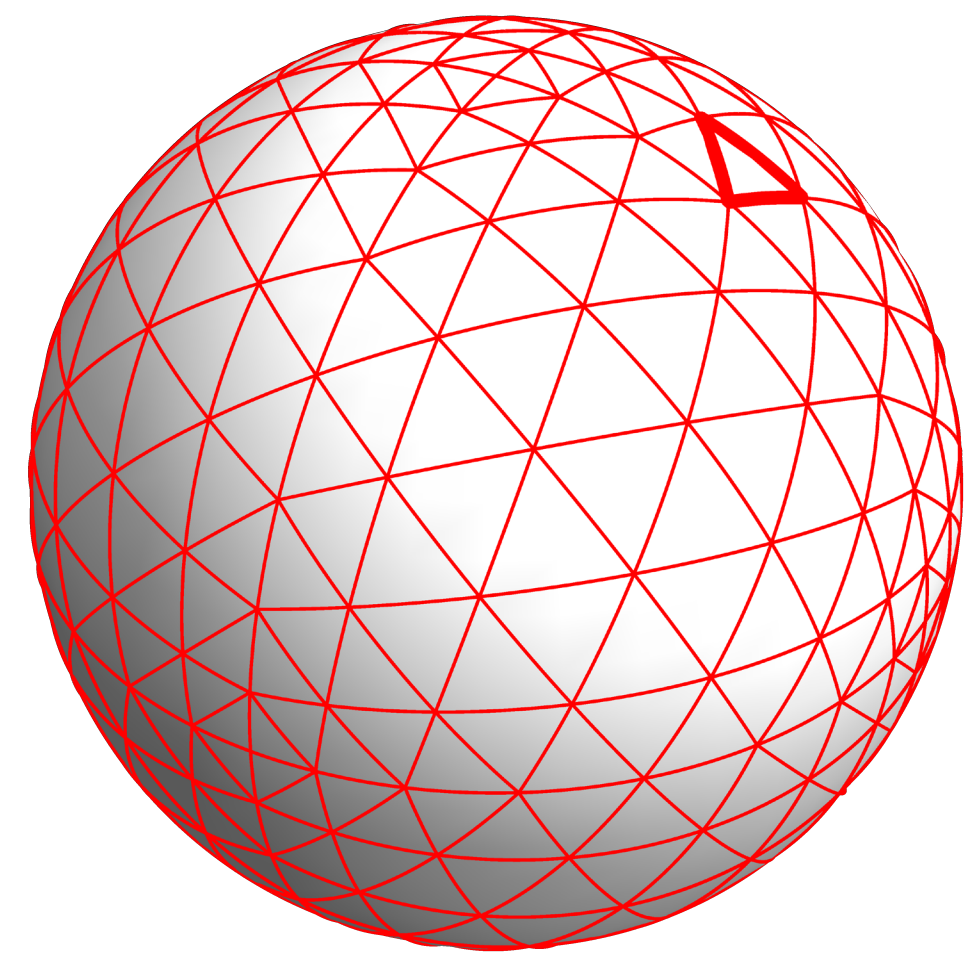


Spectral
models

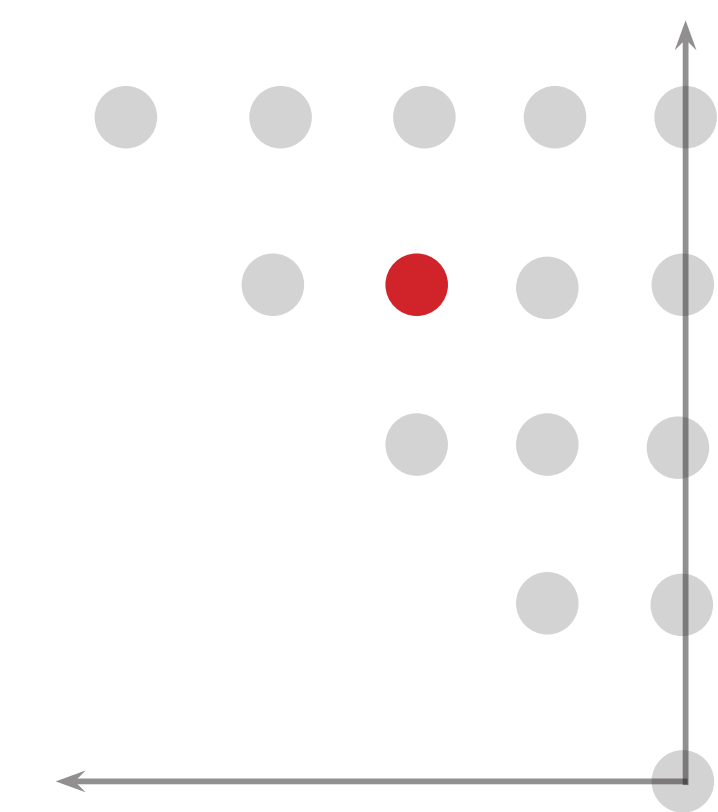
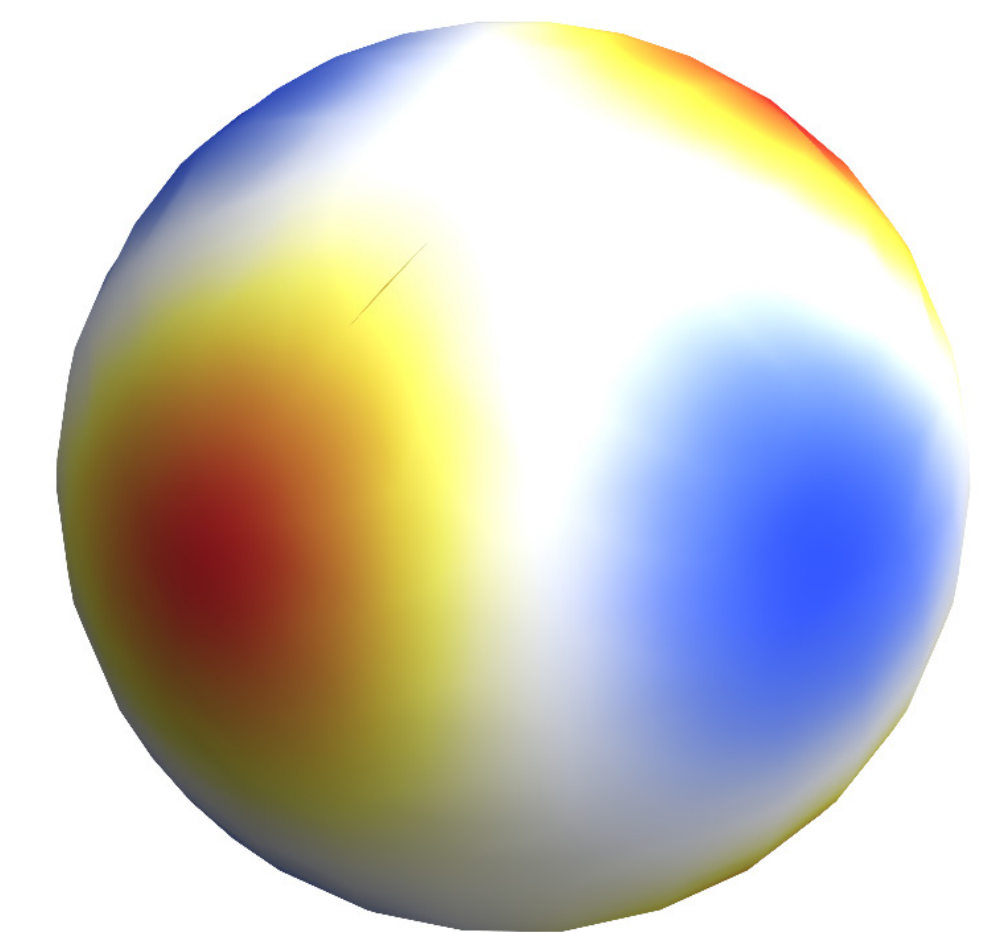


Motivation

Finite element
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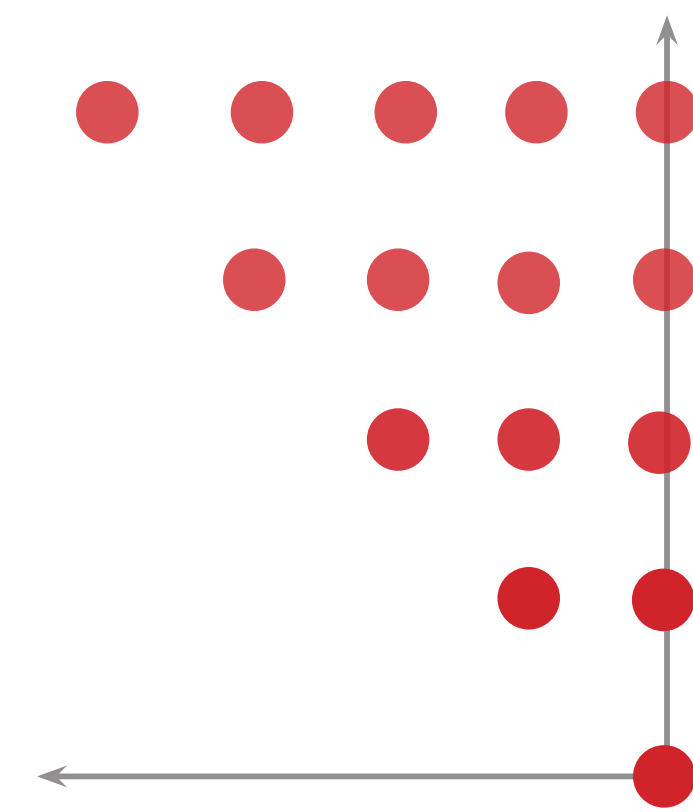
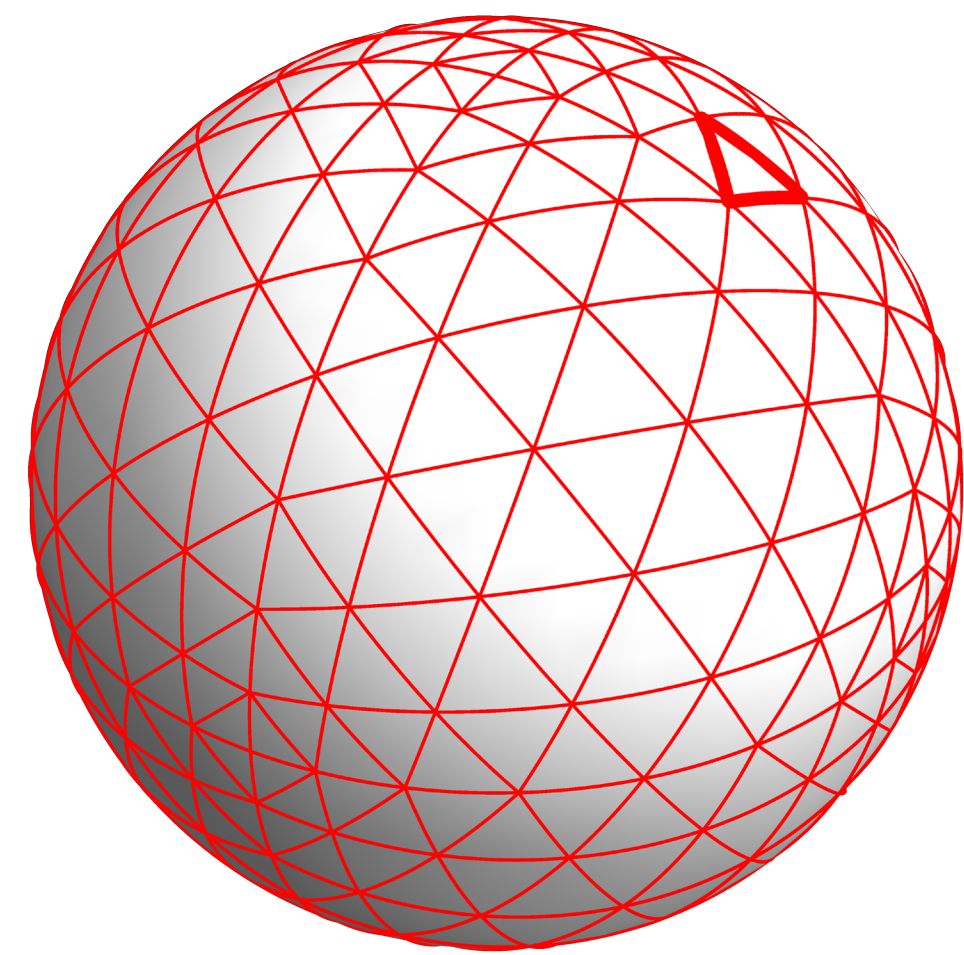


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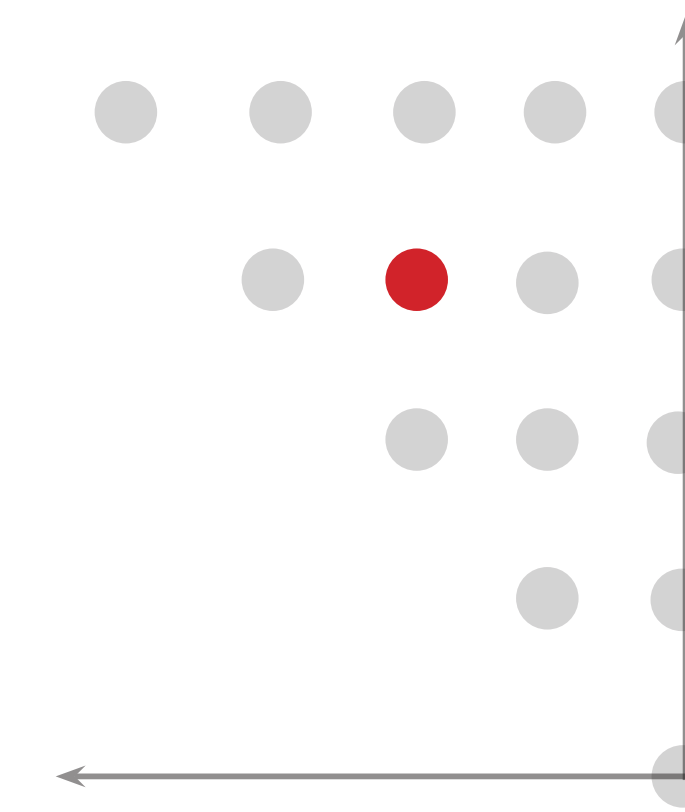
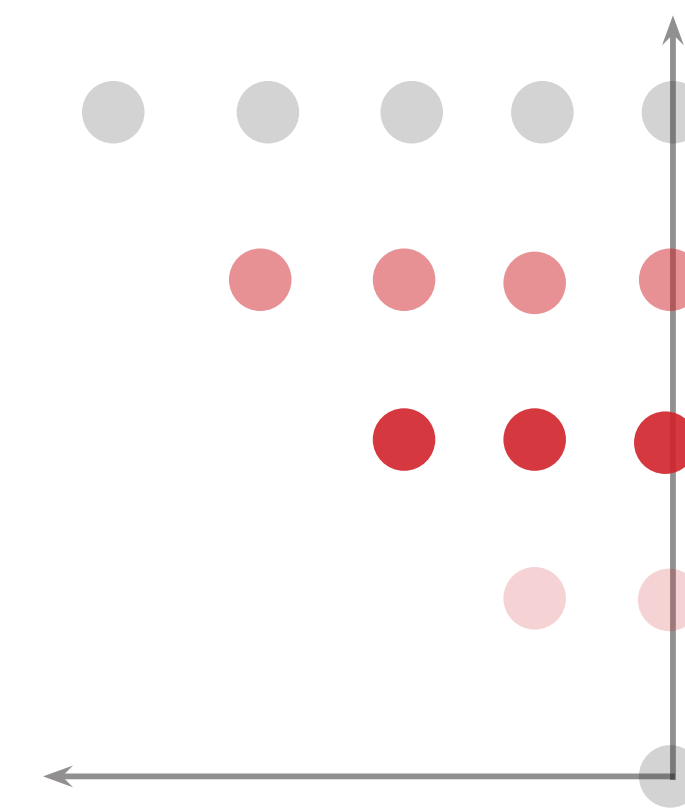
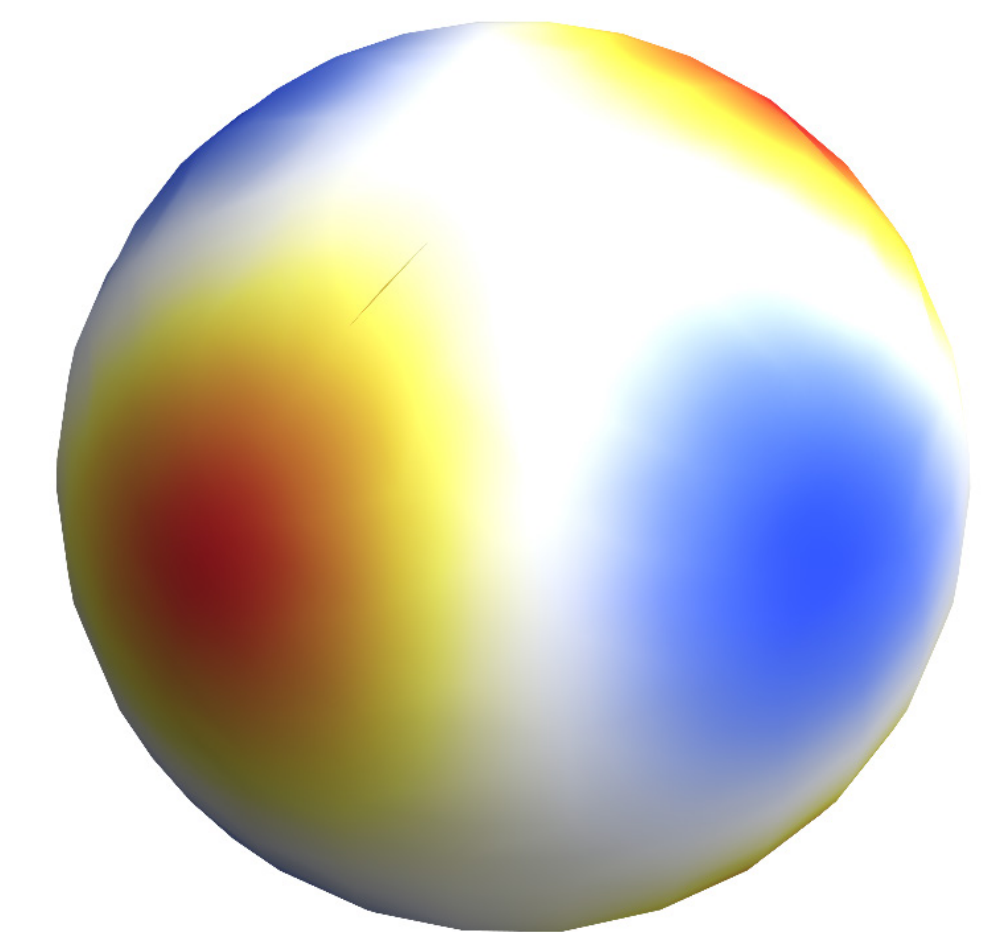


Motivation

Finite element
type models

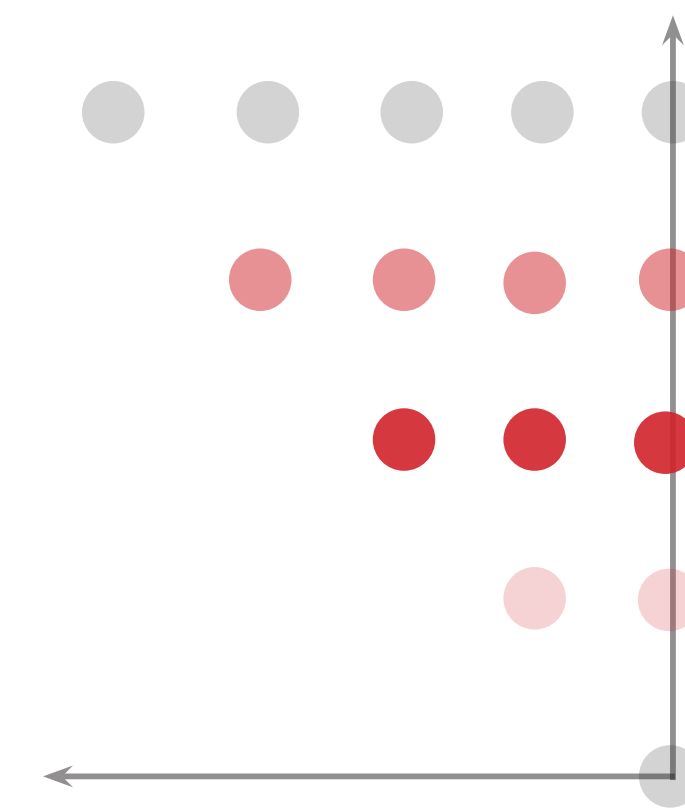
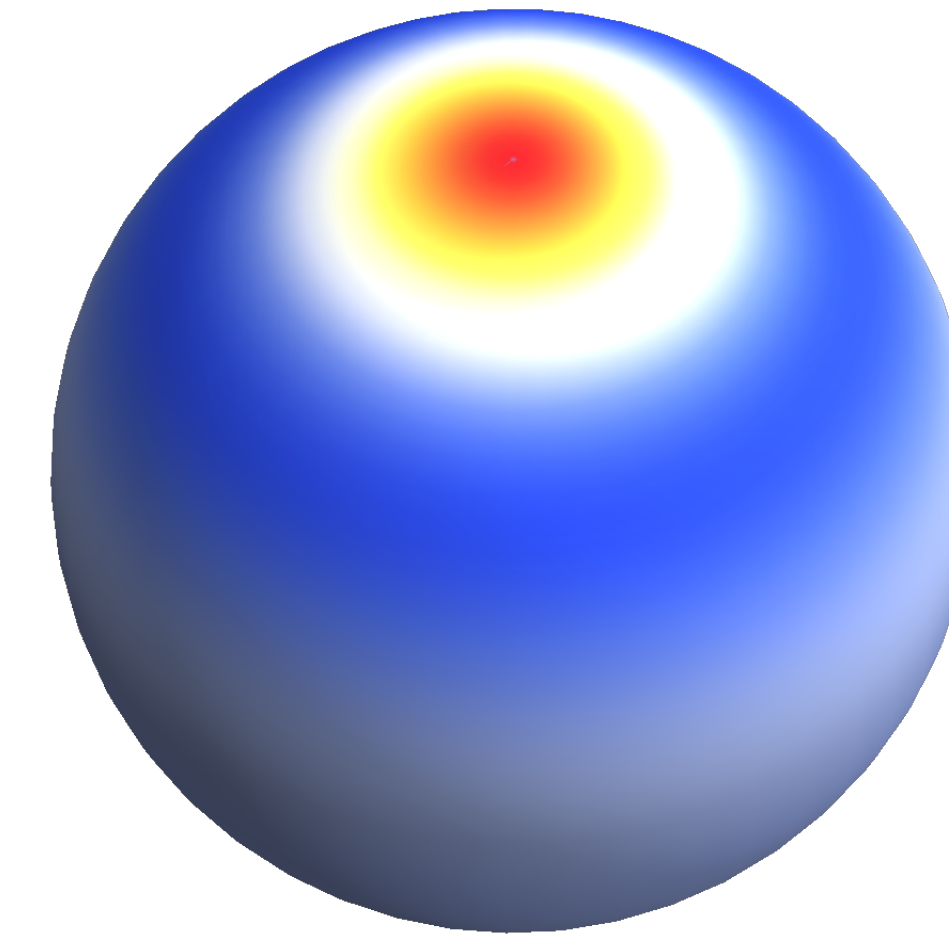
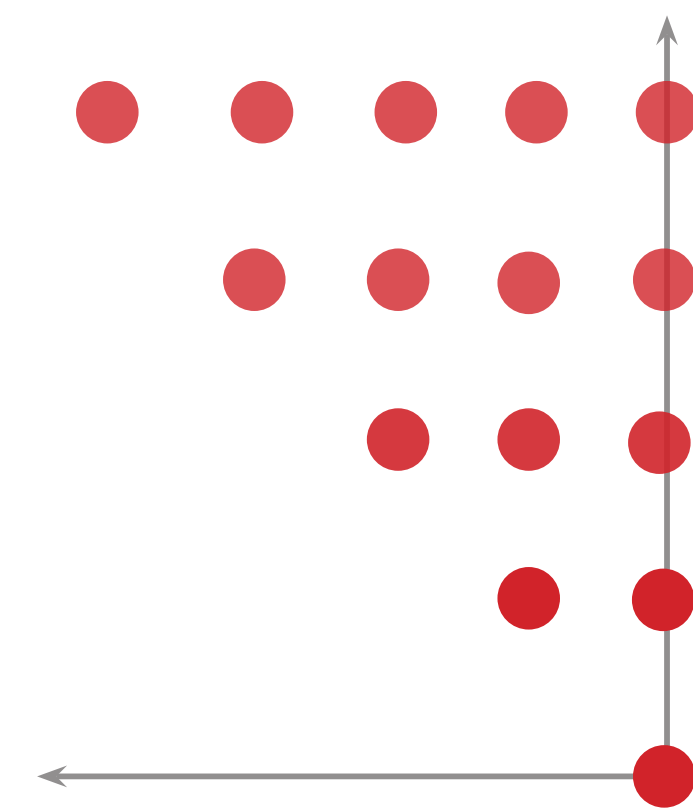
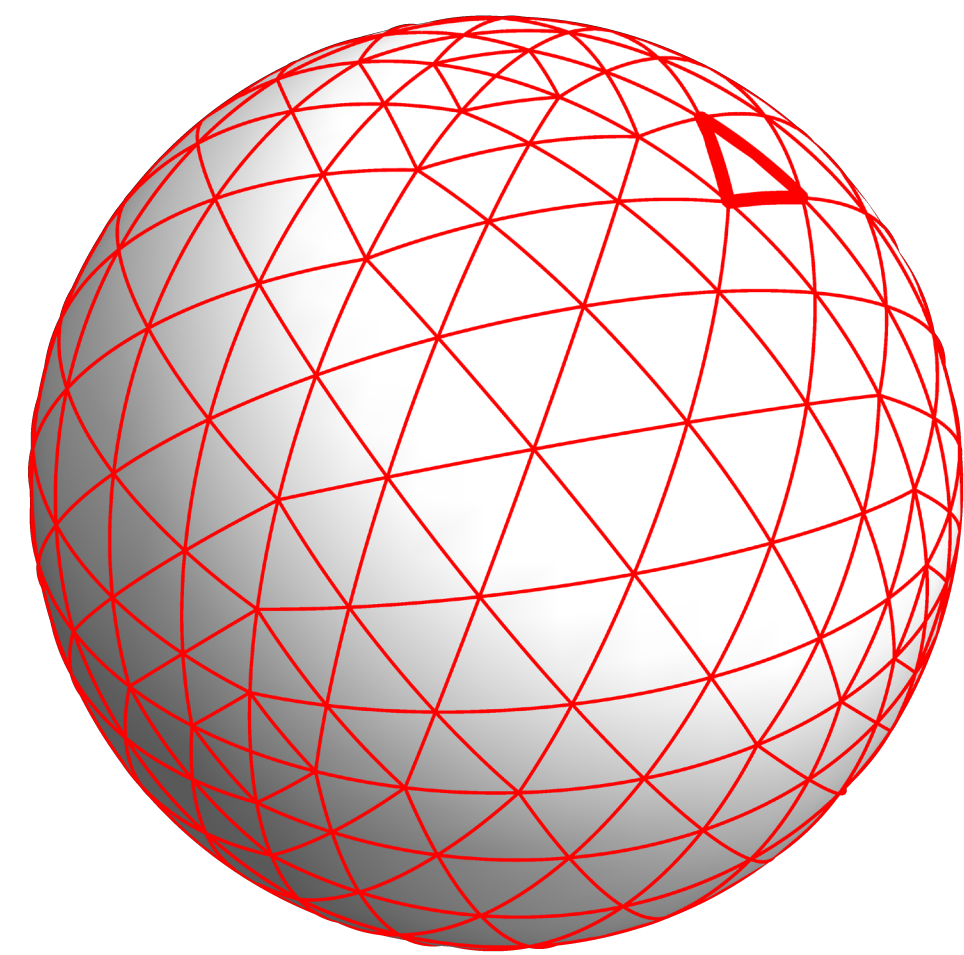


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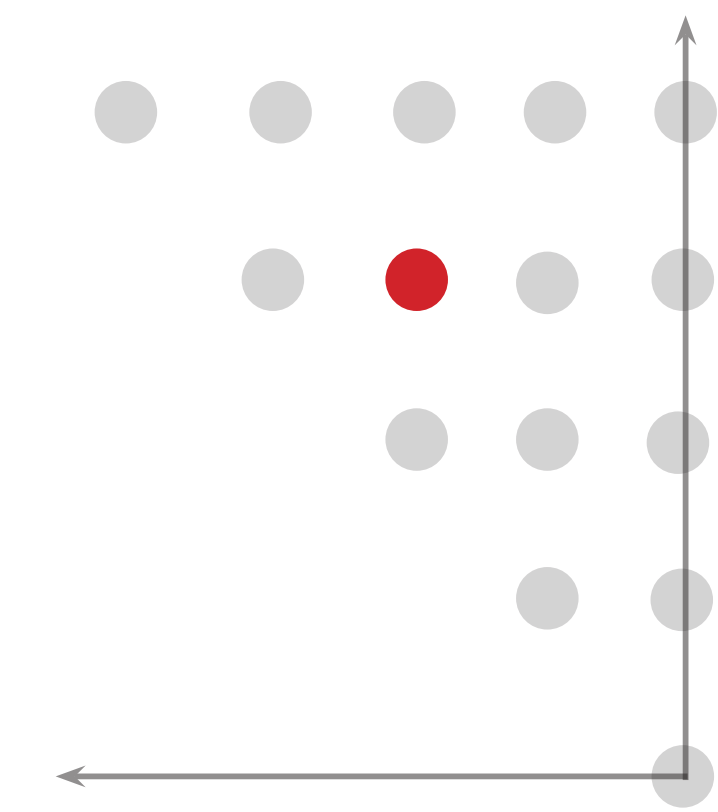
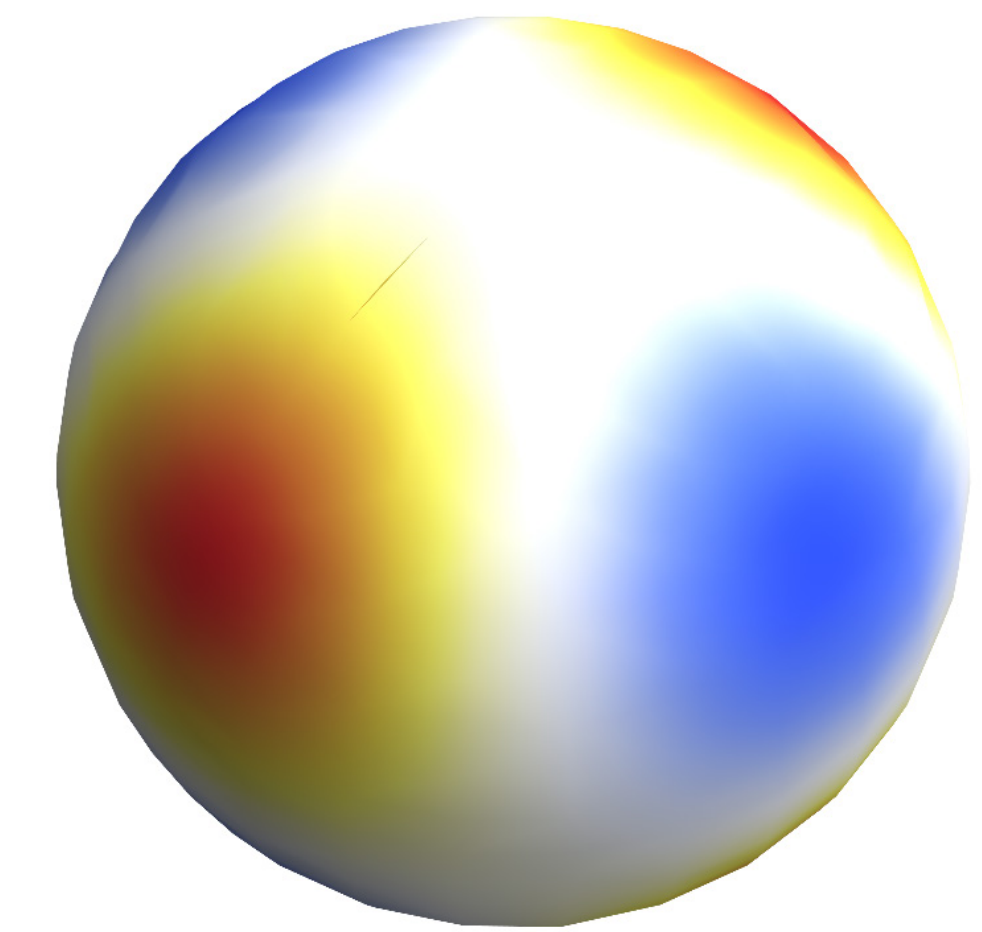


Motivation

Finite element
type models

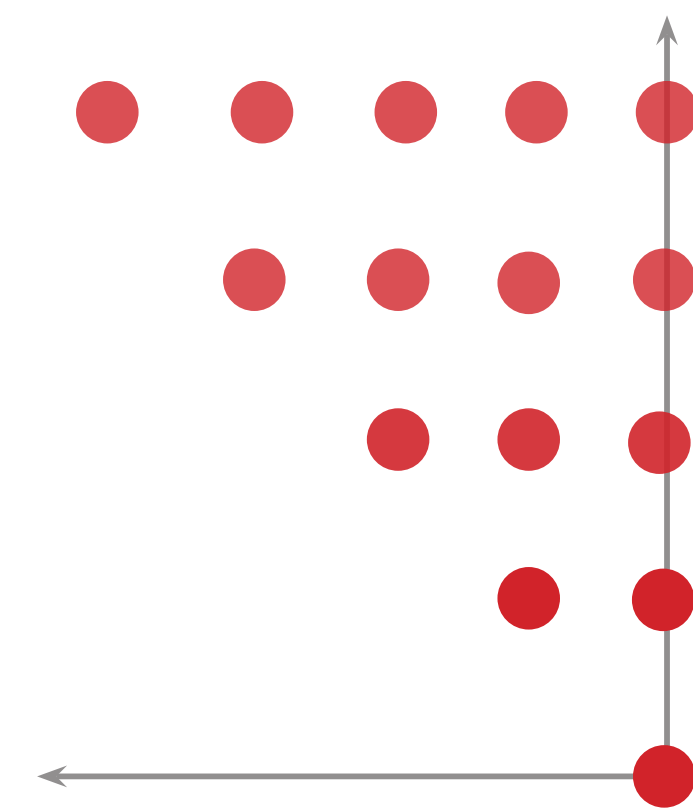
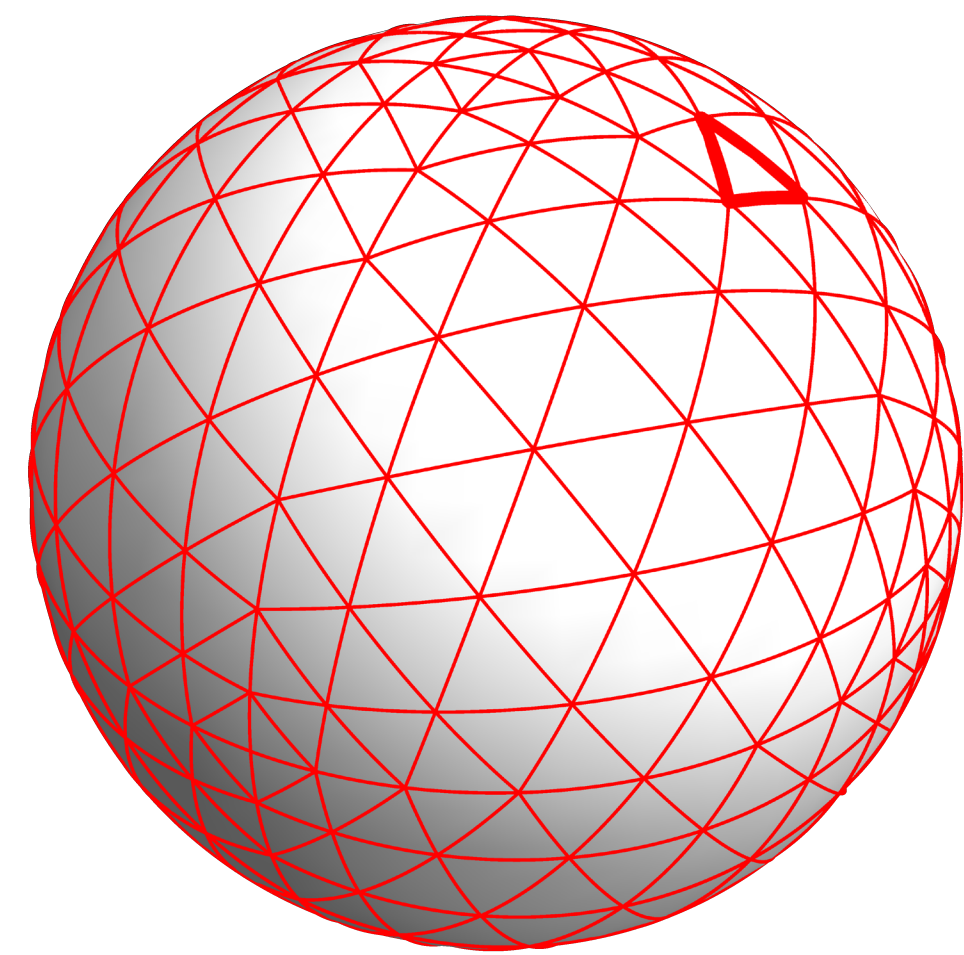


Spectral
models

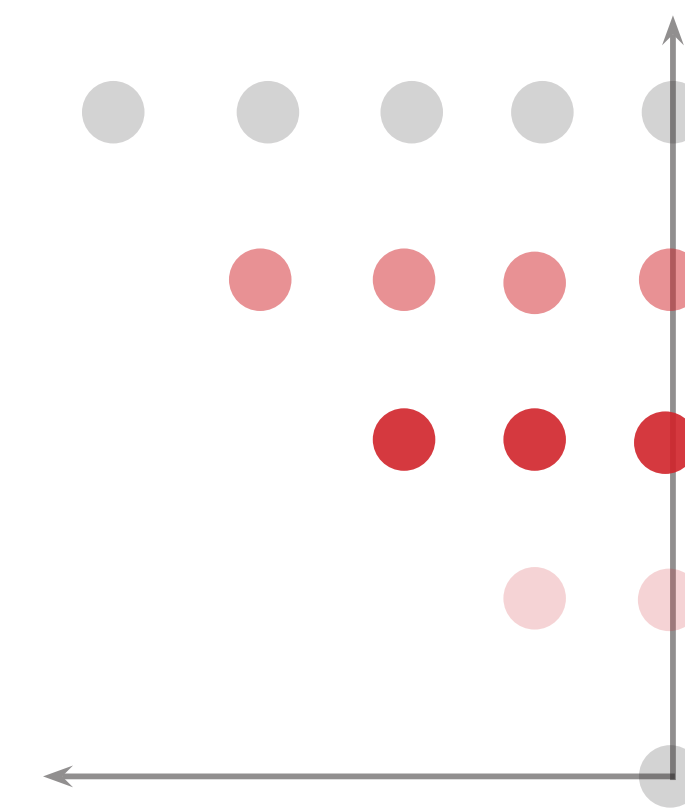
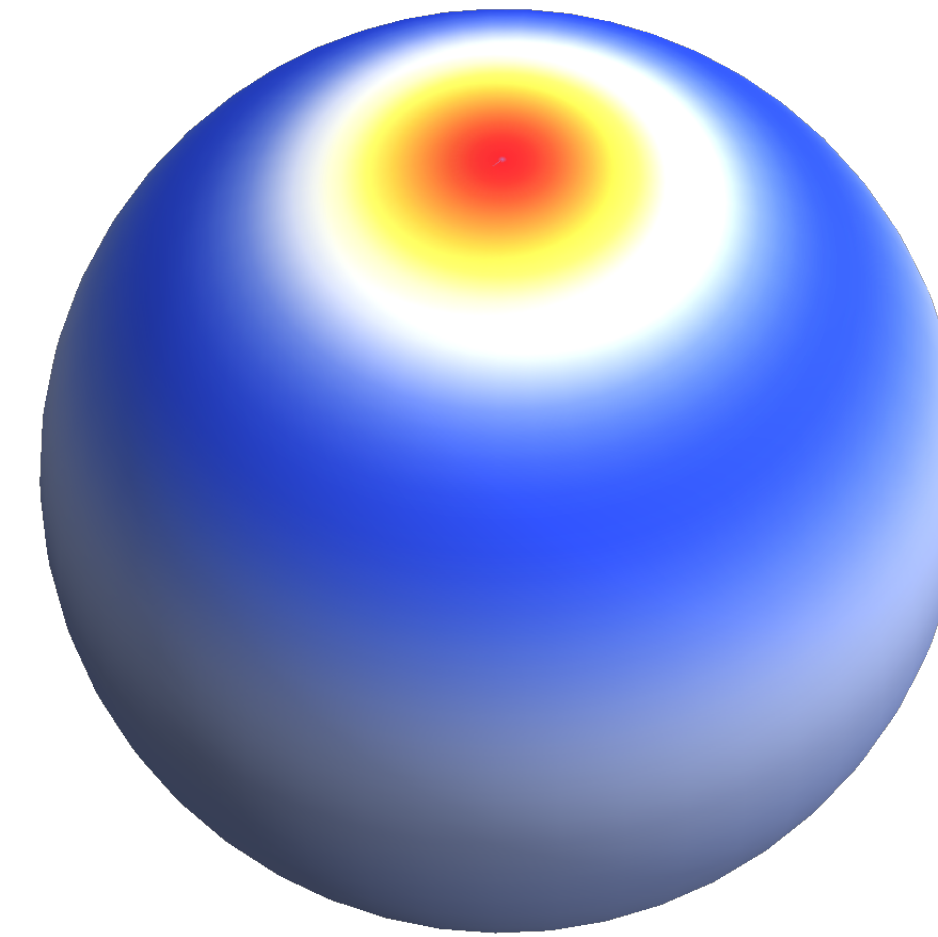


Motivation

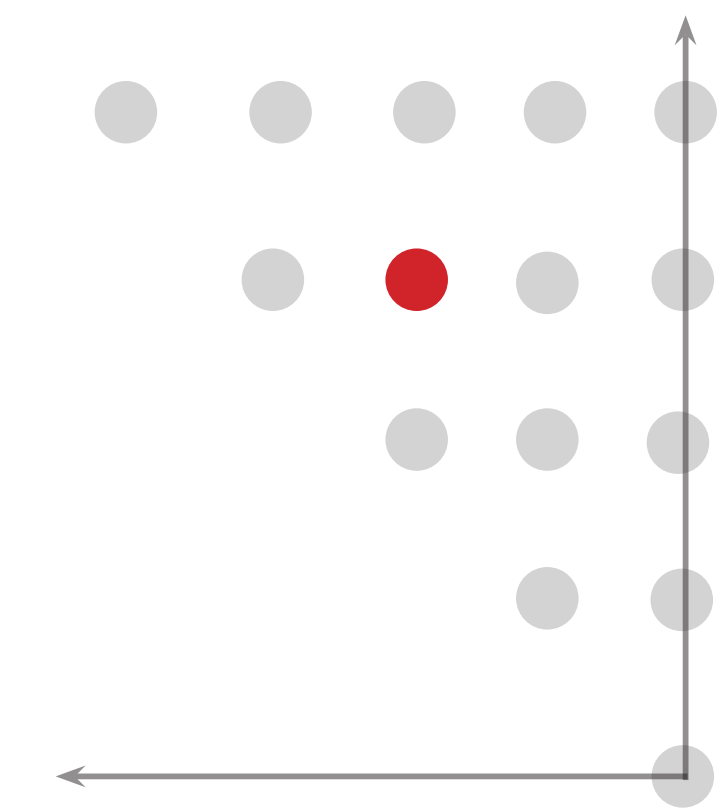
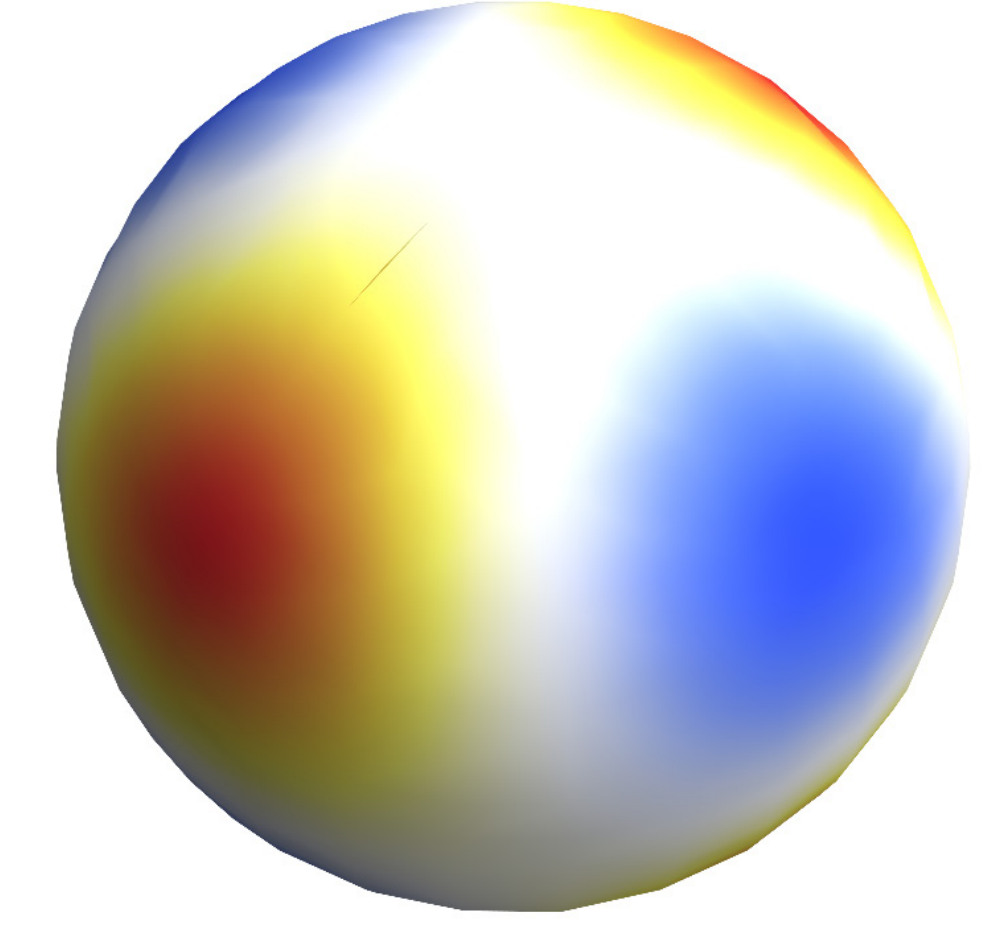
Finite element
type models



Local spectral
model

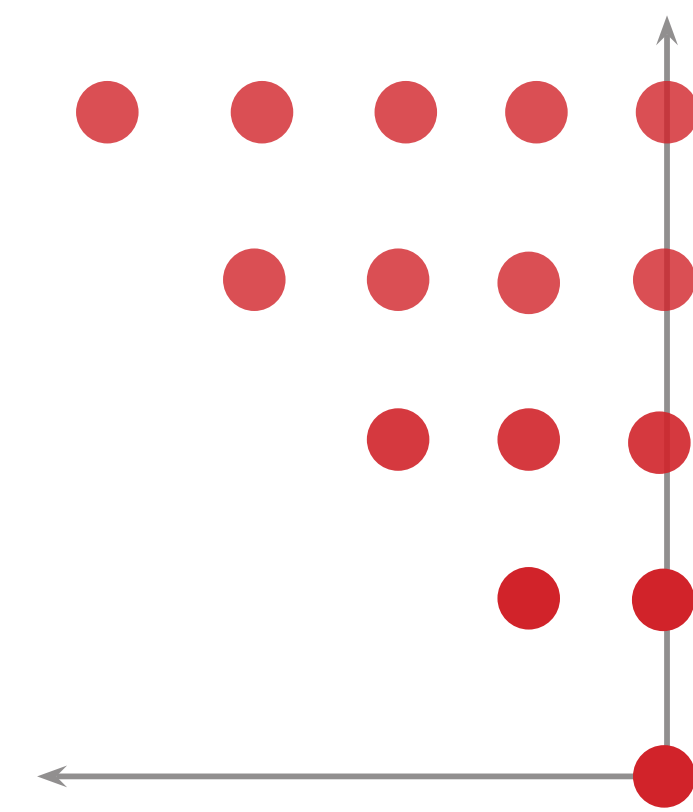
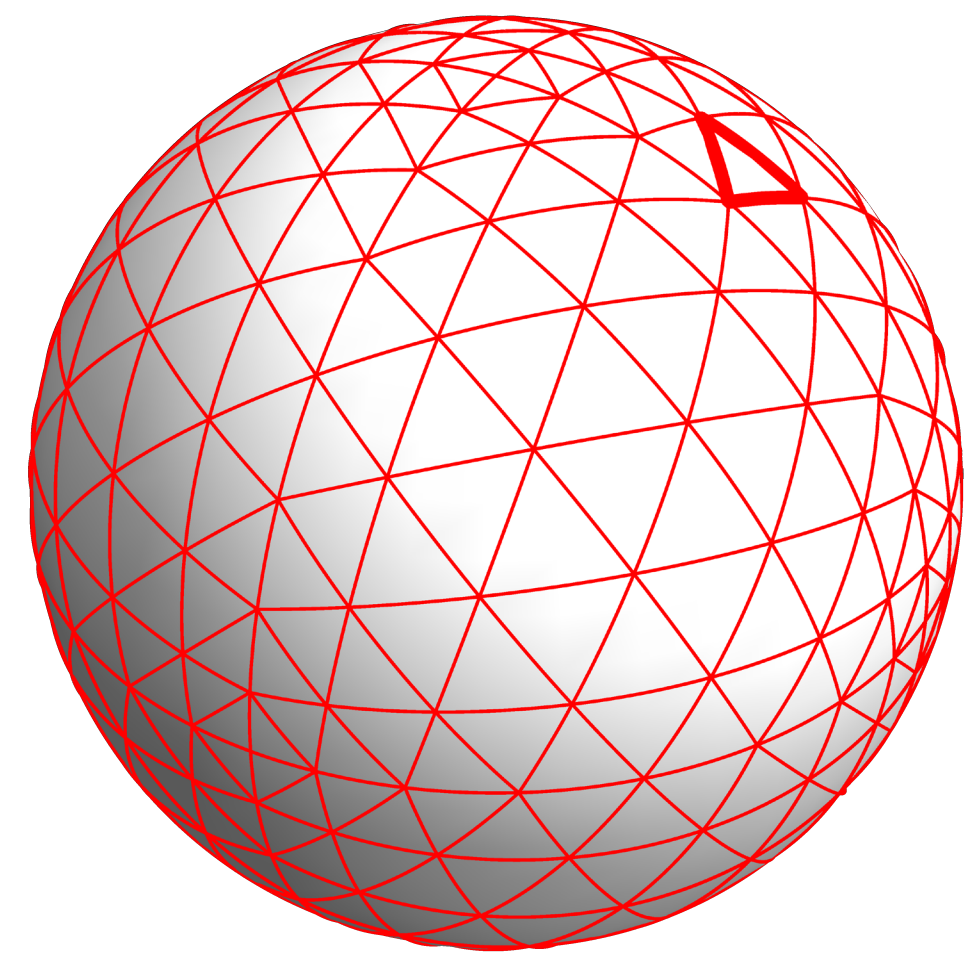


Spectral
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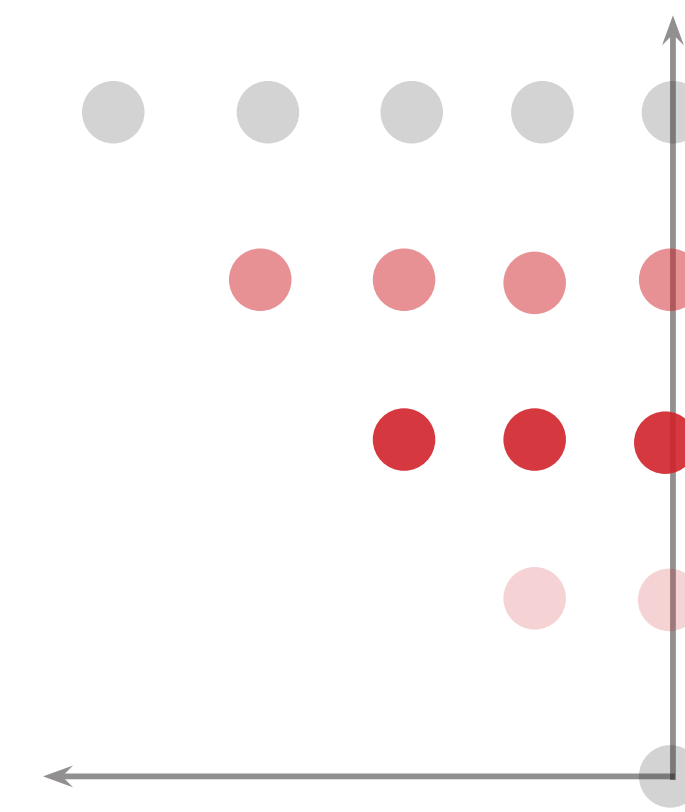
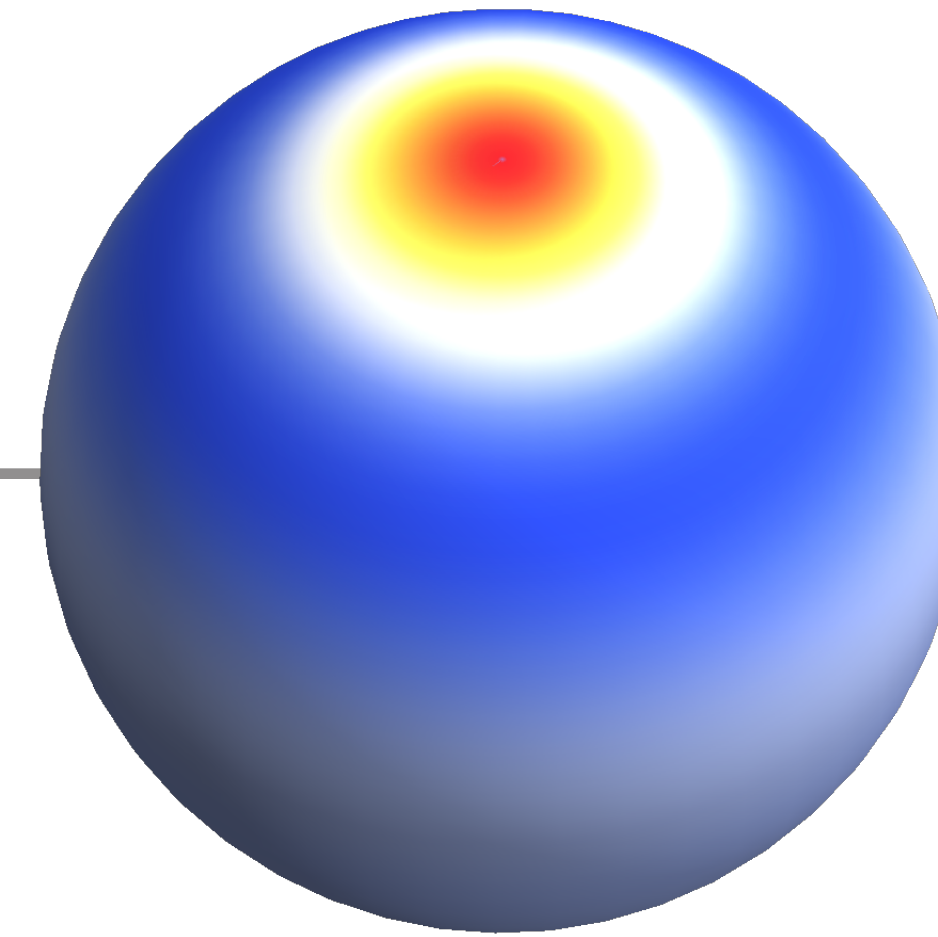


Motivation

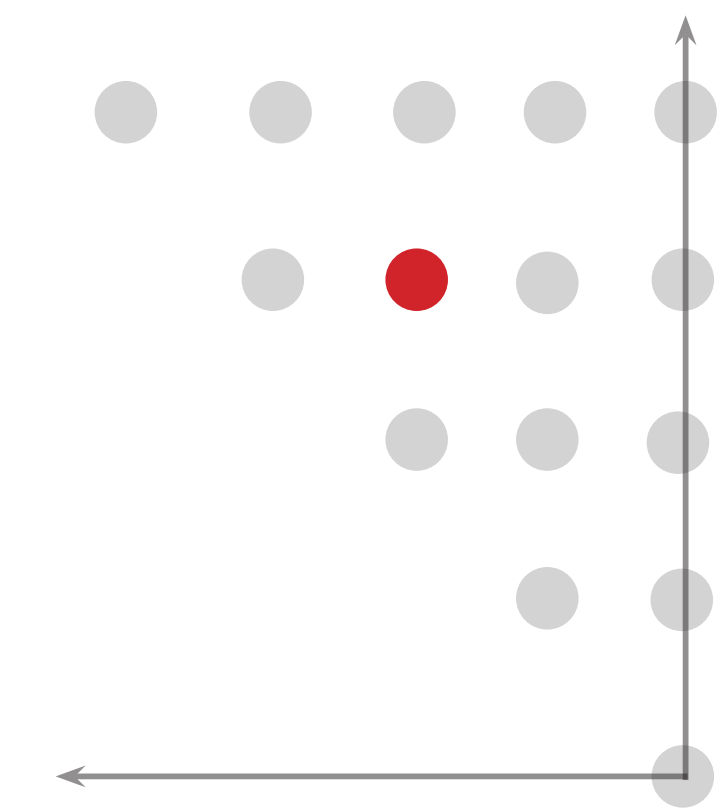
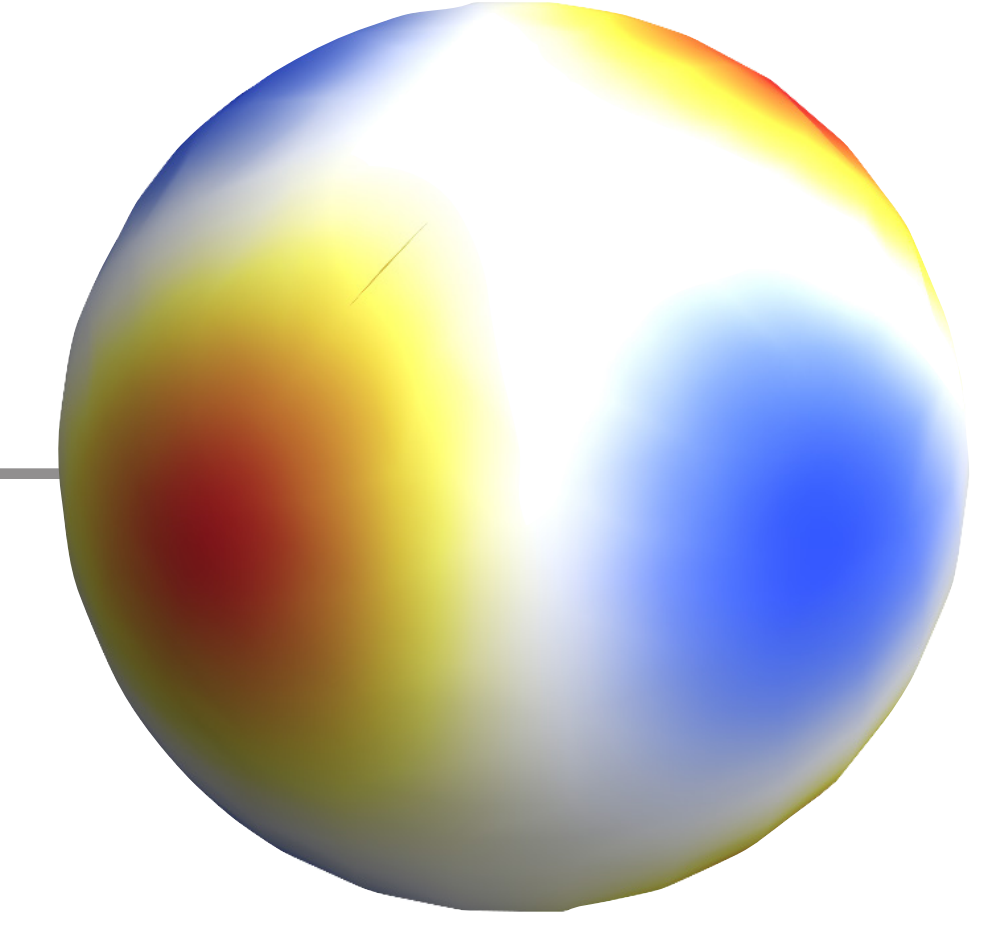
Finite element
type models



Local spectral
model

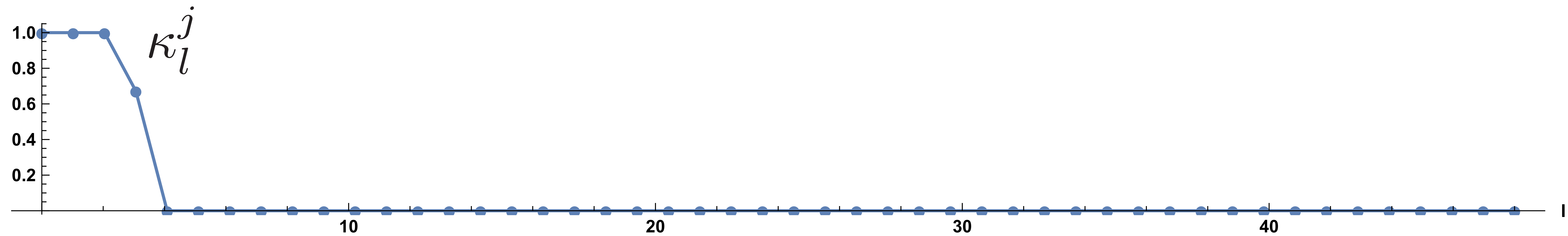


Spectral
models



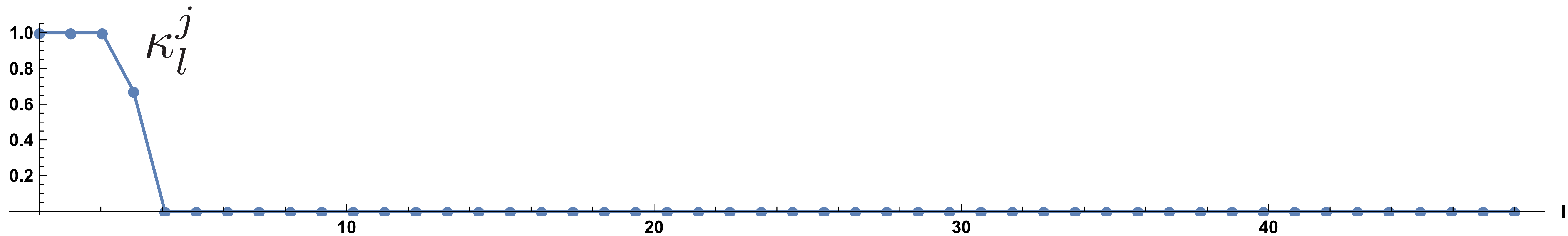
Local spectral representation

Local spectral representation

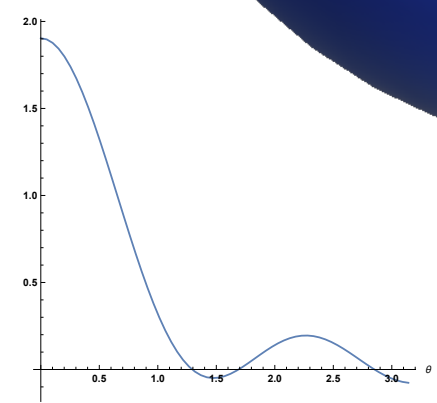
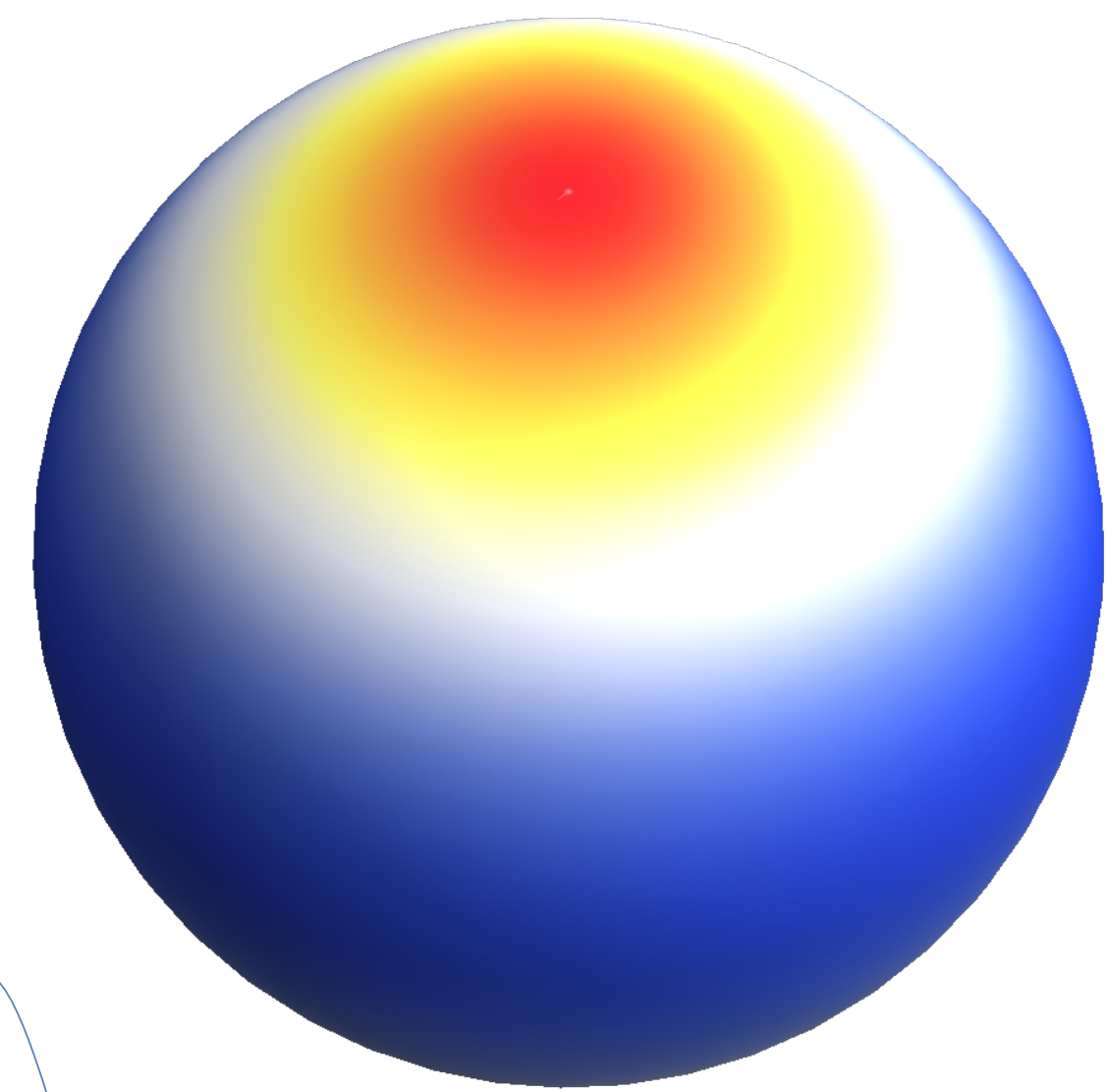


Local spectral representation

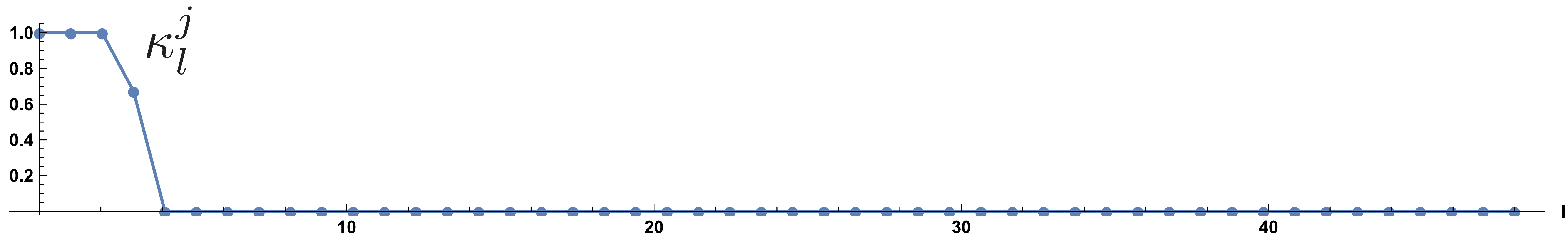
$$\phi_j(\omega) = \sum_{l=0}^{2^j-1} \kappa_l^j y_{l0}(\omega)$$



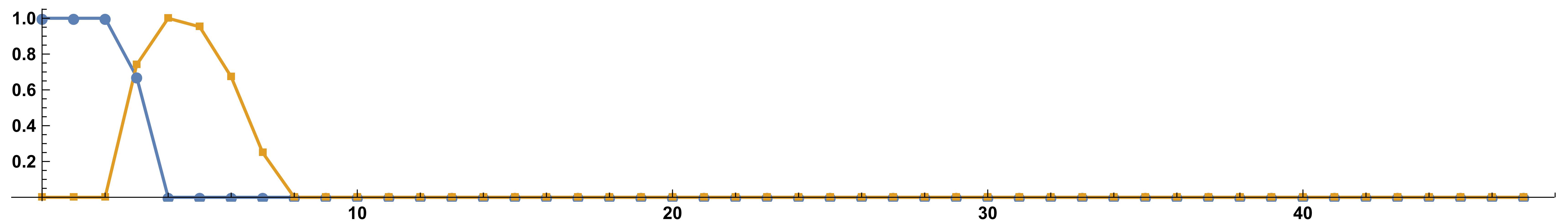
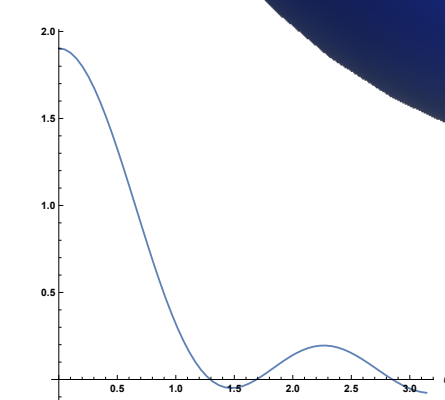
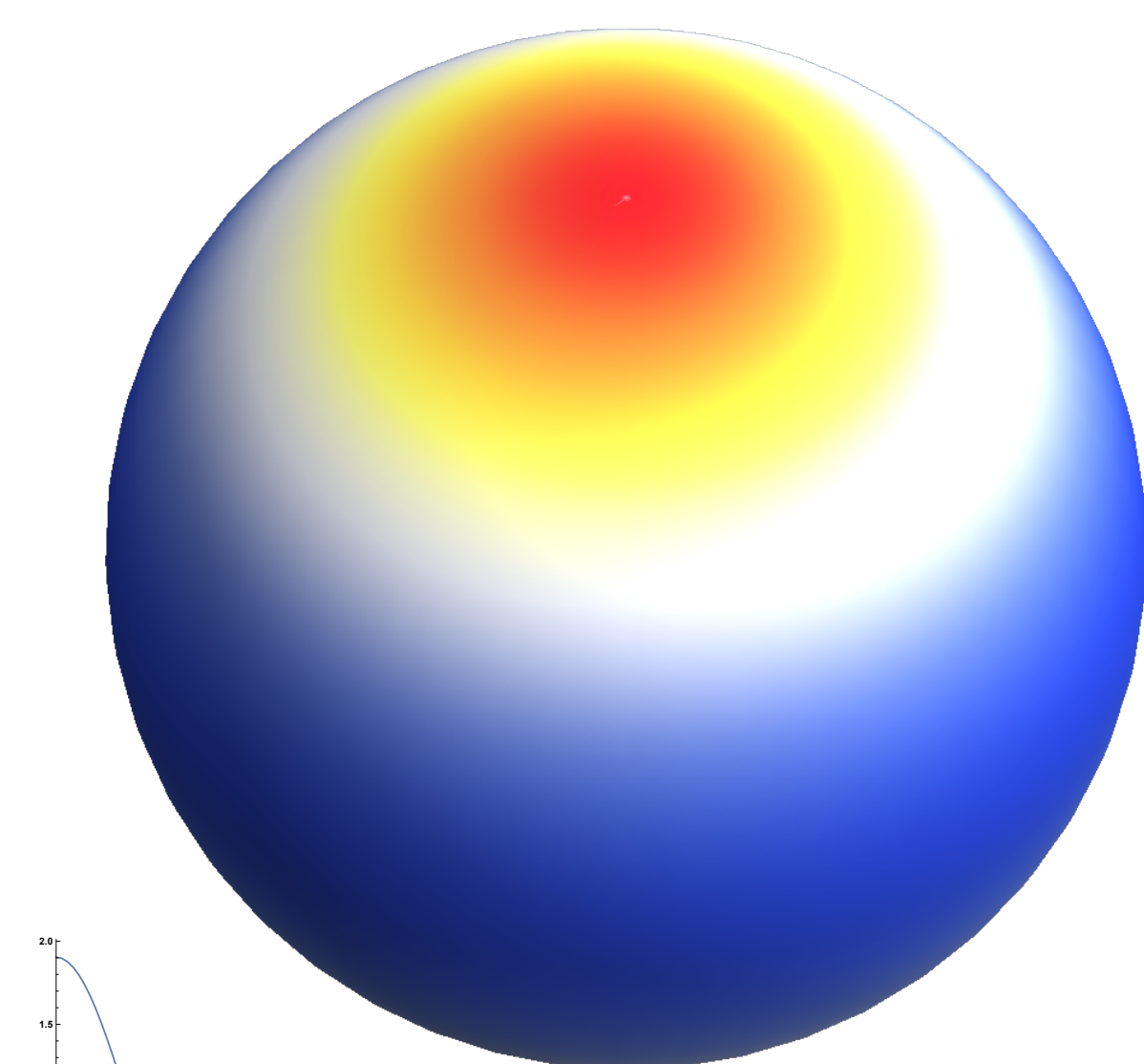
Local spectral representation



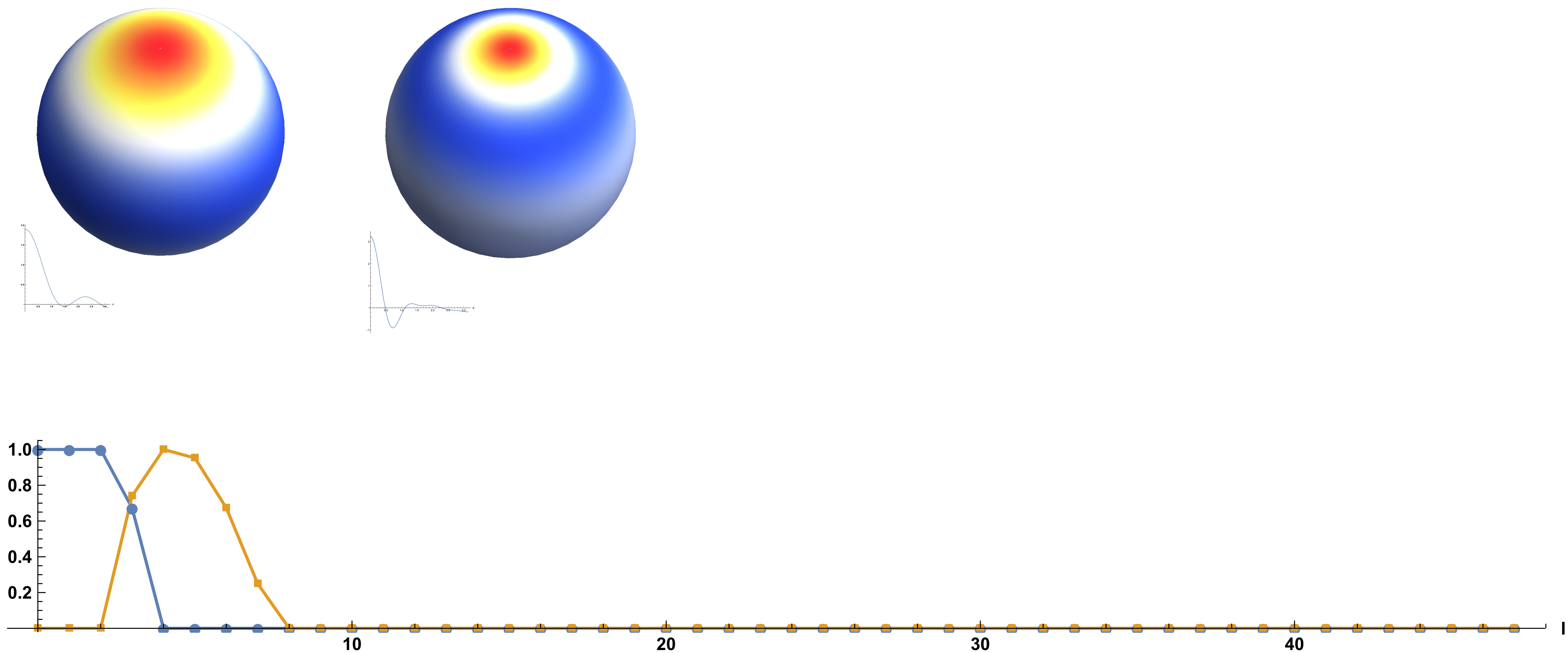
$$\phi_j(\omega) = \sum_{l=0}^{2^j-1} \kappa_l^j y_{l0}(\omega)$$



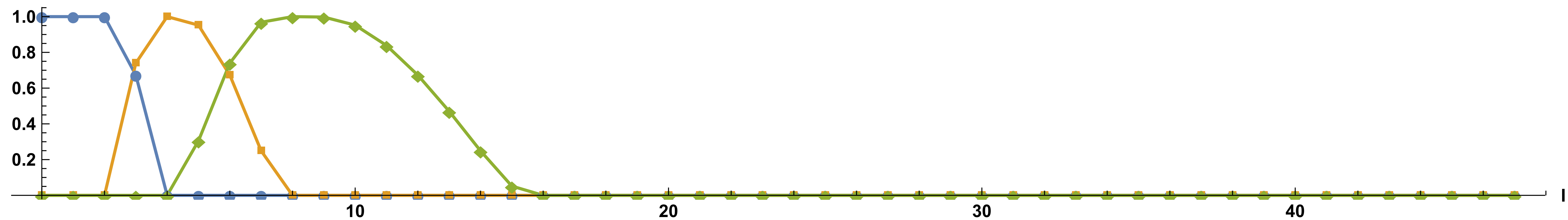
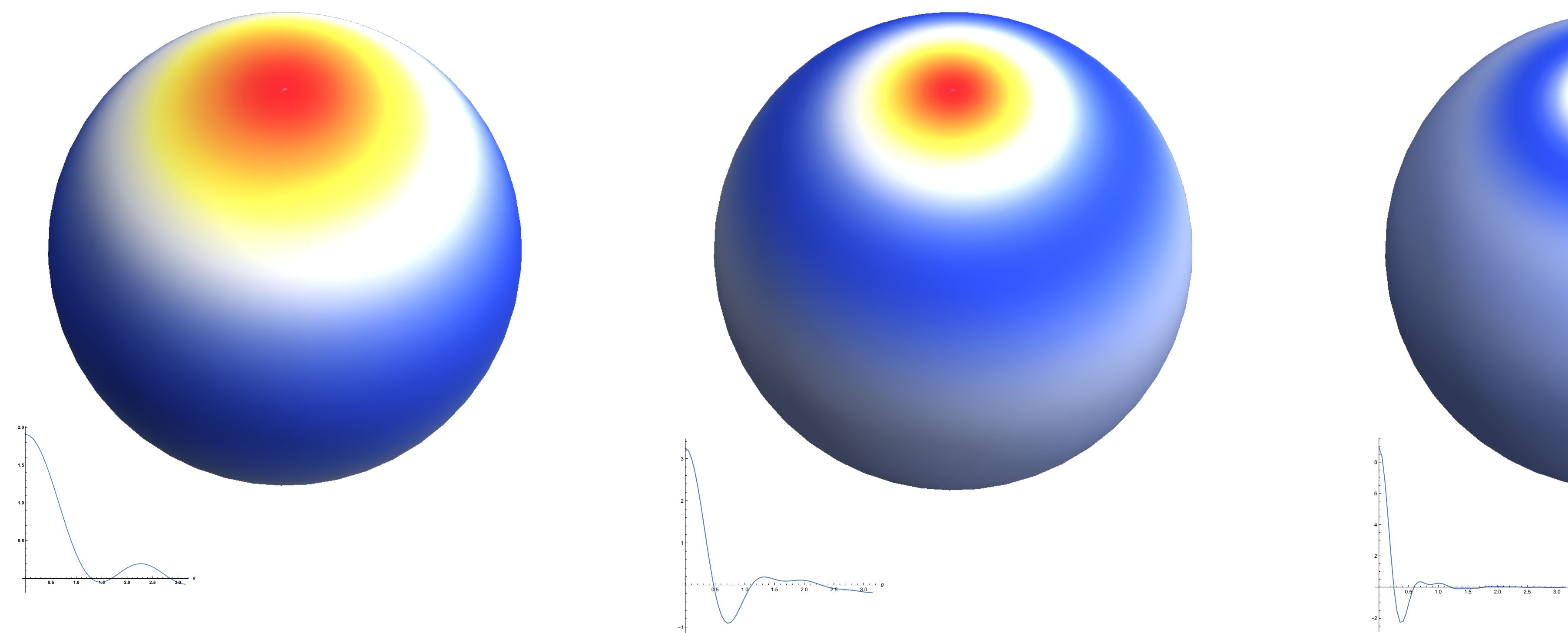
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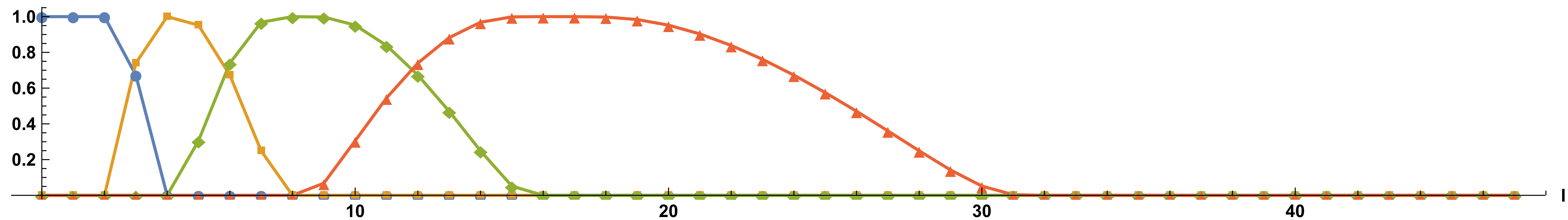
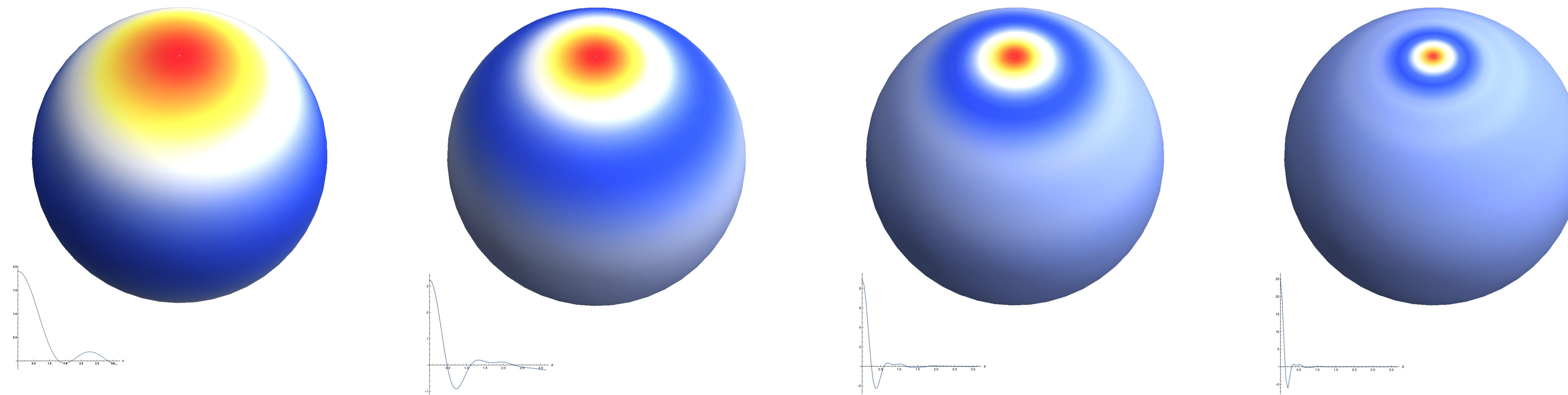
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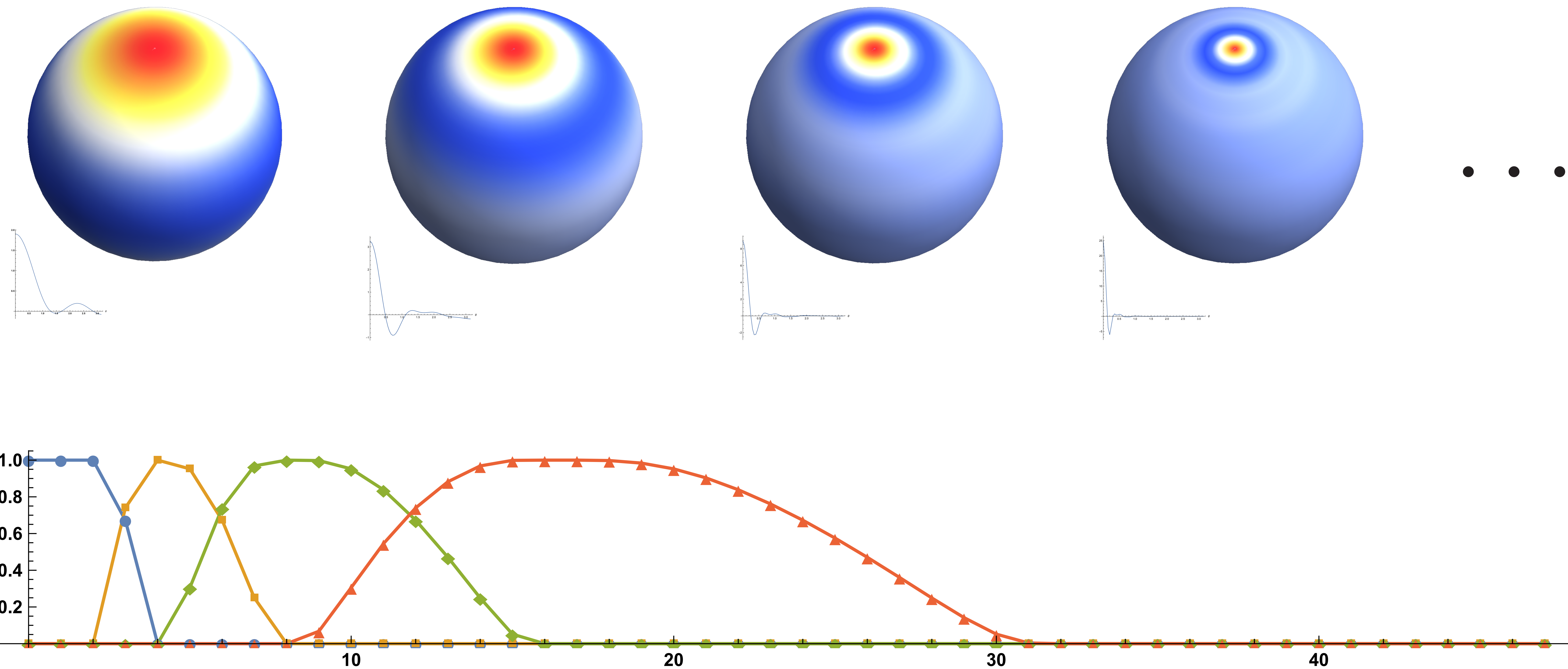
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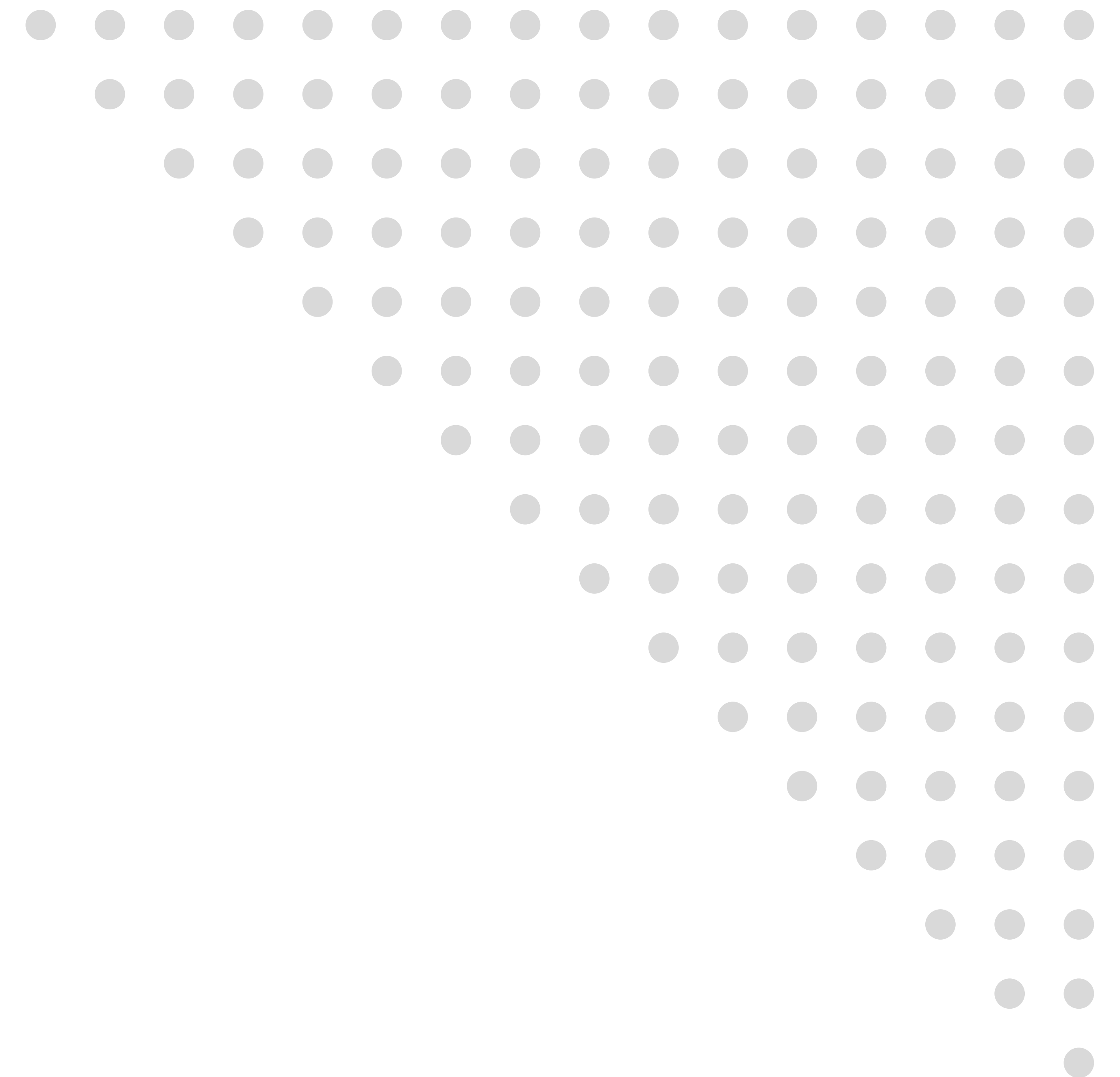
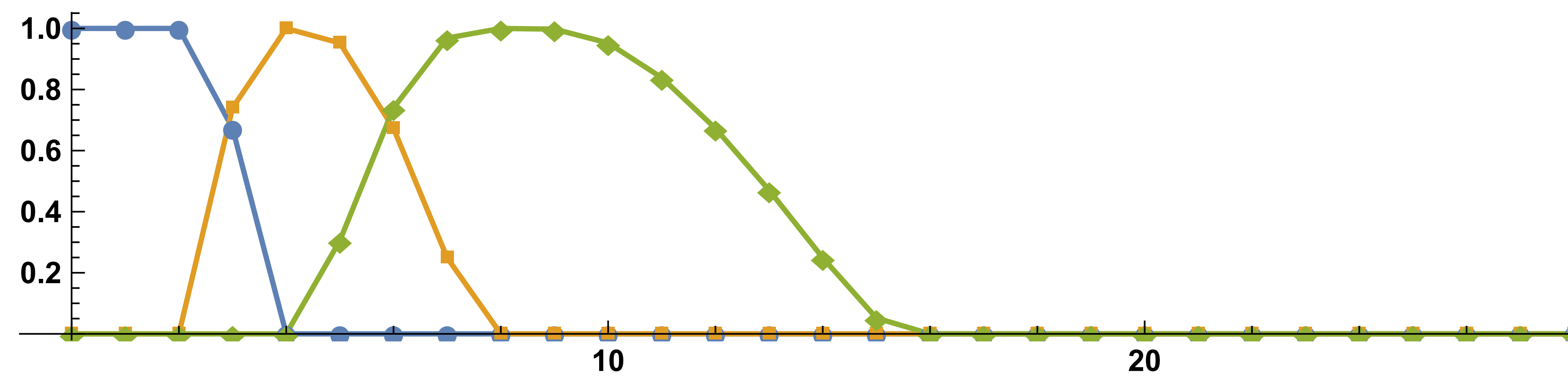
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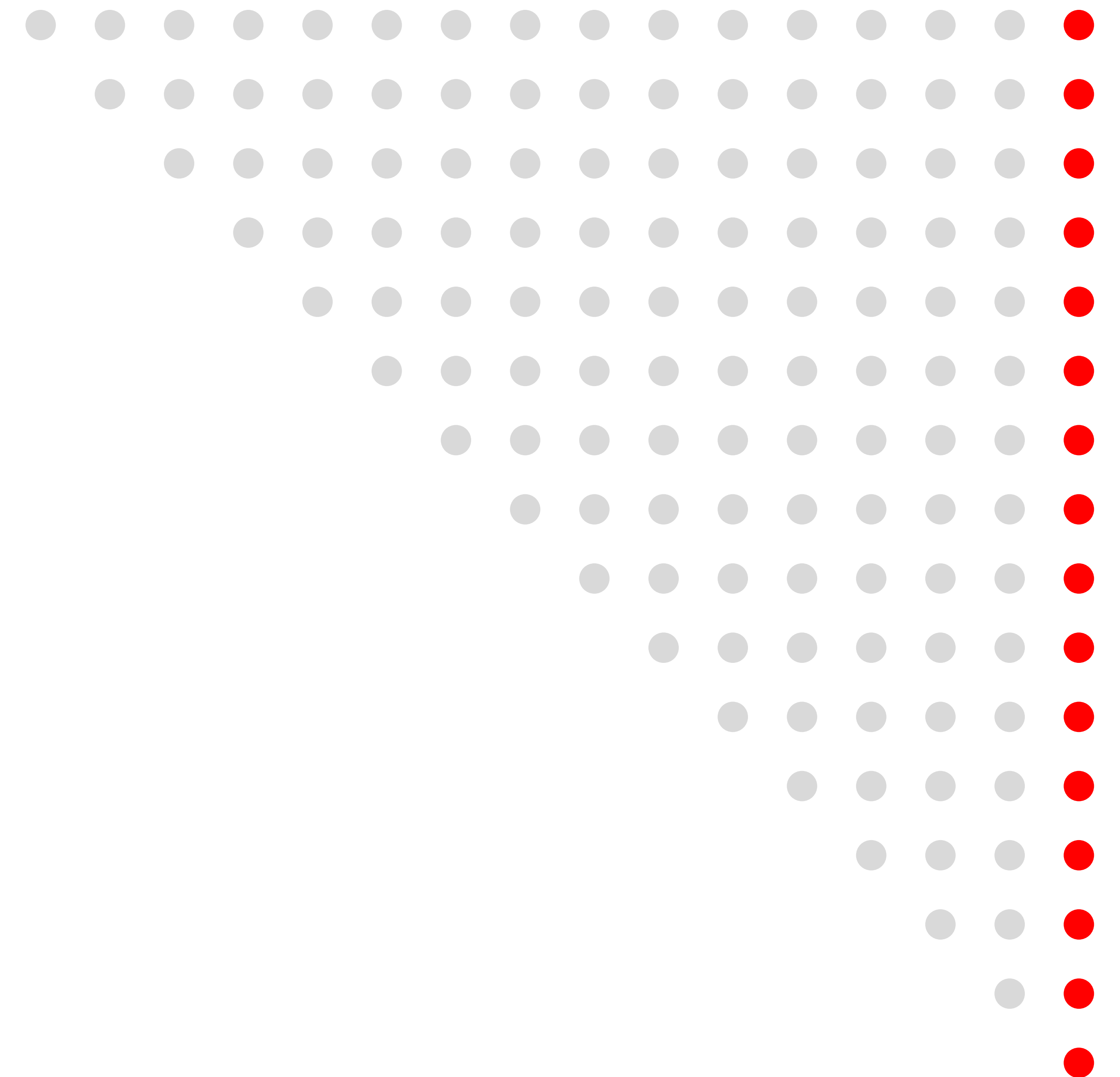
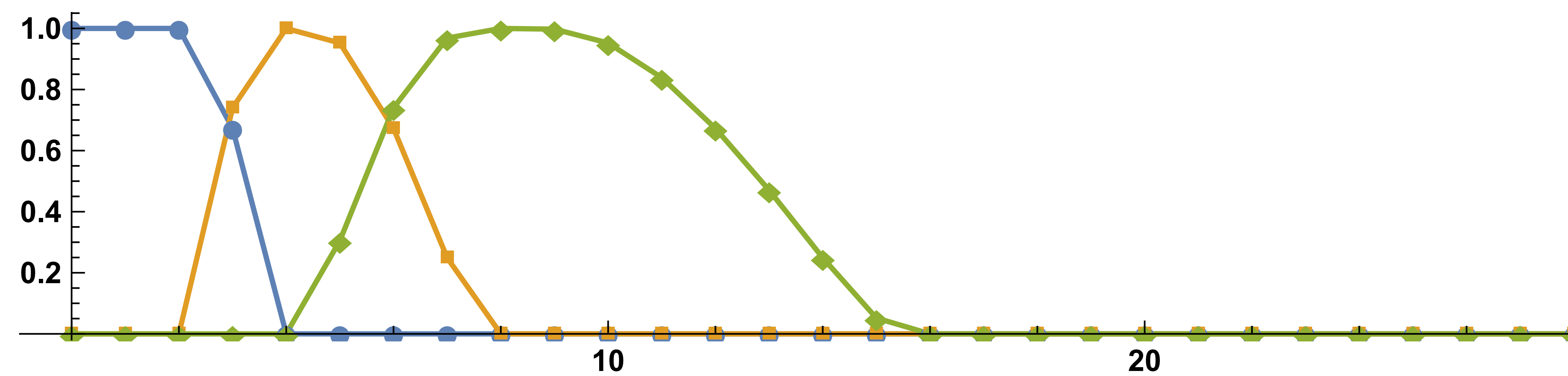
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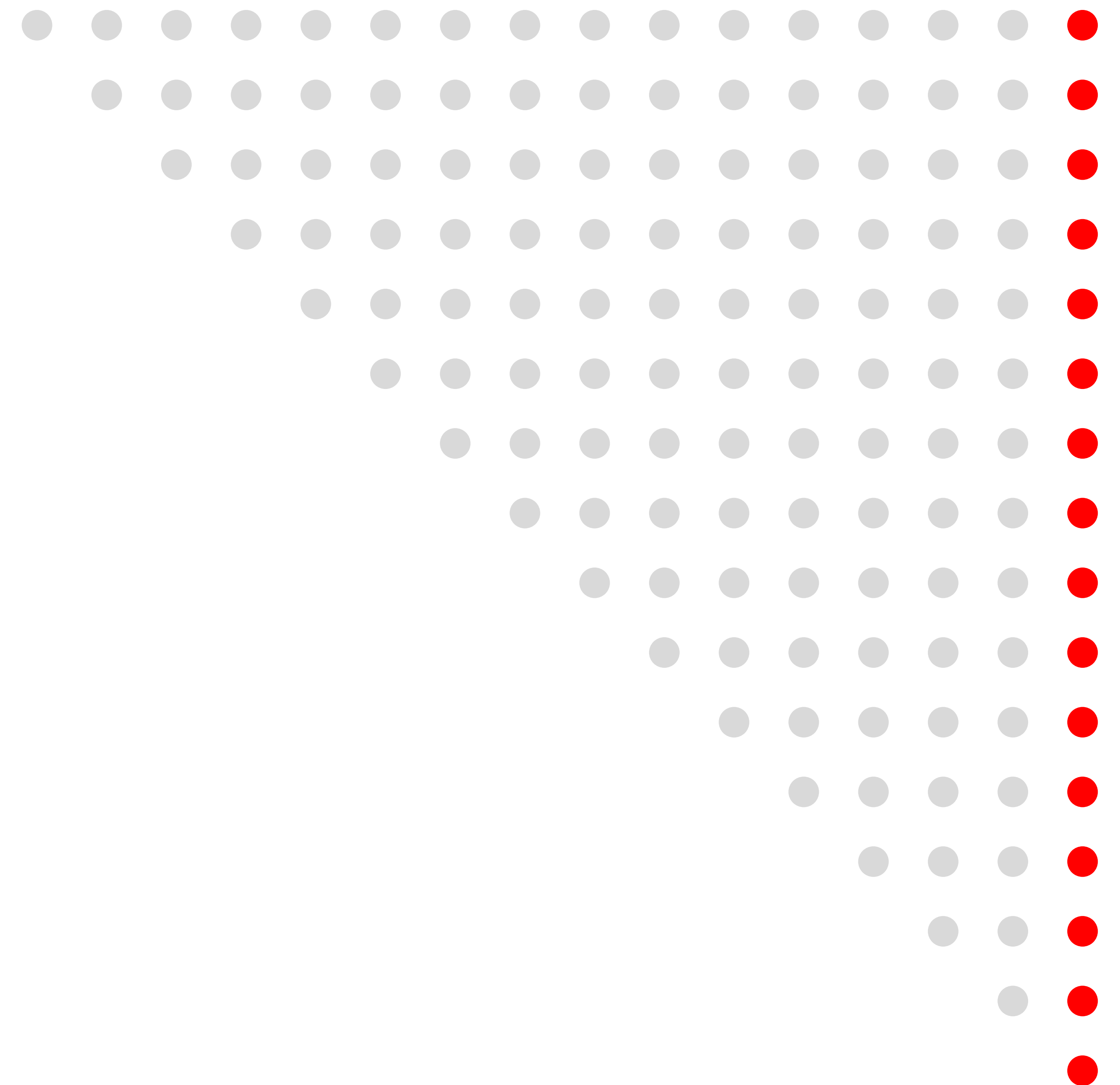
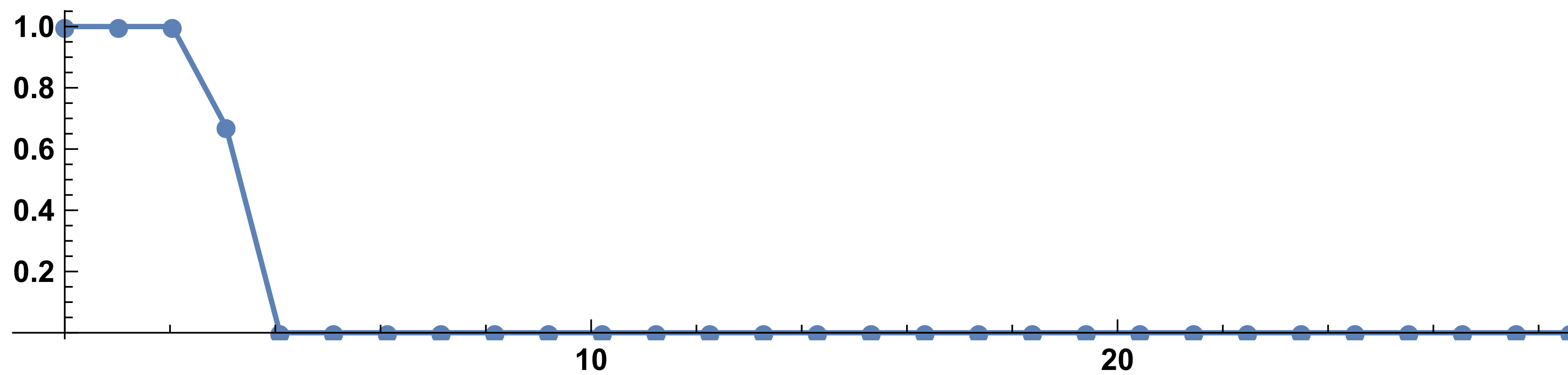
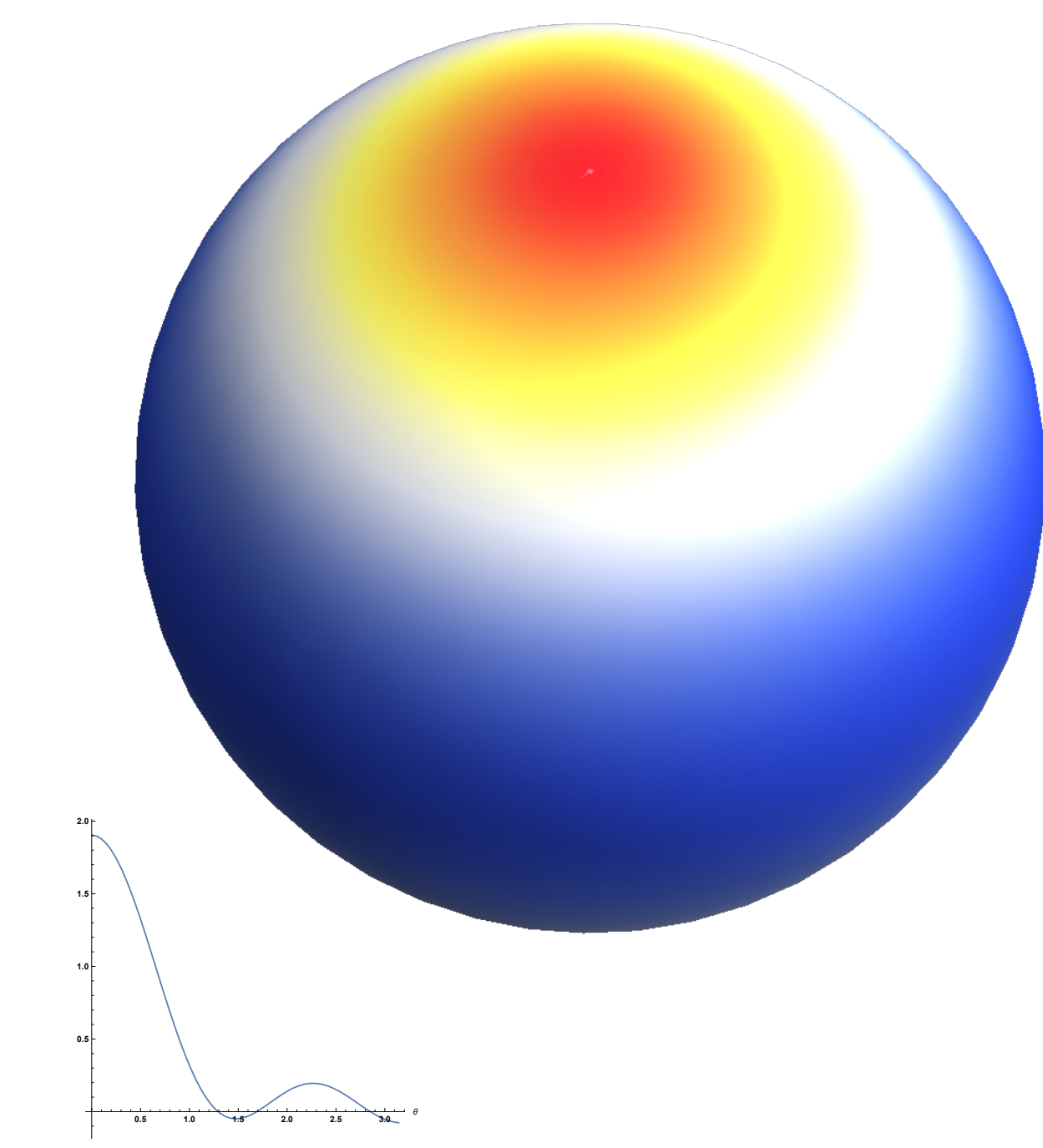
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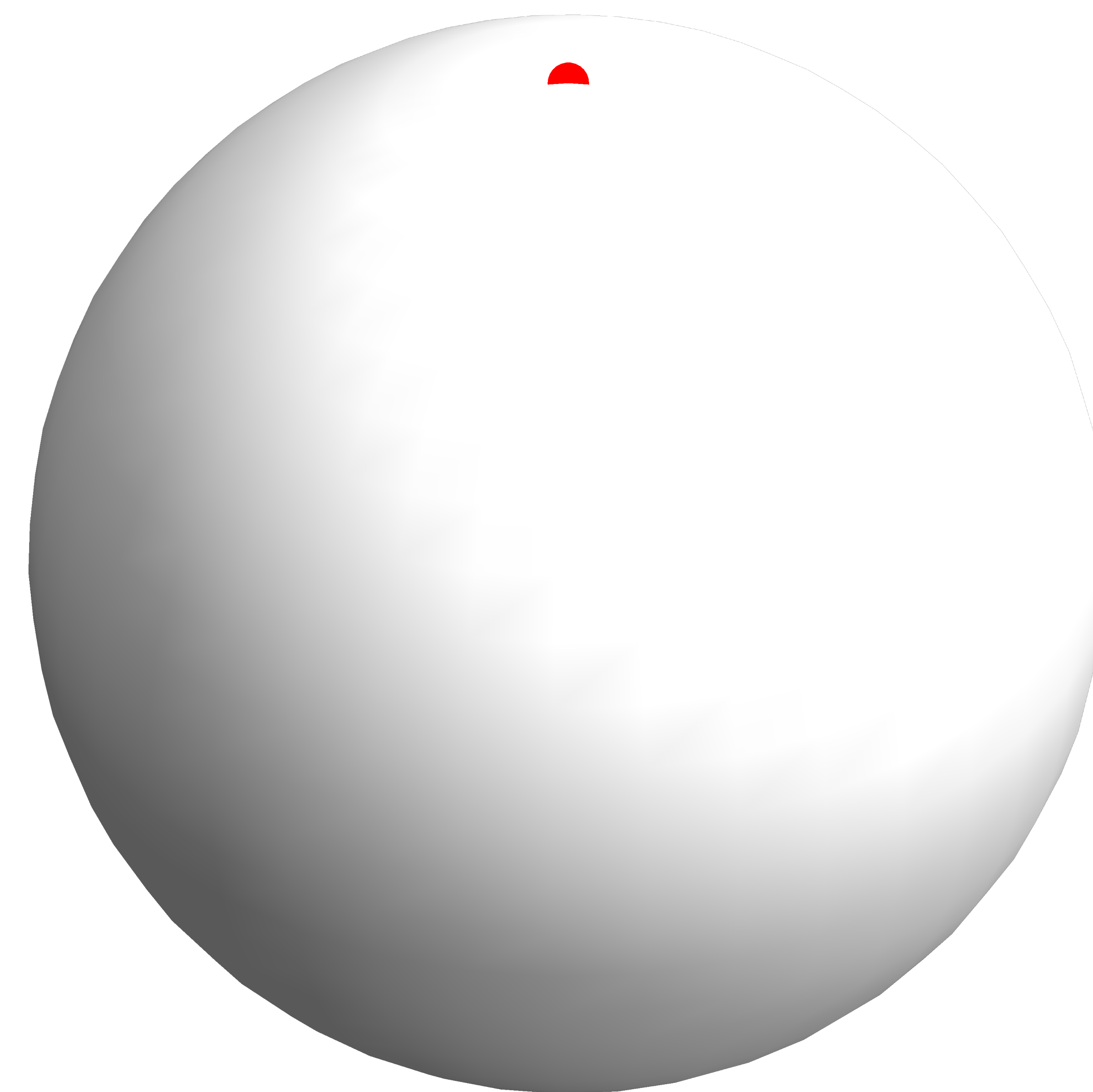
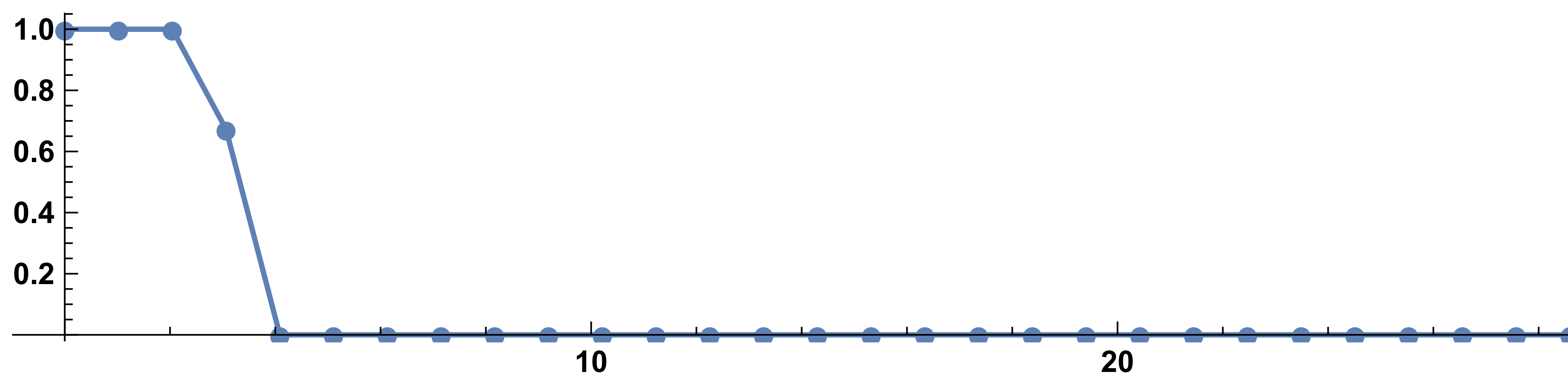
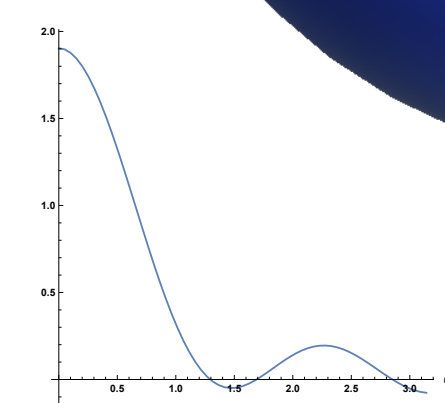
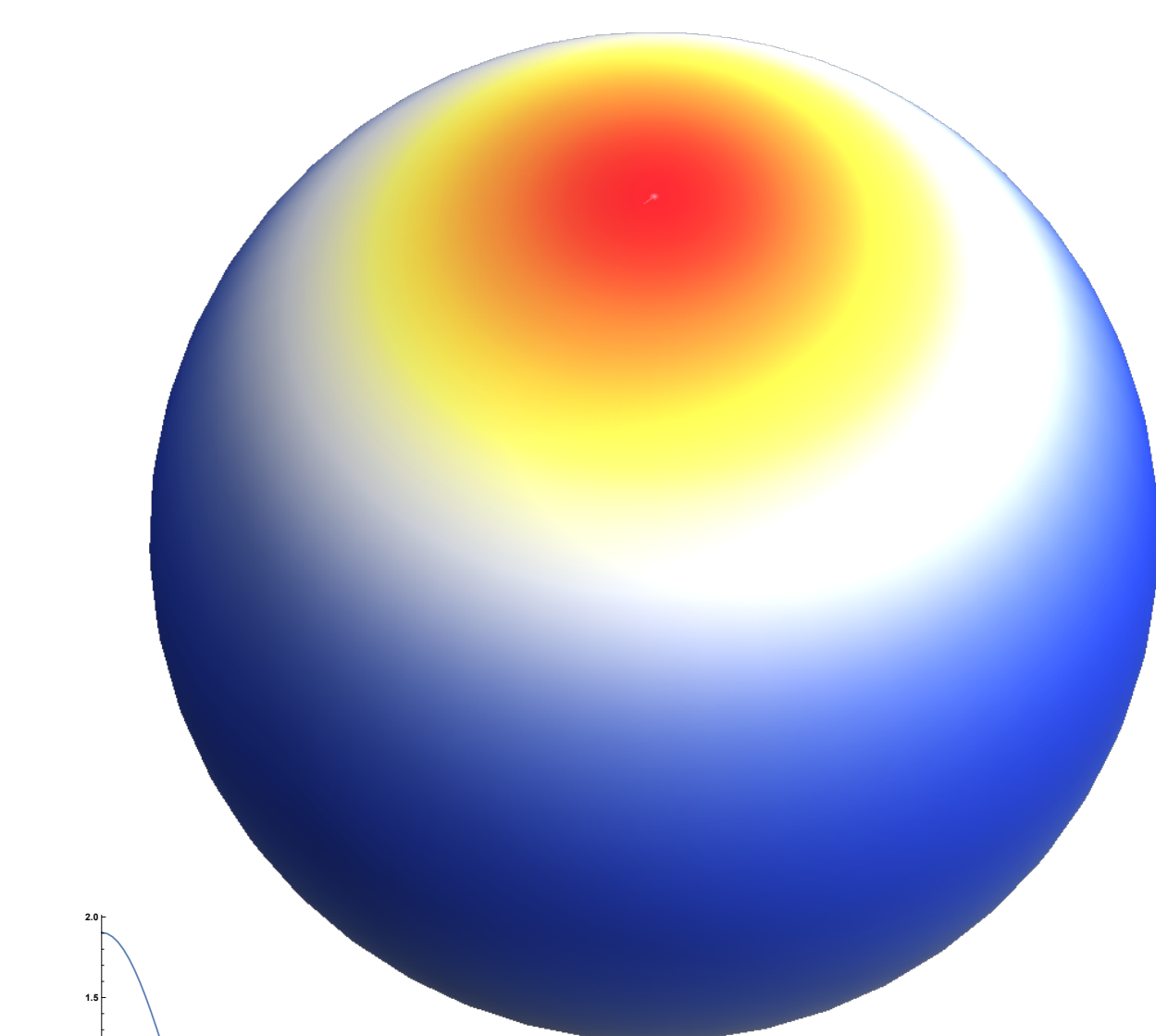
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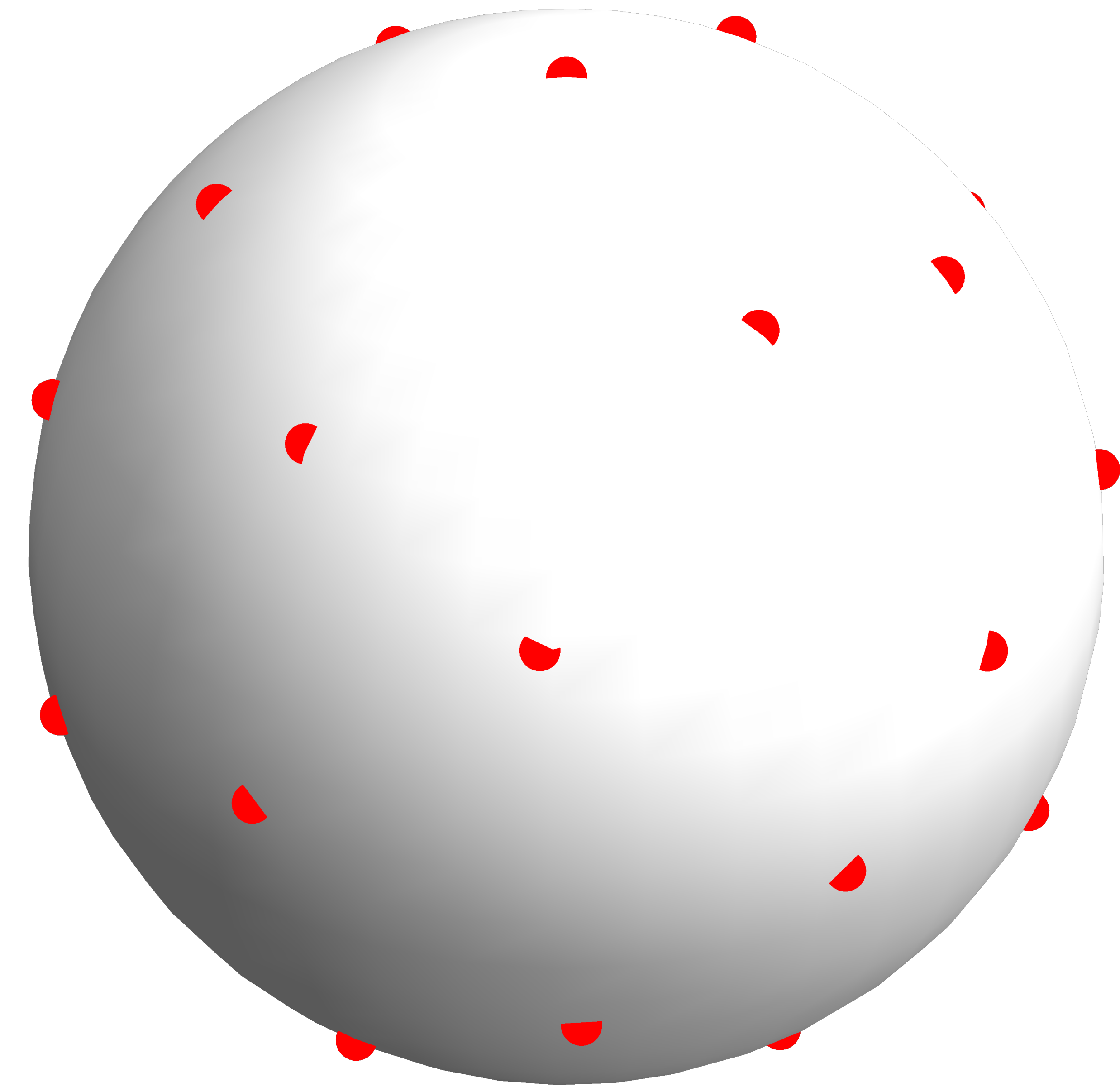
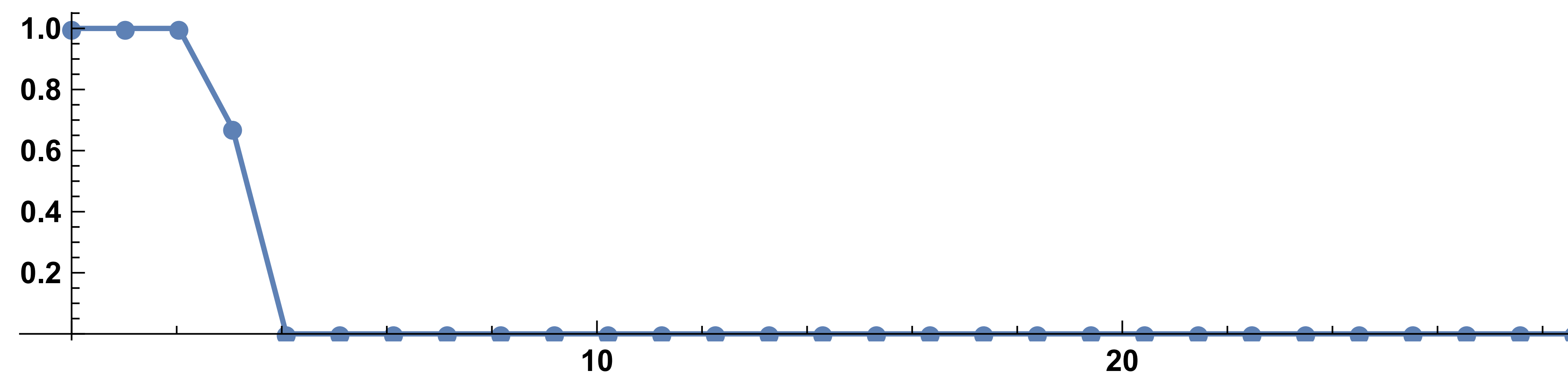
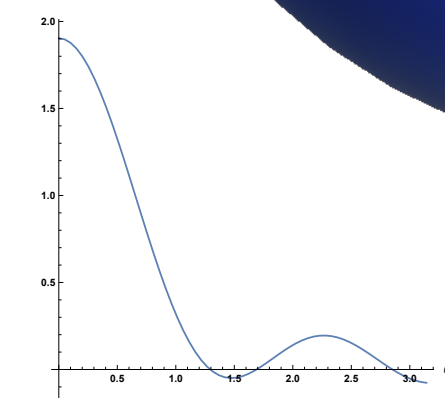
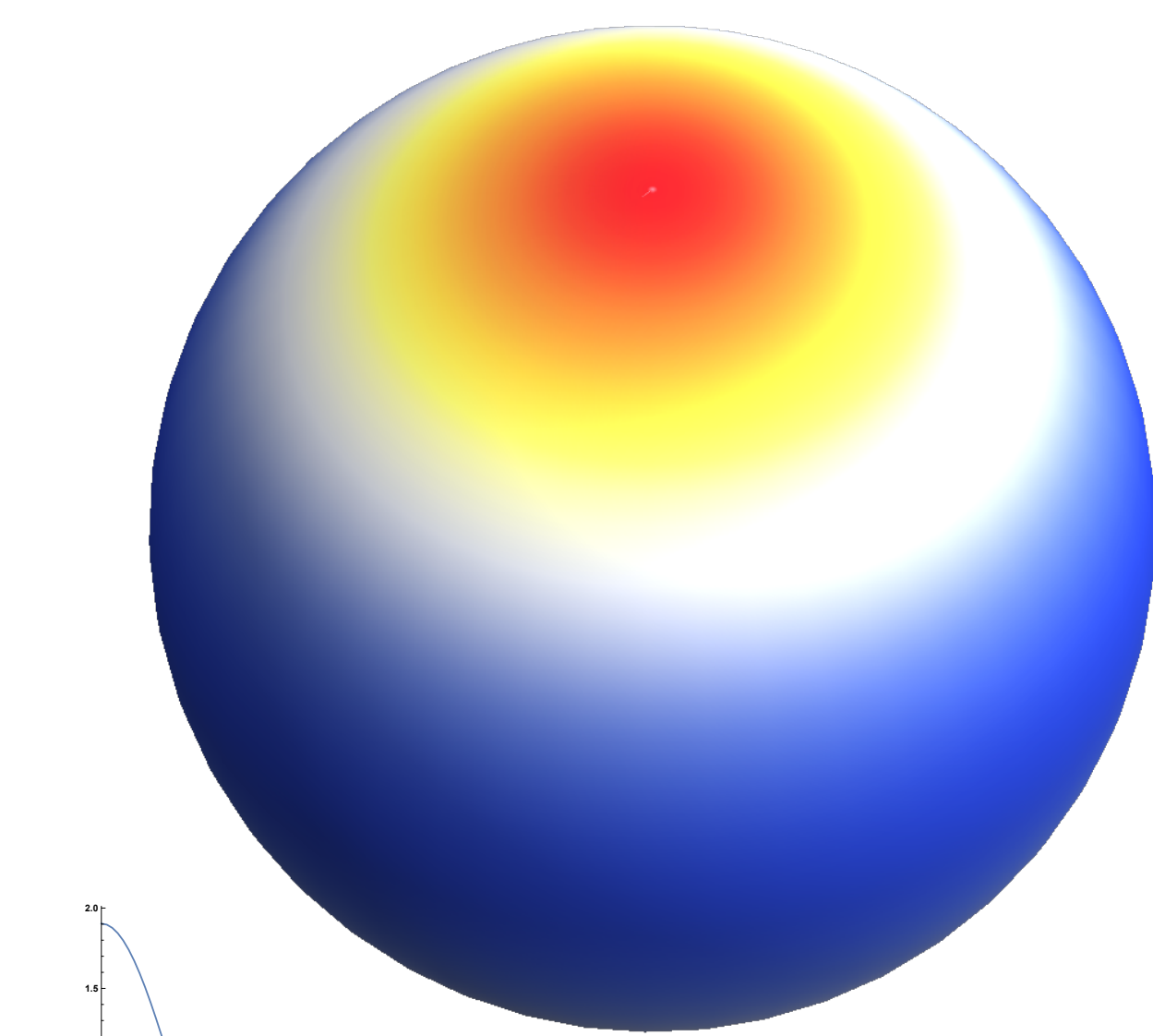
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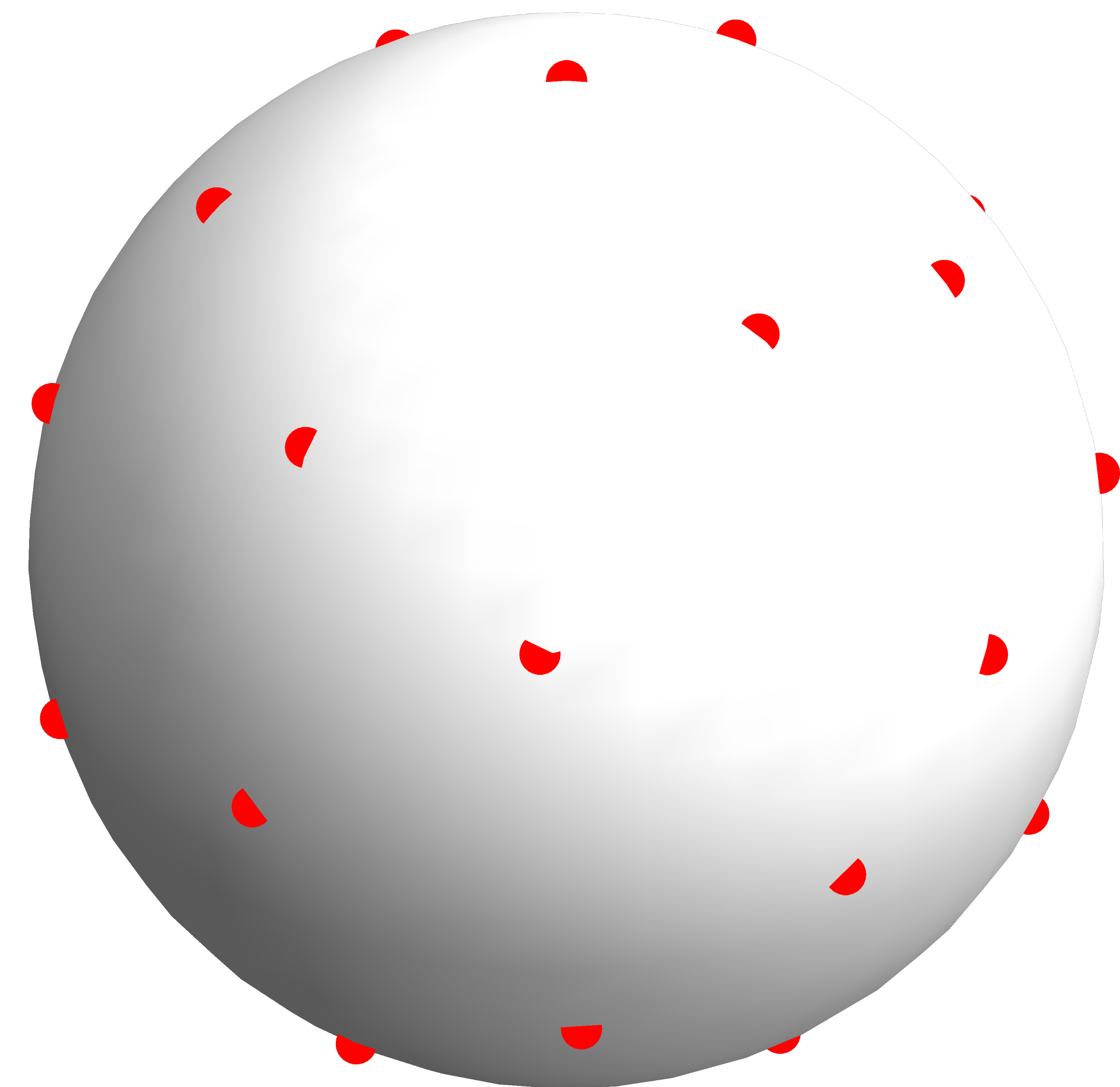
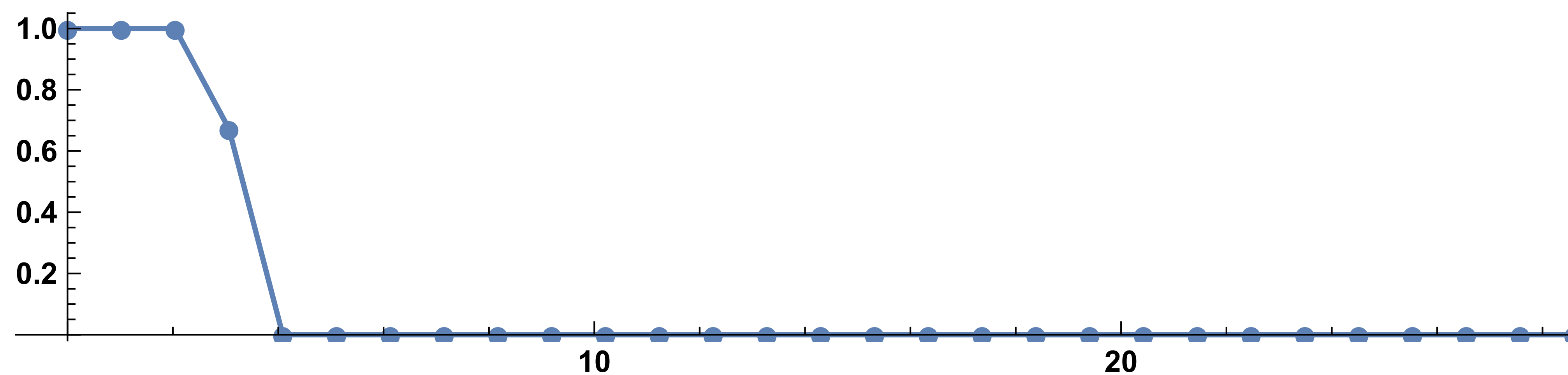
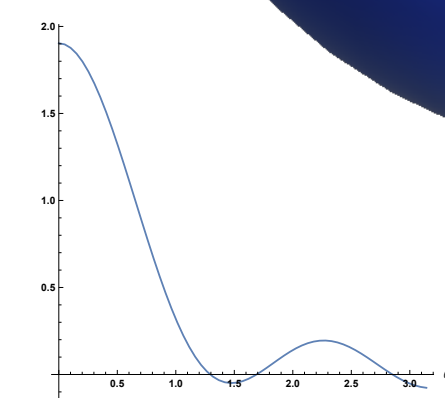
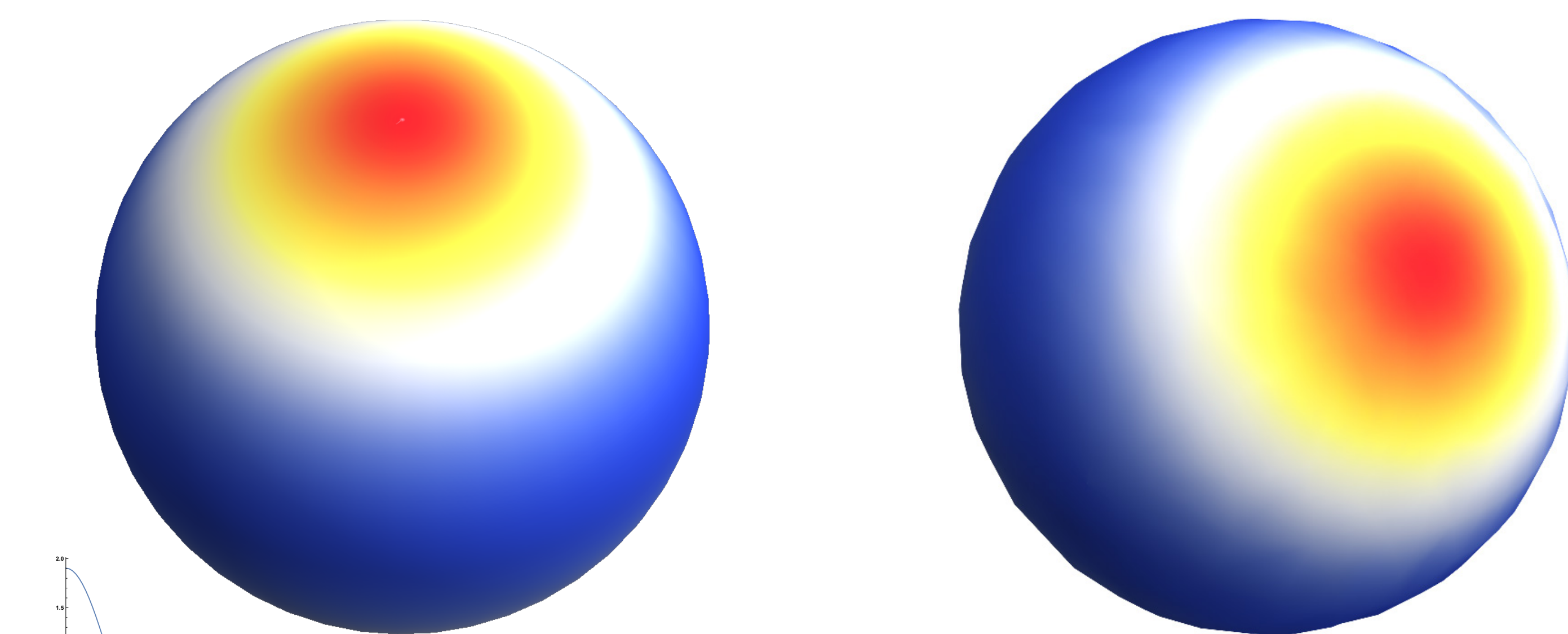
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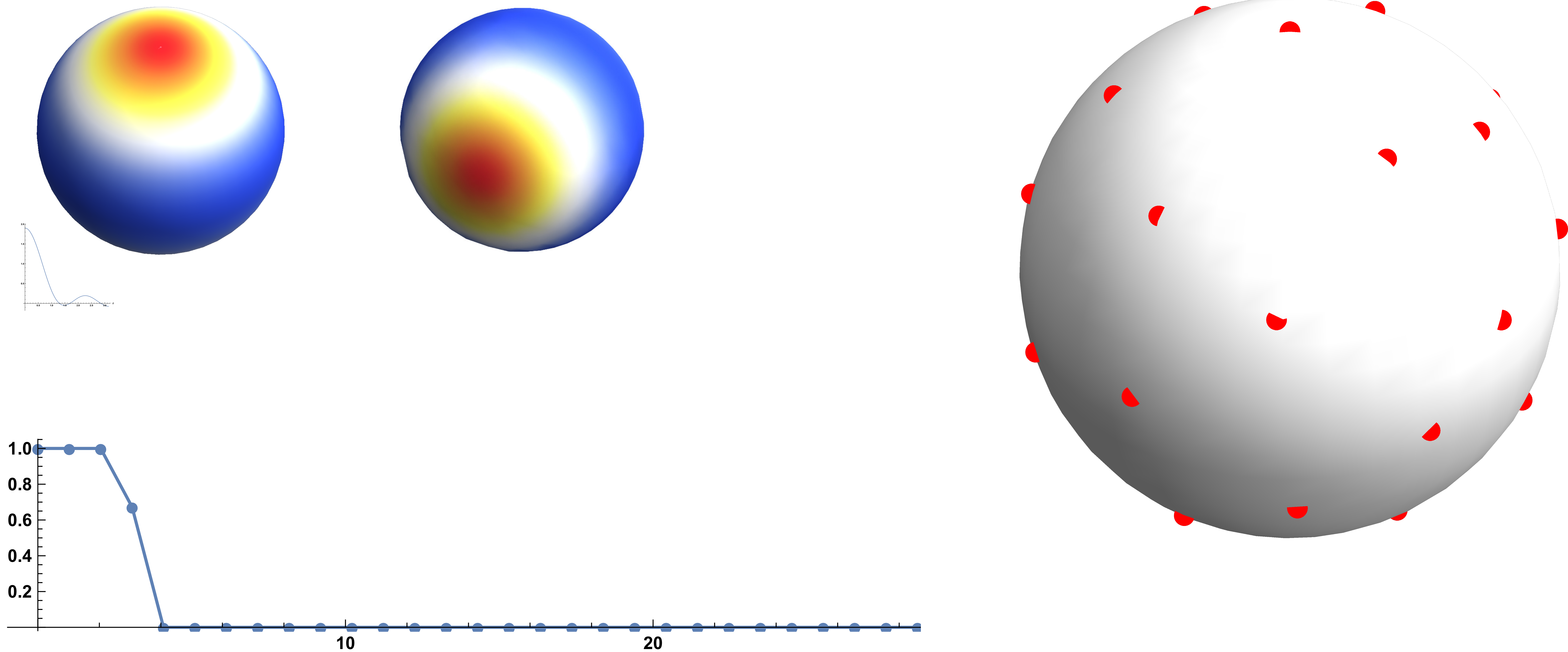
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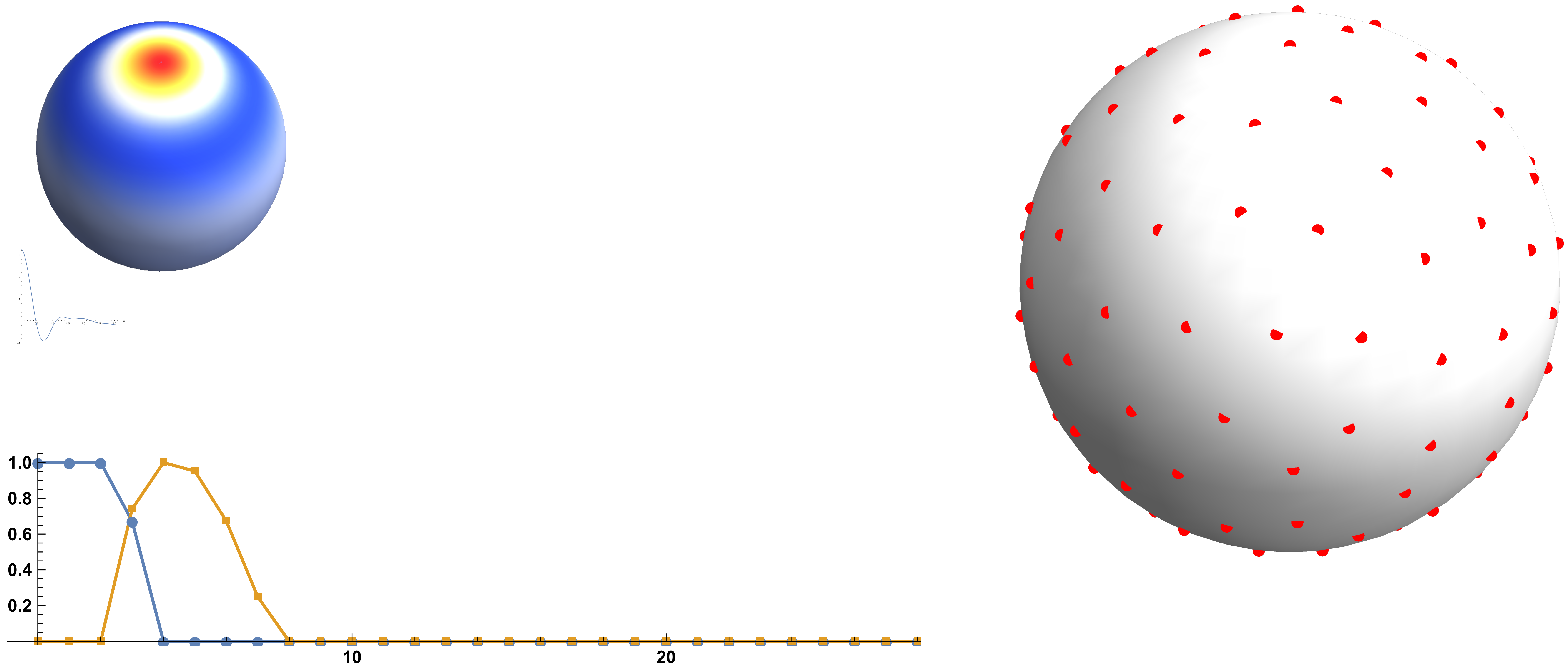
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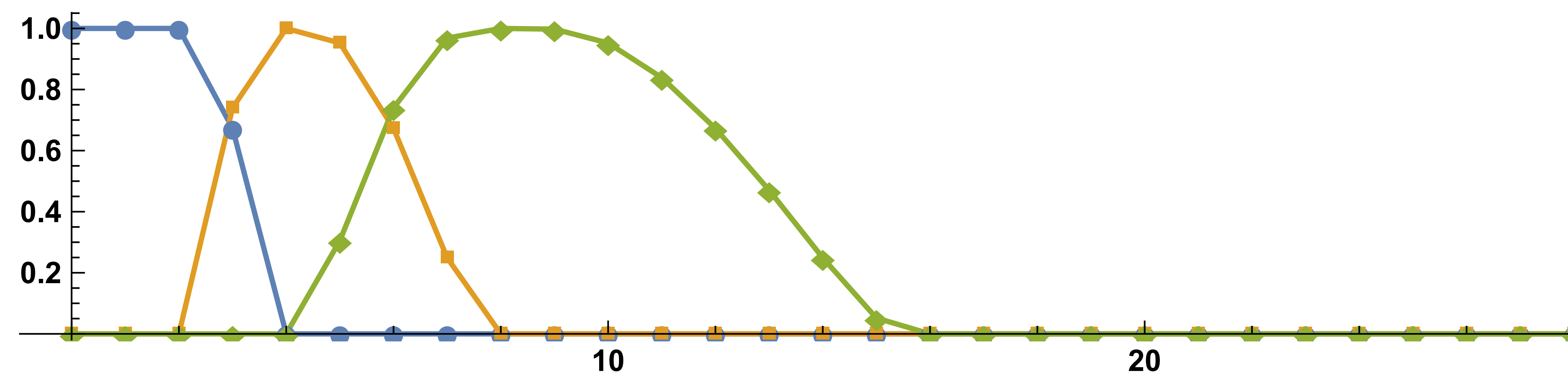
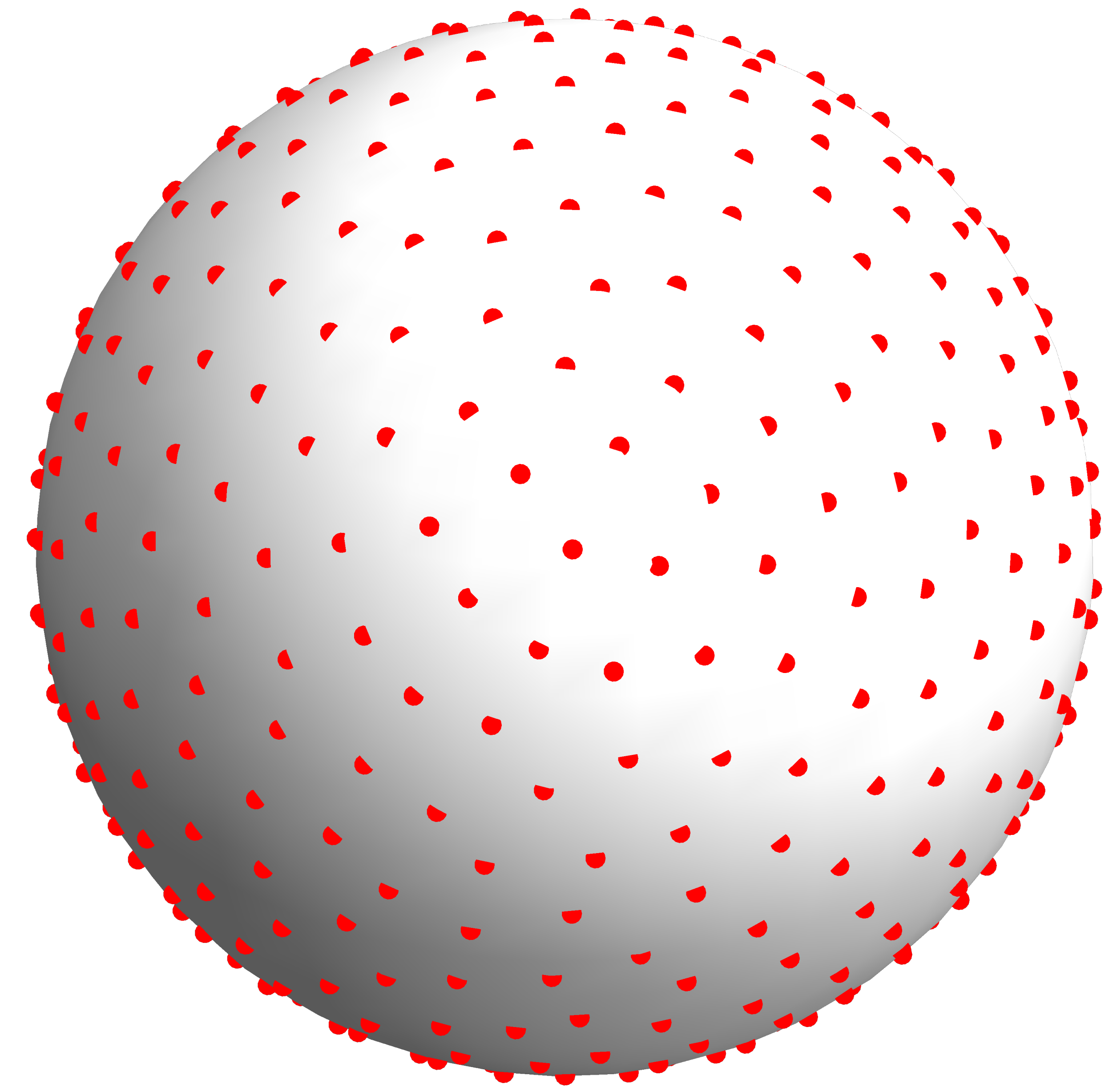
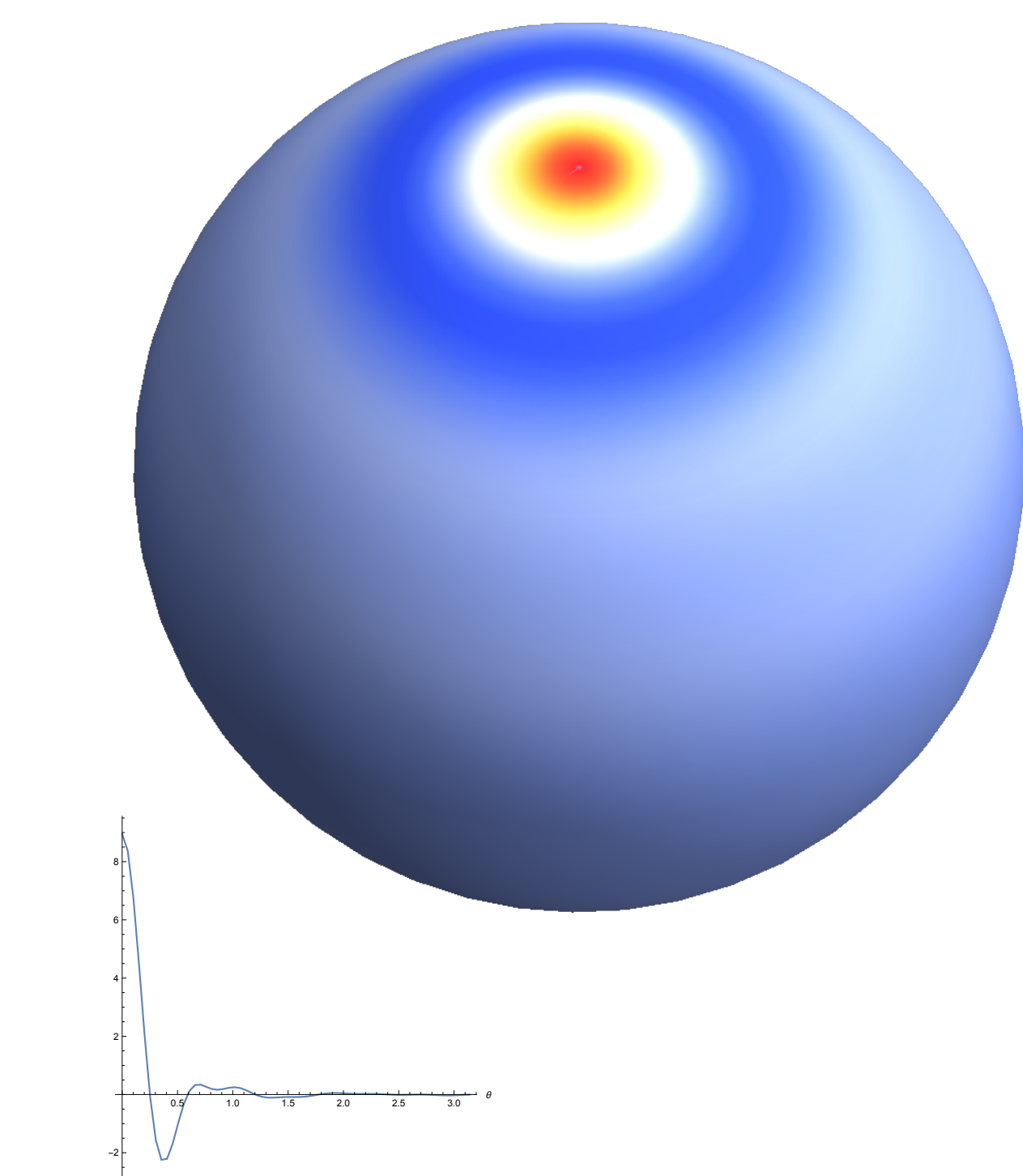
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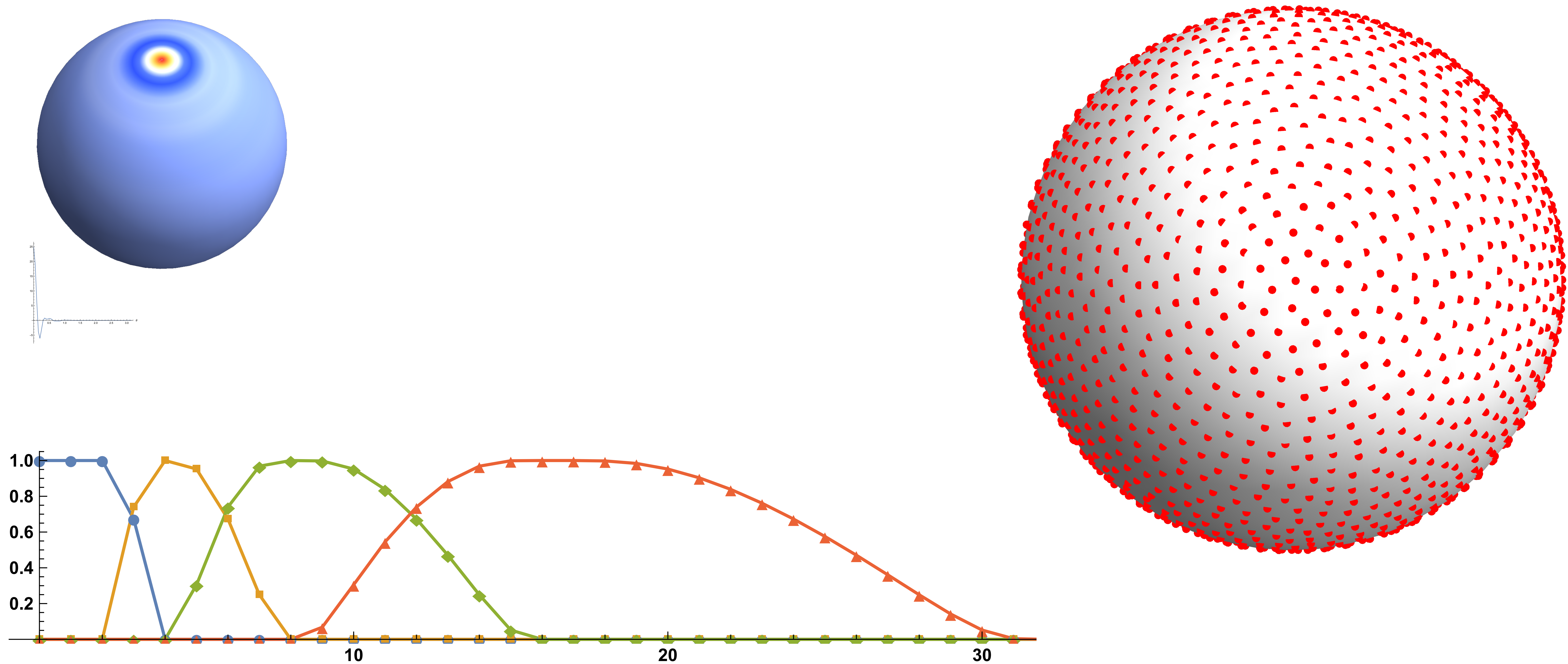
Local spectral representation



Local spectral representation



Local spectral representation

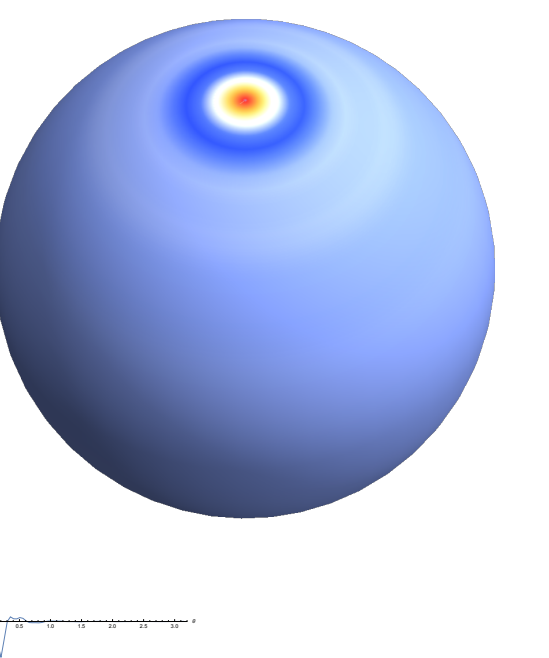
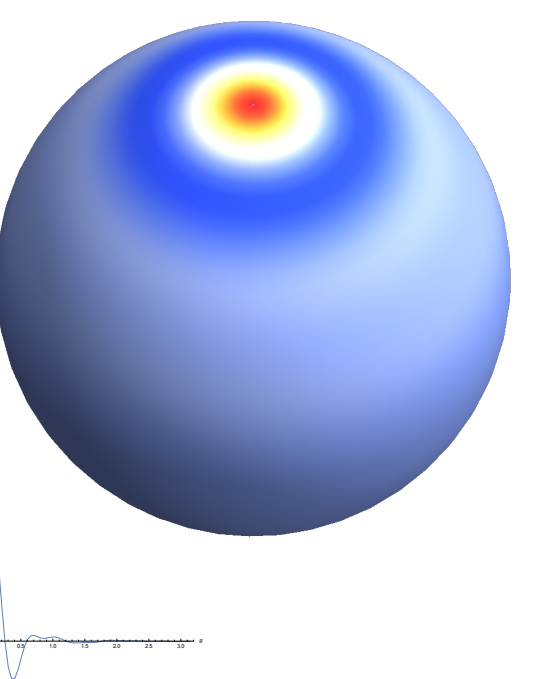
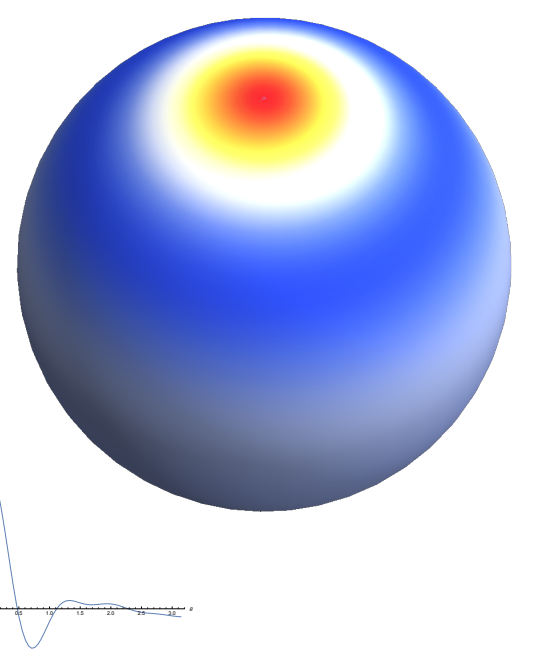
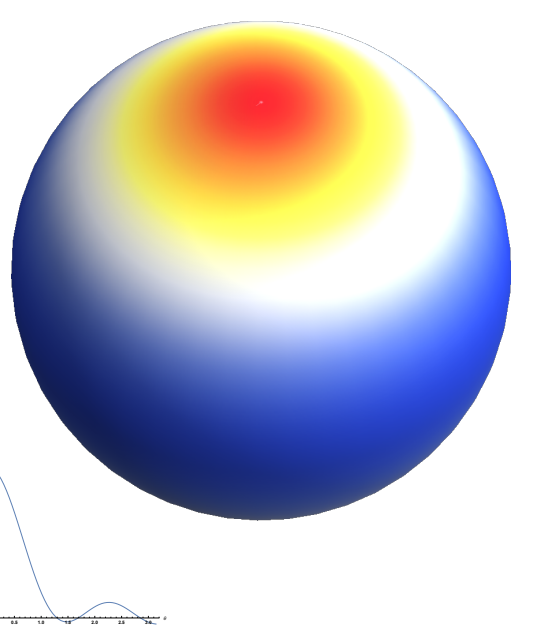


Local spectral representation

- Parseval tight frame for $L_2(\mathbb{S}^2)$:

$$f(\omega) = \sum_{j=0}^{\infty} \sum_{k=0}^{K_j} \langle f(\eta), \psi_{jk}(\eta) \rangle \psi_{jk}(\omega)$$

- Overcomplete analogue of orthonormal basis with Parseval identity being valid.

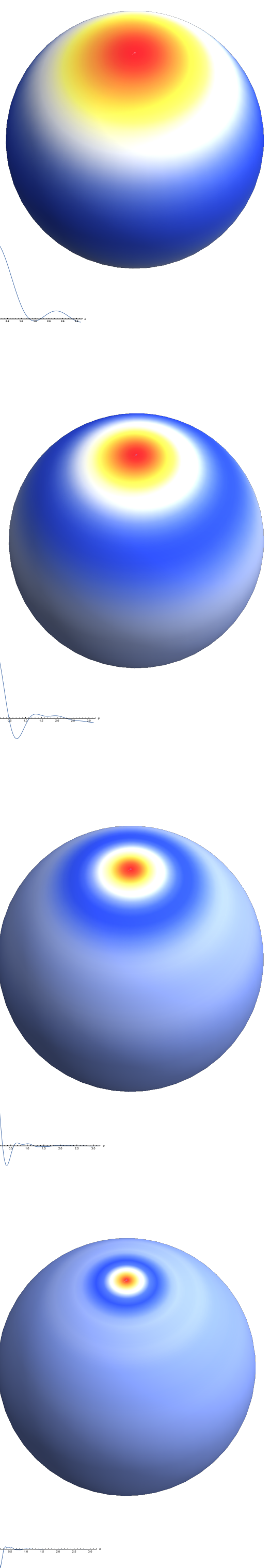


Local spectral representation

- Perfect quadrature rule:

$$\langle f(\eta) , \psi_{jk}(\eta) \rangle = \sum_{i=0}^{K_j} f(\lambda_i) \psi_{jk}(\lambda_i)$$

- Locations λ_i are chosen such that no quadrature weights are required (for $f \in V_j$).



Local spectral representation

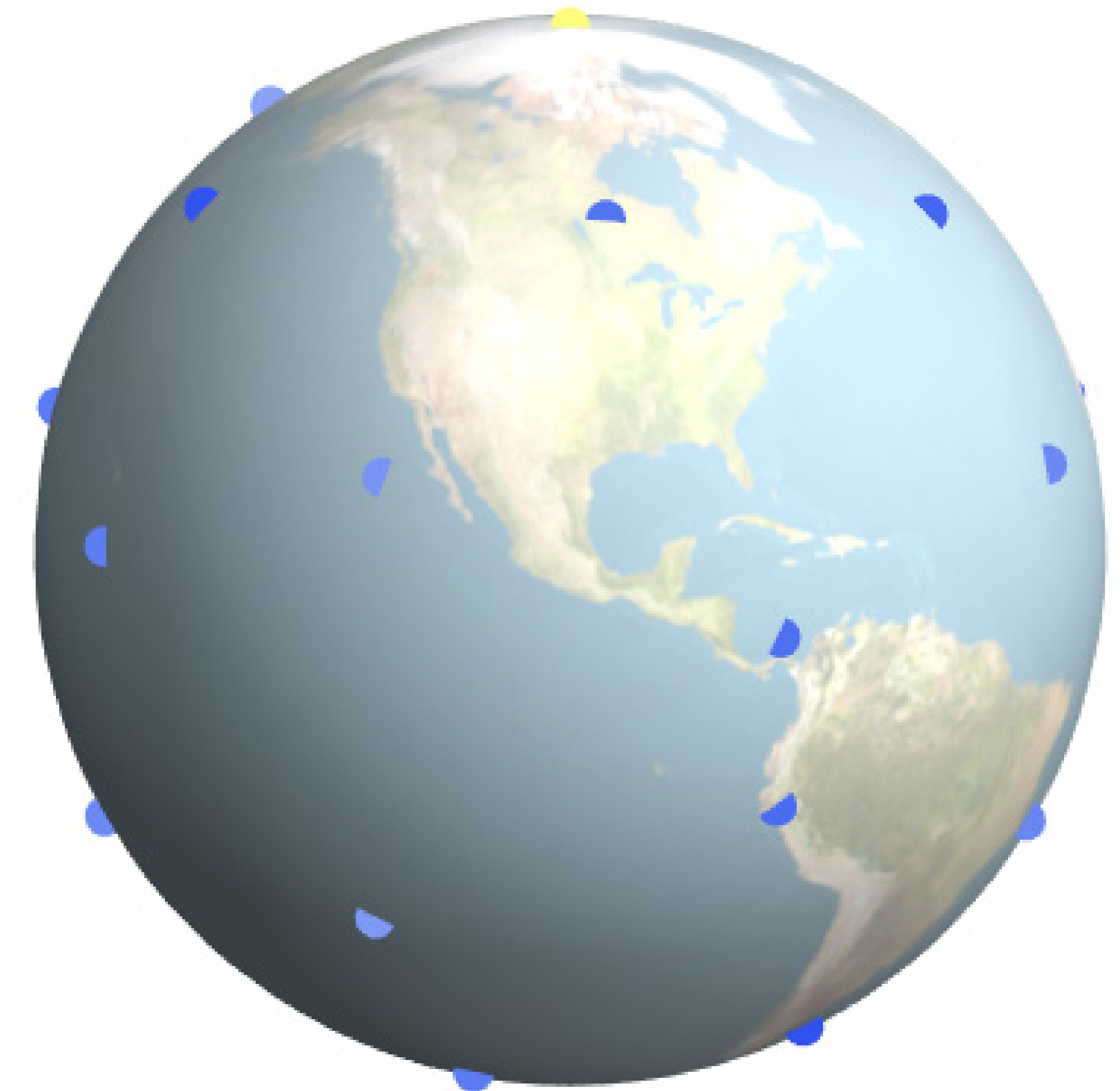
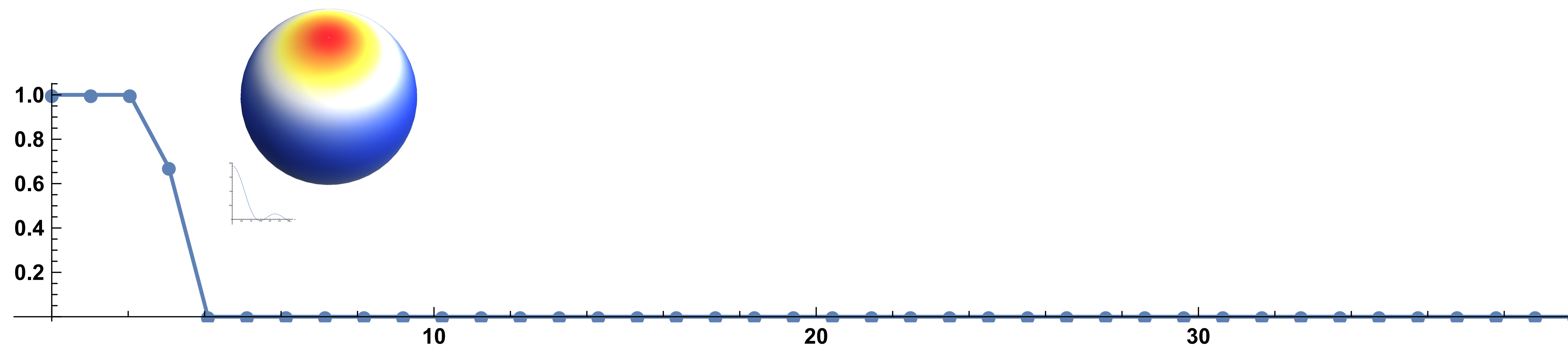
- Representation of vorticity:

$$\zeta = \sum_{jk} \zeta_{jk} \psi_{jk}$$

Local spectral representation

- Representation of vorticity:

$$\zeta = \sum_{jk} \zeta_{jk} \psi_{jk}$$

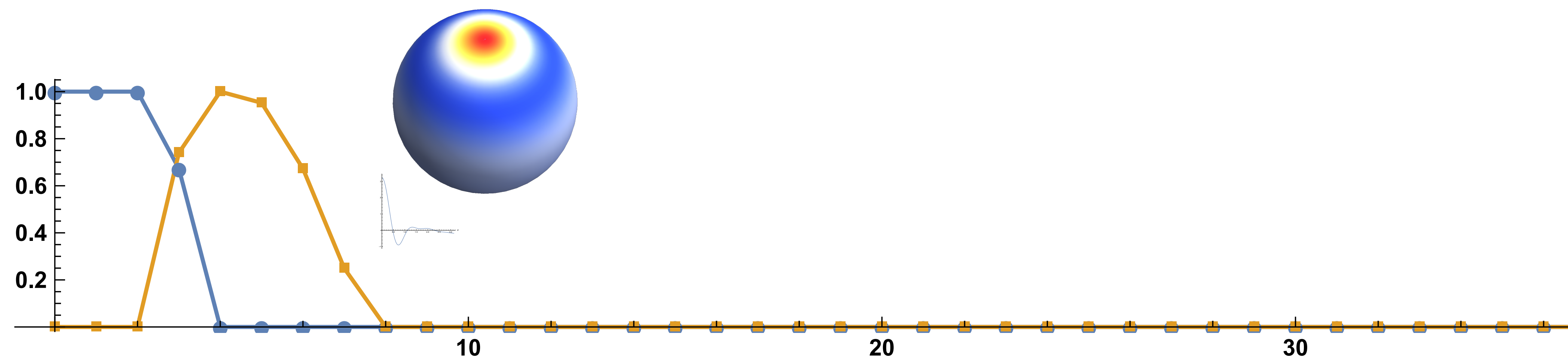


$$j = -1$$

Local spectral representation

- Representation of vorticity:

$$\zeta = \sum_{jk} \zeta_{jk} \psi_{jk}$$

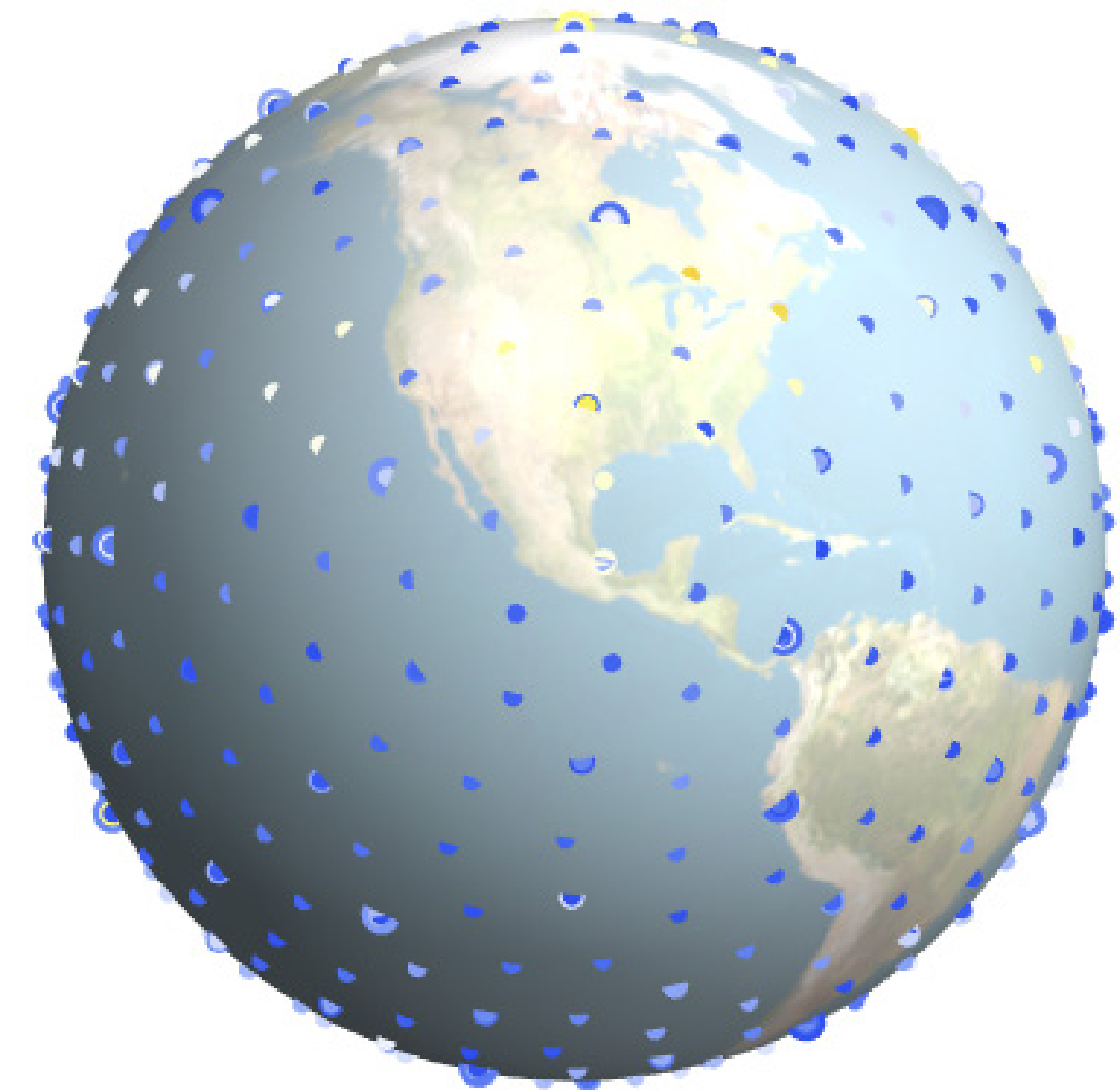
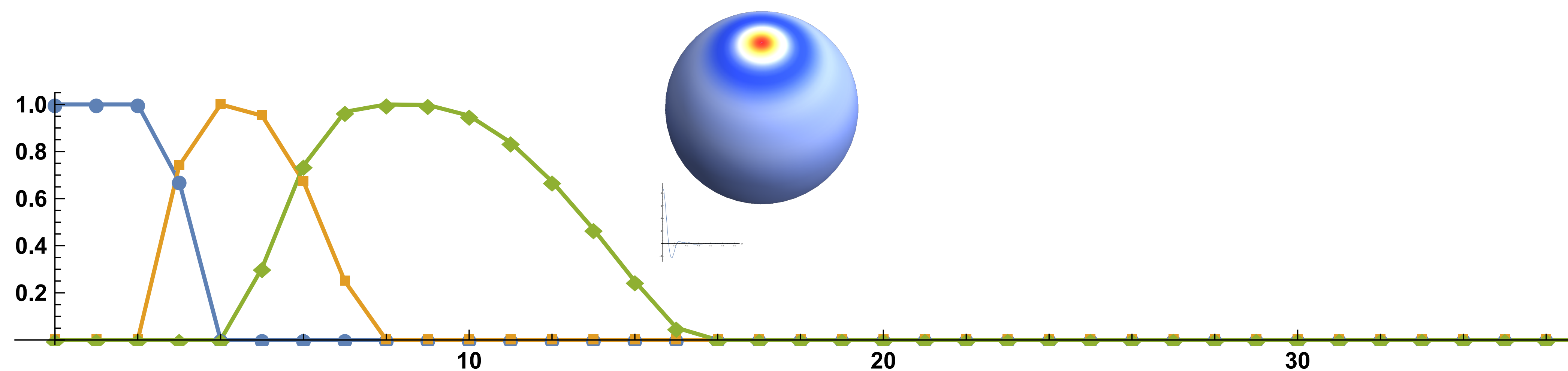


$$j \leq 0$$

Local spectral representation

- Representation of vorticity:

$$\zeta = \sum_{jk} \zeta_{jk} \psi_{jk}$$

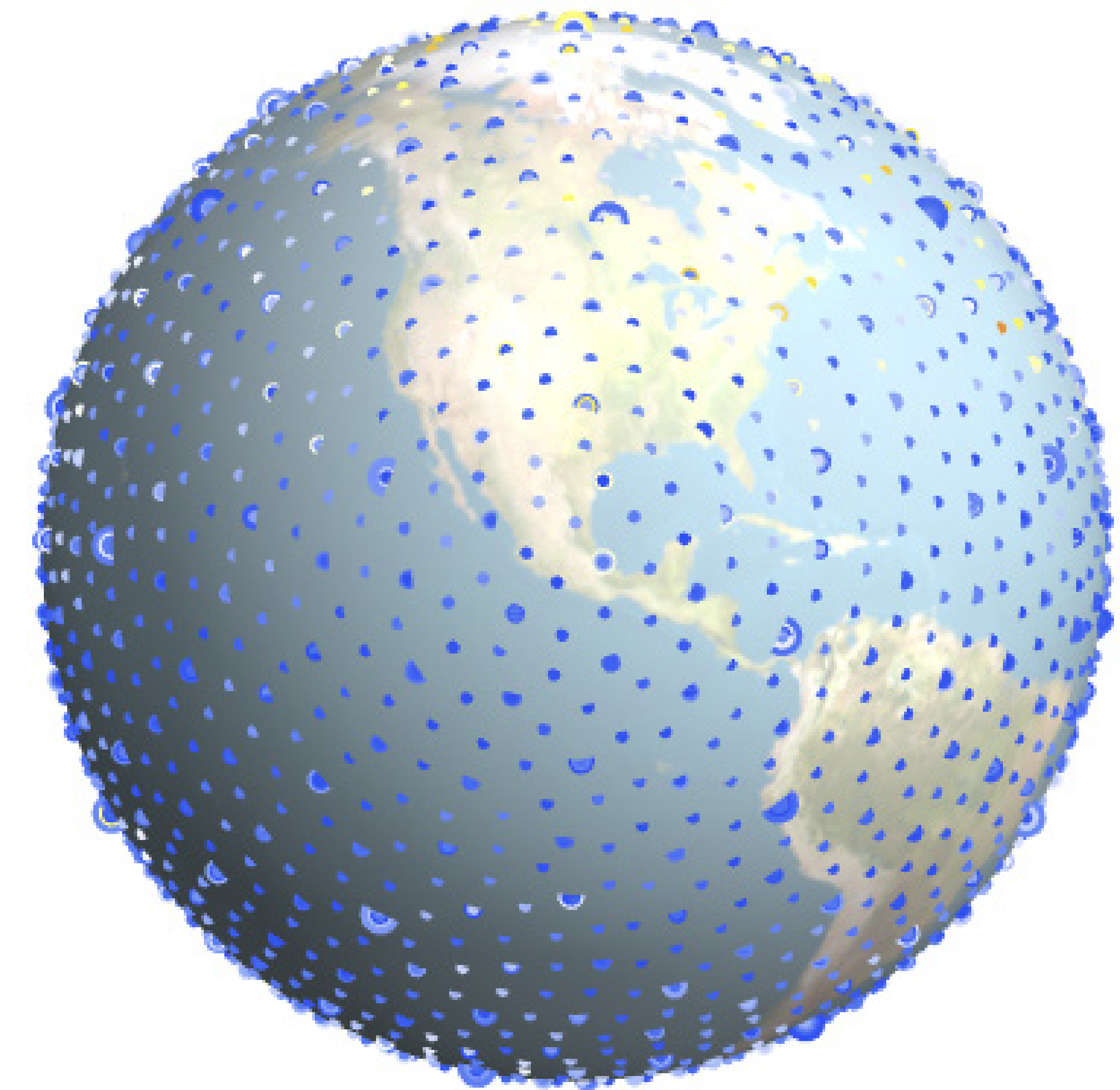
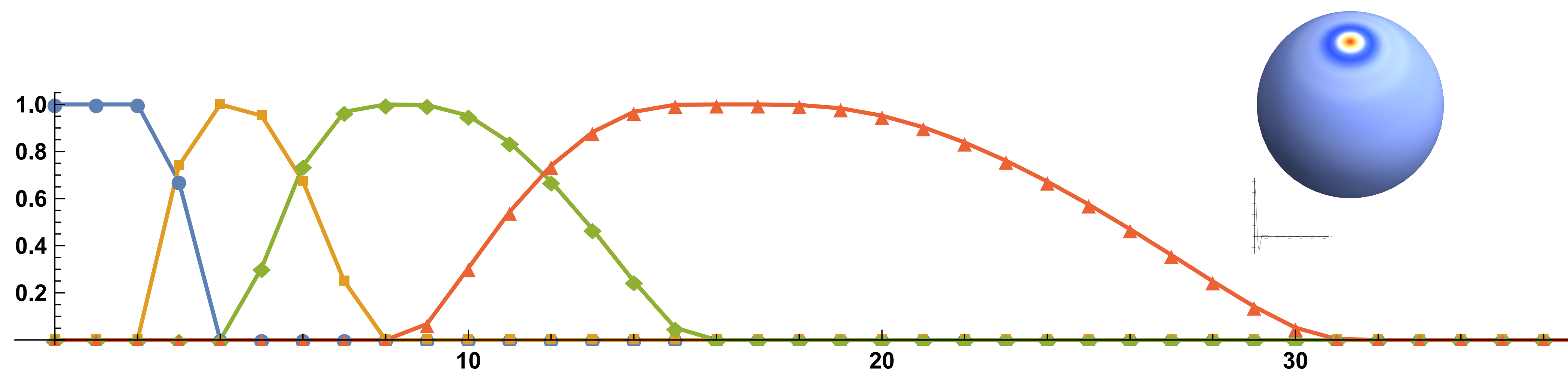


$$j \leq 1$$

Local spectral representation

- Representation of vorticity:

$$\zeta = \sum_{jk} \zeta_{jk} \psi_{jk}$$

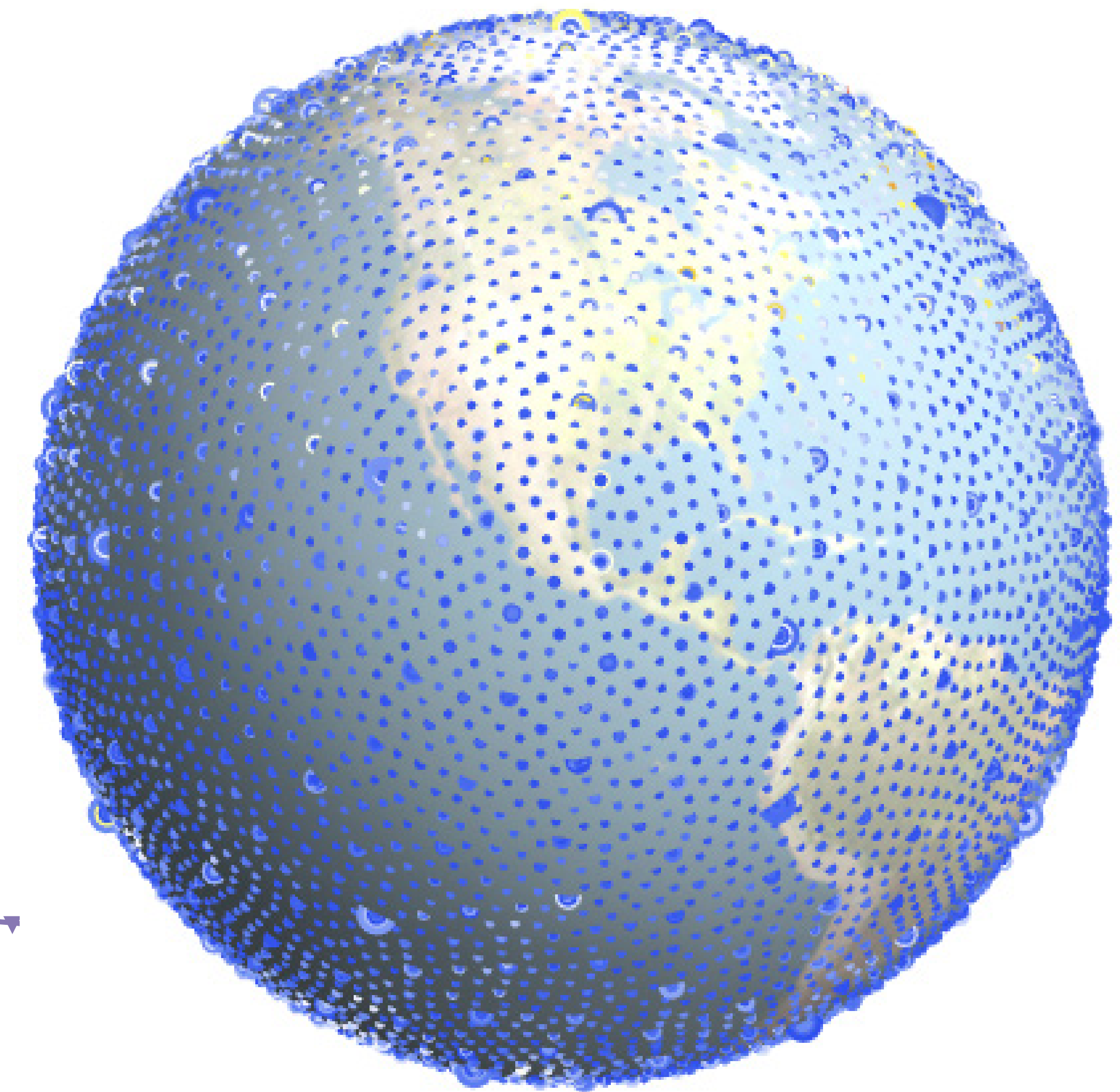


$$j \leq 2$$

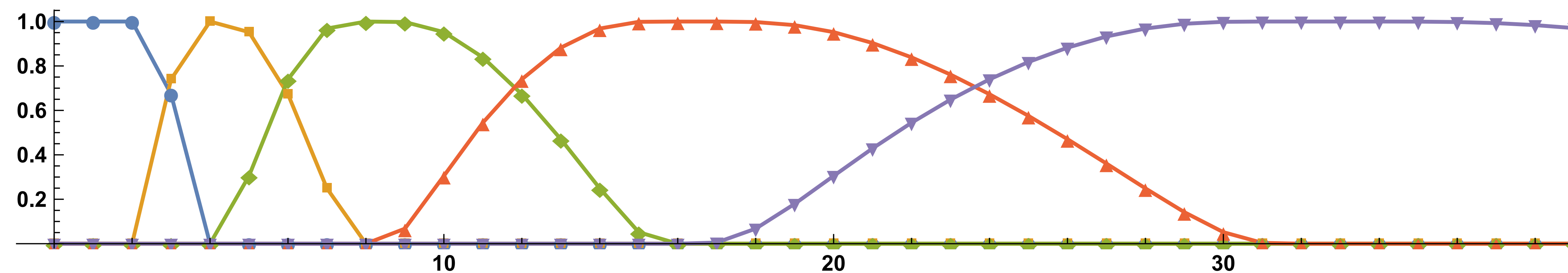
Local spectral representation

- Representation of vorticity:

$$\zeta = \sum_{jk} \zeta_{jk} \psi_{jk}$$



$j \leq 3$



Primitive equations

$$\frac{\partial \zeta}{\partial t} = -\nabla \cdot (\zeta + f)\vec{u} - \nabla \times \left(\dot{\sigma} \frac{\partial \vec{u}}{\partial \sigma} + RT' \nabla q \right)$$

$$\frac{\partial \delta}{\partial t} = \nabla \times (\zeta + f)\vec{u} - \nabla \times \left(\dot{\sigma} \frac{\partial \vec{u}}{\partial \sigma} + RT' \nabla q \right) - \nabla^2 \left(\phi + \frac{\vec{u} \cdot \vec{u}}{2} + R\bar{T}q \right)$$

$$\frac{\partial T}{\partial t} = \frac{\kappa T}{\sigma p_s} \frac{\partial(\sigma p_s)}{\partial t}$$

Primitive equations

$$\frac{\partial \zeta}{\partial t} = -\nabla \cdot (\zeta + f)\vec{u} - \nabla \times \left(\dot{\sigma} \frac{\partial \vec{u}}{\partial \sigma} + RT' \nabla q \right)$$

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$$\frac{\partial T}{\partial t} = \frac{\kappa T}{\sigma p_s} \frac{\partial(\sigma p_s)}{\partial t}$$

prognostic fields represented using wavelets

Primitive equations

$$\frac{\partial \zeta}{\partial t} = -\nabla \cdot (\zeta + f) \vec{u} - \nabla \times \left(\dot{\sigma} \frac{\partial \vec{u}}{\partial \sigma} + RT' \nabla q \right)$$

$$\frac{\partial \delta}{\partial t} = \nabla \times (\zeta + f) \vec{u} - \nabla \times \left(\dot{\sigma} \frac{\partial \vec{u}}{\partial \sigma} + RT' \nabla q \right) - \nabla^2 \left(\phi + \frac{\vec{u} \cdot \vec{u}}{2} + R\bar{T}q \right)$$

$$\frac{\partial T}{\partial t} = \frac{\kappa T}{\sigma p_s} \frac{\partial(\sigma p_s)}{\partial t}$$

velocity needed to implement fast transform method

prognostic fields represented using wavelets

Local spectral representation

$$\vec{u} = J \nabla \xi + \nabla \chi$$

$$\xi = \Delta^{-1} \zeta$$

$$\chi = \Delta^{-1} \delta$$

Local spectral representation

$$\vec{u} = J \nabla \xi + \nabla \chi$$

$$\xi = \Delta^{-1} \zeta$$

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$$\xi = \sum_{jk} \xi_{jk} \psi_{jk}$$

$$\chi = \sum_{jk} \chi_{jk} \psi_{jk}$$

Local spectral representation

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$$J \nabla \xi$$

Local spectral representation

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$$J\nabla \xi = \sum_{jk} \xi_{jk} J\nabla \psi_{jk}$$

Local spectral representation

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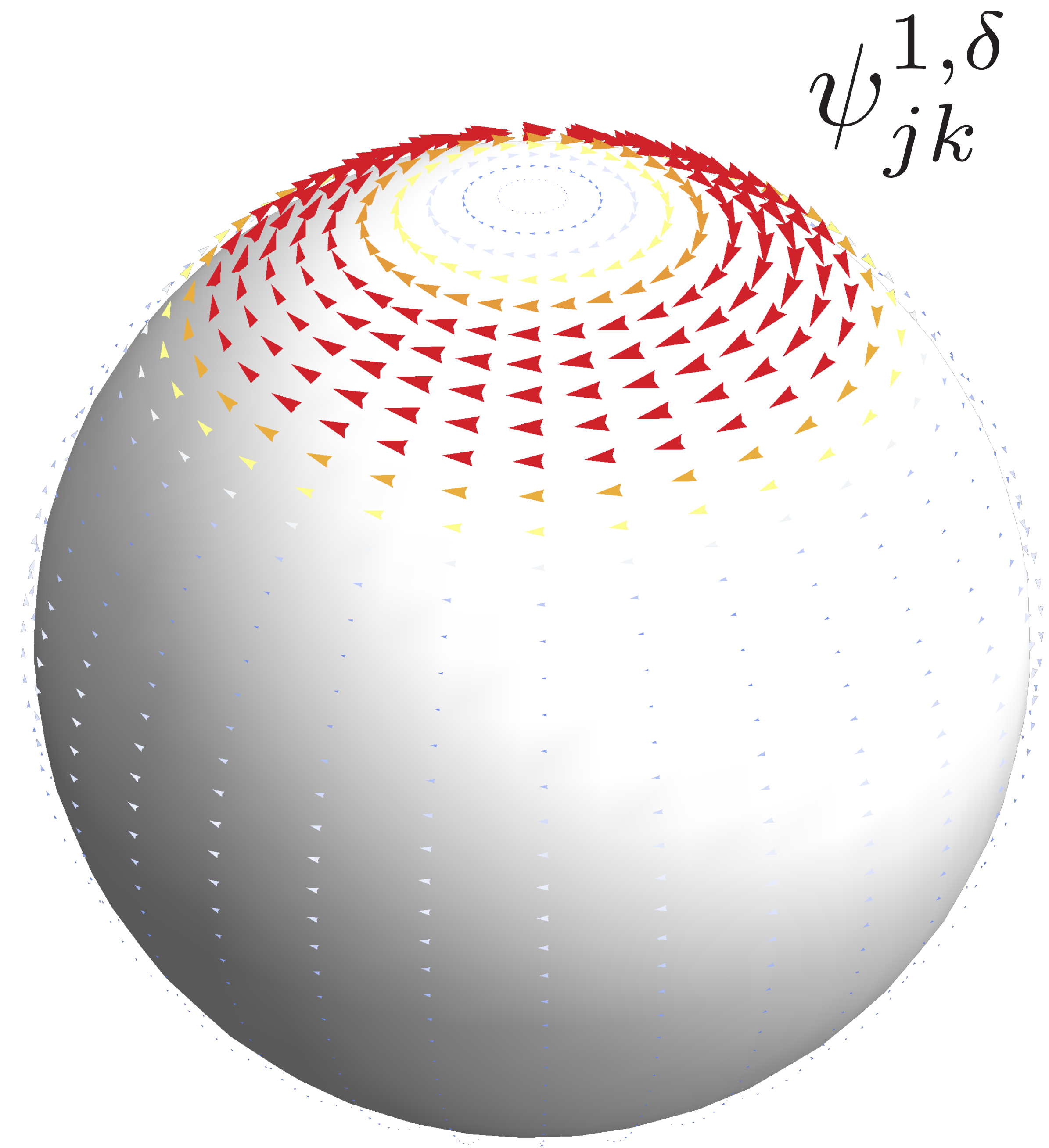
$$\Rightarrow \psi_{jk}^{1,\delta} = J\nabla \psi_{jk}$$

Local spectral representation

$$\xi = \sum_{jk} \xi_{jk} \psi_{jk}$$

$$J\nabla \xi = \sum_{jk} \xi_{jk} J\nabla \psi_{jk}$$

$$\Rightarrow \psi_{jk}^{1,\delta} = J\nabla \psi_{jk}$$



Local spectral representation

$$\chi = \sum_{jk} \chi_{jk} \psi_{jk}$$

Local spectral representation

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$$\nabla \chi$$

Local spectral representation

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Local spectral representation

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$$\nabla \chi = \sum_{jk} \chi_{jk} \nabla \psi_{jk}$$

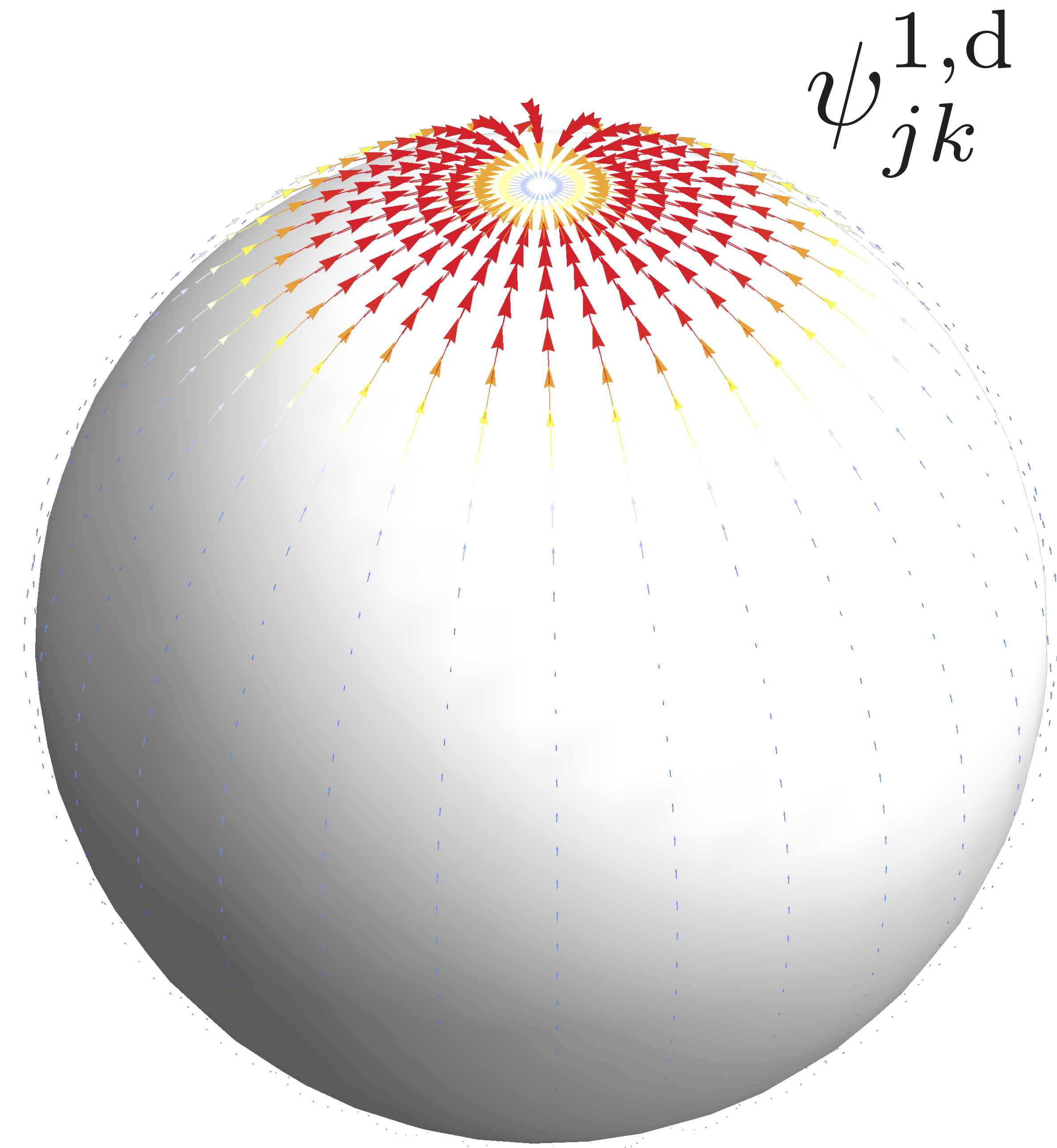
$$\Rightarrow \psi_{jk}^{1,d} = \nabla \psi_{jk}$$

Local spectral representation

$$\chi = \sum_{jk} \chi_{jk} \psi_{jk}$$

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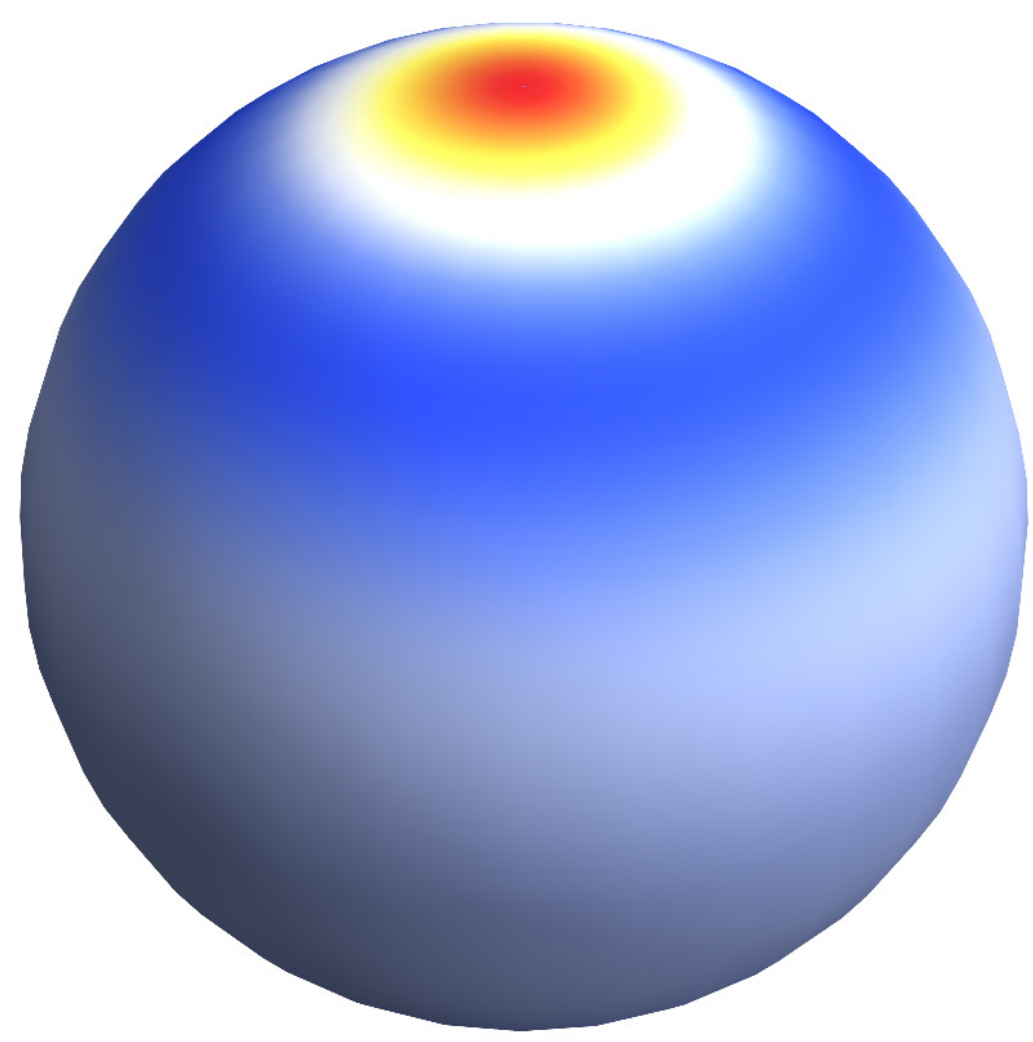
$$\xi = \sum_{jk} \boxed{\xi_{jk}} \psi_{jk}$$

$$\chi = \sum_{jk} \boxed{\chi_{jk}} \psi_{jk}$$

$$\nabla \chi = \sum_{jk} \boxed{\chi_{jk}} \psi_{jk}^{1,d}$$

Ψ_{ec} : local spectral exterior calculus

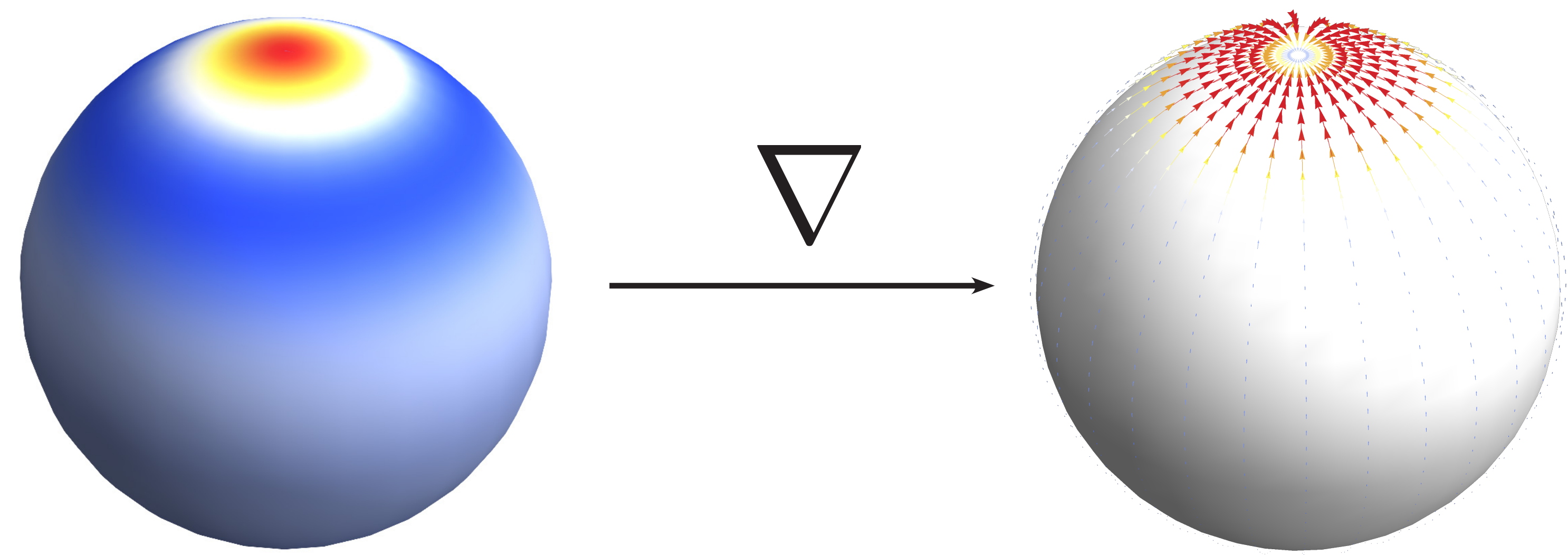
ψ_{jk}



Ψ_{ec} : local spectral exterior calculus

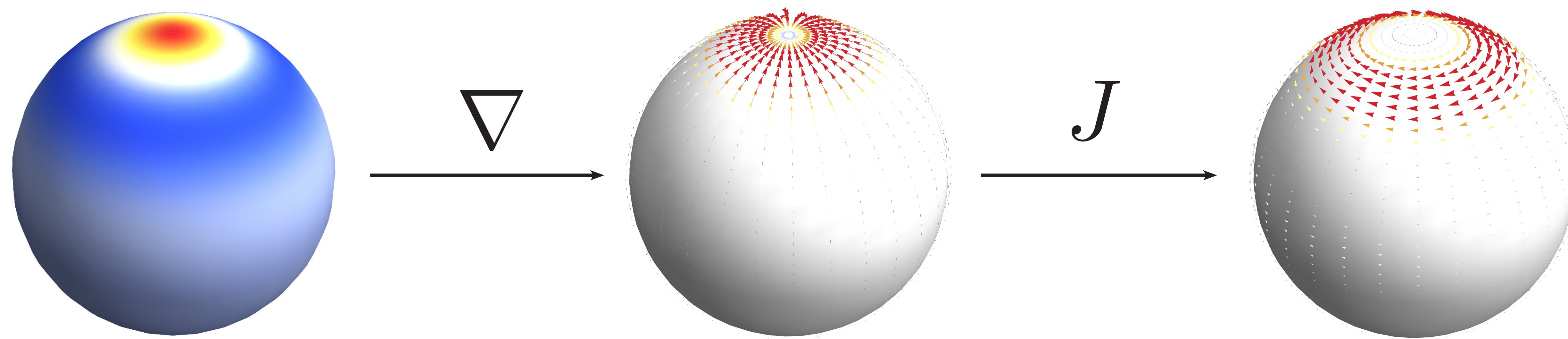
$$\psi_{jk}$$

$$\psi_{jk}^{1,d} = \nabla \psi_{jk}$$



Ψ_{ec} : local spectral exterior calculus

$$\psi_{jk} \qquad \psi_{jk}^{1,d} = \nabla \psi_{jk} \qquad \psi_{jk}^{1,\delta} = J \nabla \psi_{jk}$$



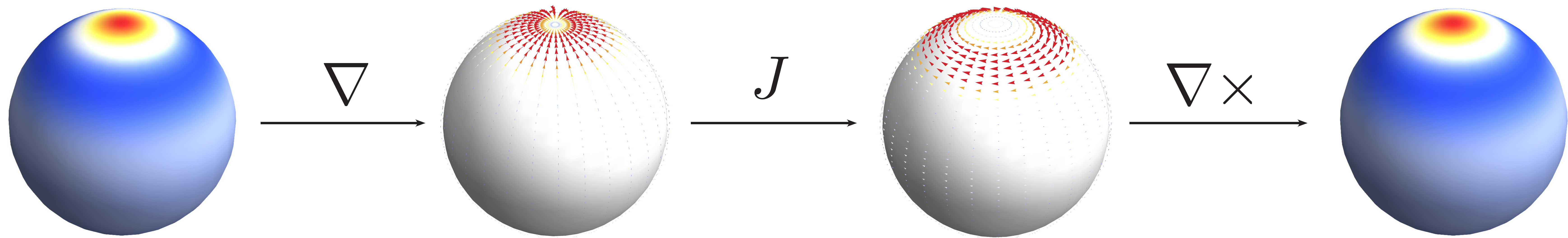
Ψ_{ec} : local spectral exterior calculus

$$\psi_{jk}$$

$$\psi_{jk}^{1,d} = \nabla \psi_{jk}$$

$$\psi_{jk}^{1,\delta} = J \nabla \psi_{jk}$$

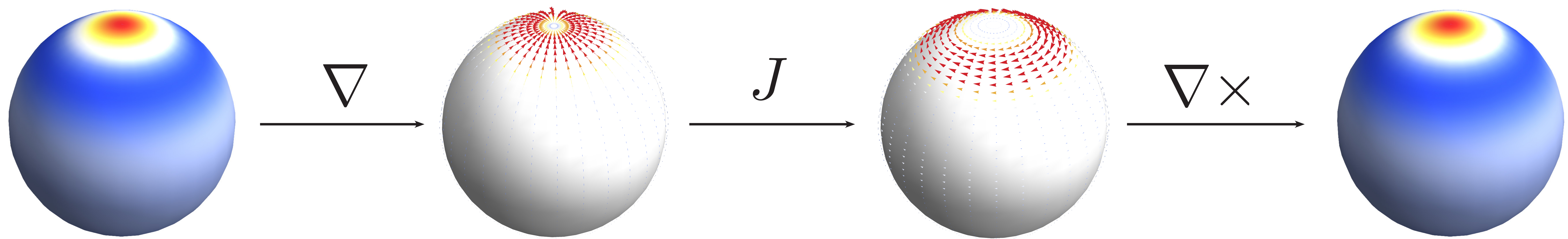
$$\psi_{jk}$$



Ψ_{ec} : local spectral exterior calculus

$$\Omega^0 \xrightarrow{d} \Omega^1_d \xrightarrow{\star} \Omega^1_\delta \xrightarrow{d} \Omega^2$$

$$\psi_{jk} \qquad \psi_{jk}^{1,d} = \nabla \psi_{jk} \qquad \psi_{jk}^{1,\delta} = J \nabla \psi_{jk} \qquad \psi_{jk}$$



Primitive equations

$$\frac{\partial \zeta}{\partial t} = -\nabla \cdot (\zeta + f) \vec{u} - \nabla \times \left(\dot{\sigma} \frac{\partial \vec{u}}{\partial \sigma} + RT' \nabla q \right)$$

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prognostic fields represented using wavelets

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$$\frac{\partial T}{\partial t} = \frac{\kappa T}{\sigma p_s} \frac{\partial(\sigma p_s)}{\partial t} \quad \text{velocity reconstructed from vector-valued wavelets}$$

prognostic fields represented using wavelets

Primitive equations

$$\vec{u} = J\nabla\xi + \nabla\chi$$

$$\xi = \Delta^{-1}\zeta$$

$$\chi = \Delta^{-1}\delta$$

$$J\nabla\xi = \sum_{jk} \boxed{\xi_{jk}} \psi_{jk}^{1,\delta}$$

$$\xi = \sum_{jk} \boxed{\xi_{jk}} \psi_{jk}$$

$$\chi = \sum_{jk} \boxed{\chi_{jk}} \psi_{jk}$$

$$\nabla\chi = \sum_{jk} \boxed{\chi_{jk}} \psi_{jk}^{1,d}$$

Reconstruction of velocity potentials

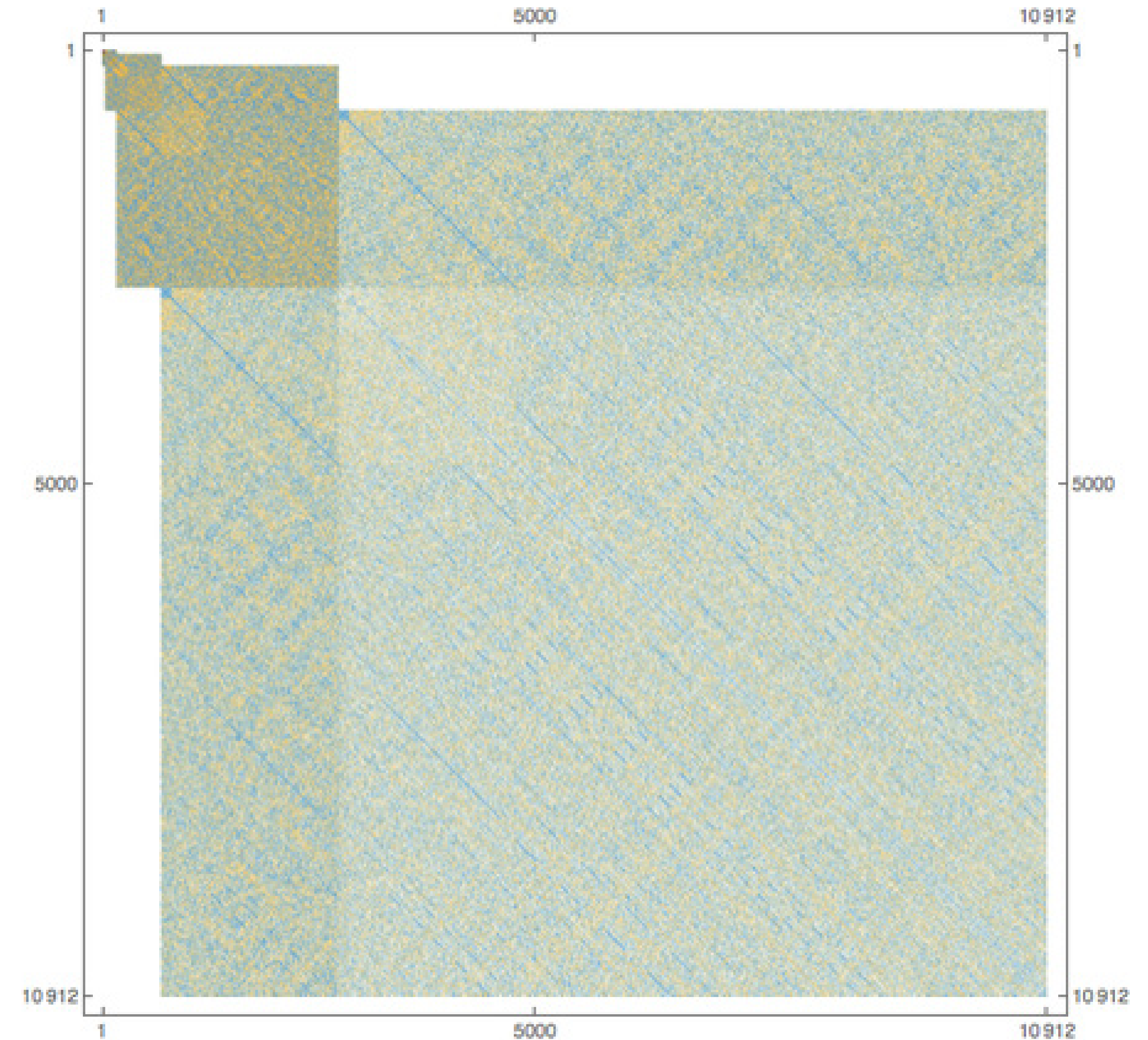
$$\xi_{jk} = \sum_{j'k'} L_{jk,j'k'}^{-1} \zeta_{j'k'}$$

$$\chi_{jk} = \sum_{j'k'} L_{jk,j'k'}^{-1} \delta_{j'k'}$$

Reconstruction of velocity potentials

$$\xi_{jk} = \sum_{j'k'} L_{jk,j'k'}^{-1} \zeta_{j'k'}$$

$$\chi_{jk} = \sum_{j'k'} L_{jk,j'k'}^{-1} \delta_{j'k'}$$



Coriolis parameter

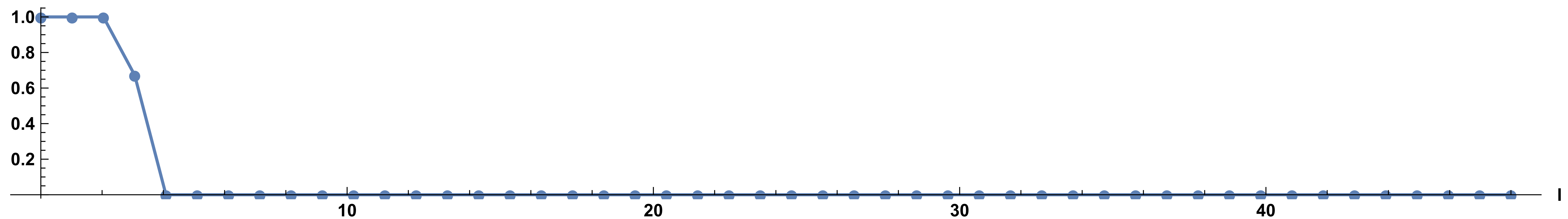
$$\eta = \zeta + f$$

$$f = 2 \Omega \cos \theta = c_f y_{10}$$

Coriolis parameter

$$\eta = \zeta + f$$

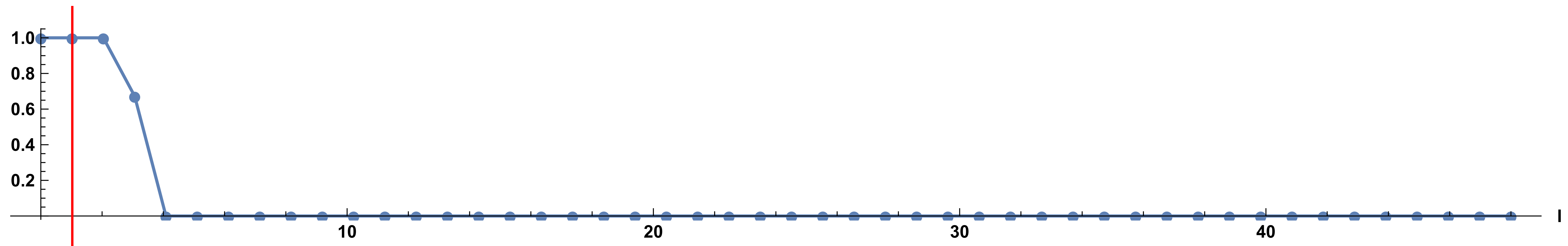
$$f = 2 \Omega \cos \theta = c_f y_{10}$$



Coriolis parameter

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$$f = 2 \Omega \cos \theta = c_f y_{10}$$

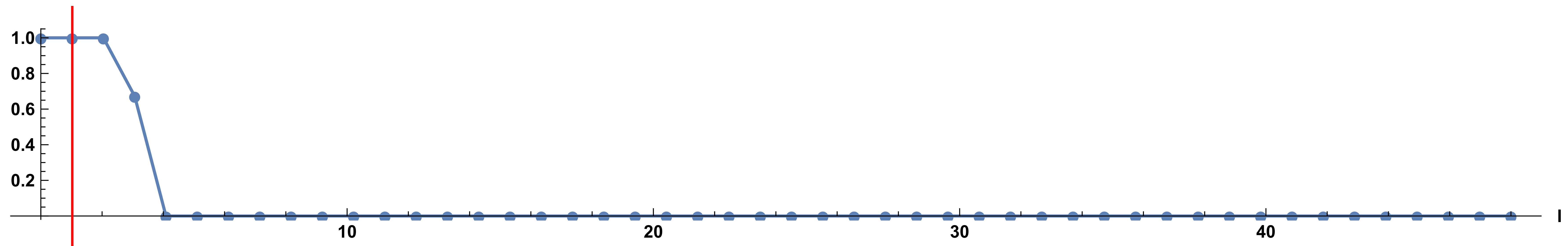


Coriolis parameter

$$\eta = \zeta + f$$

$$f = 2 \Omega \cos \theta = c_f y_{10}$$

$$\Rightarrow \eta_{0k} = \zeta_{0k} + f_{0k}$$



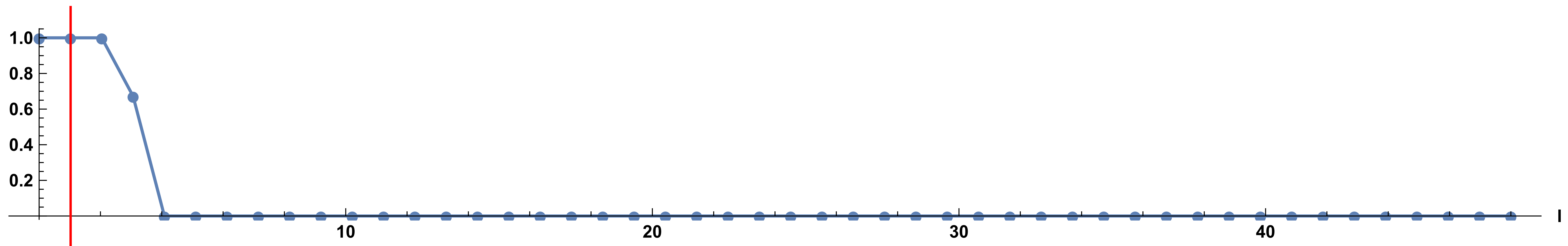
Coriolis parameter

$$\eta = \zeta + f$$

$$f = 2 \Omega \cos \theta = c_f y_{10}$$

$$\Rightarrow \eta_{0k} = \zeta_{0k} + f_{0k}$$

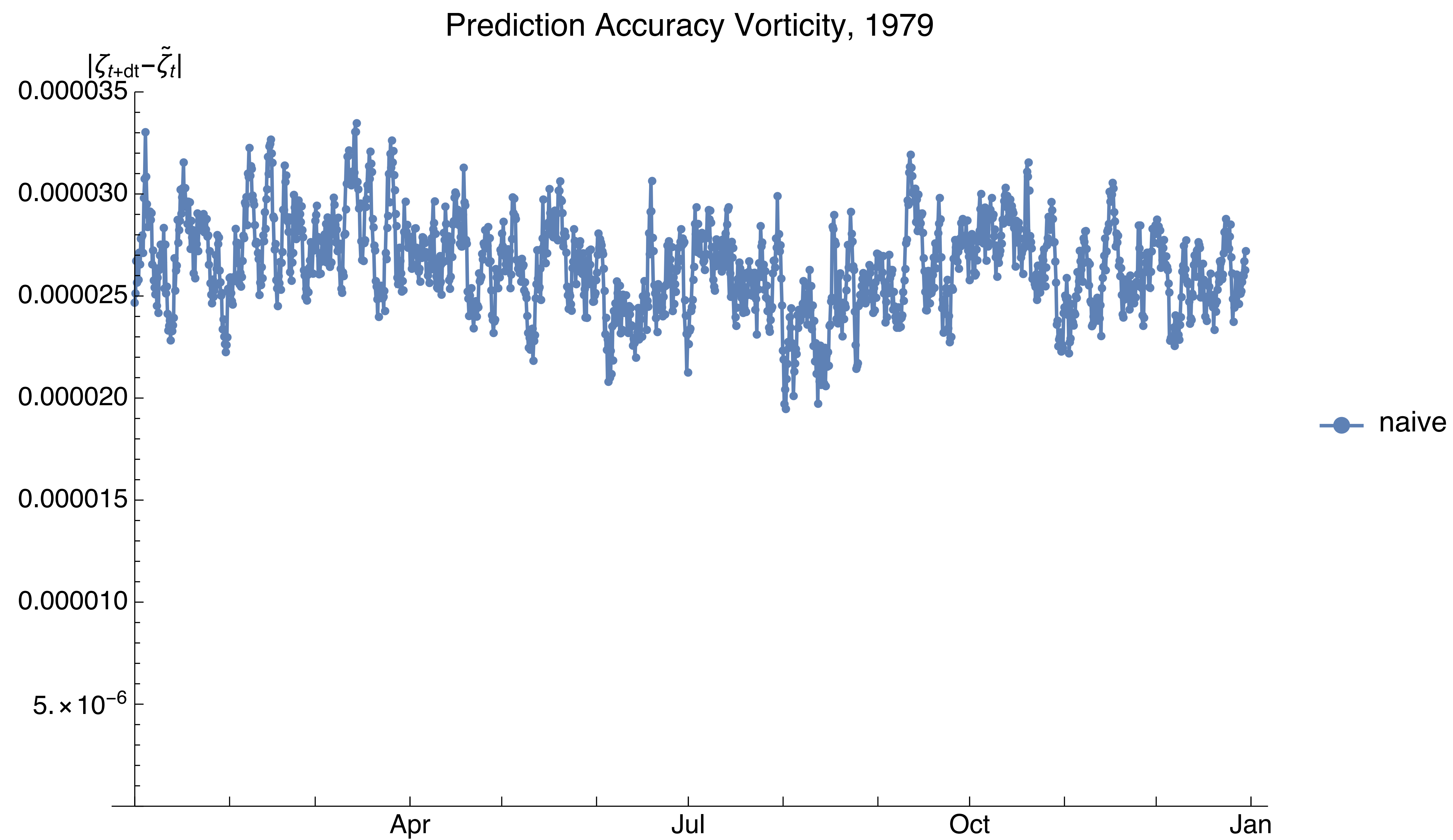
32 coefficients are affected



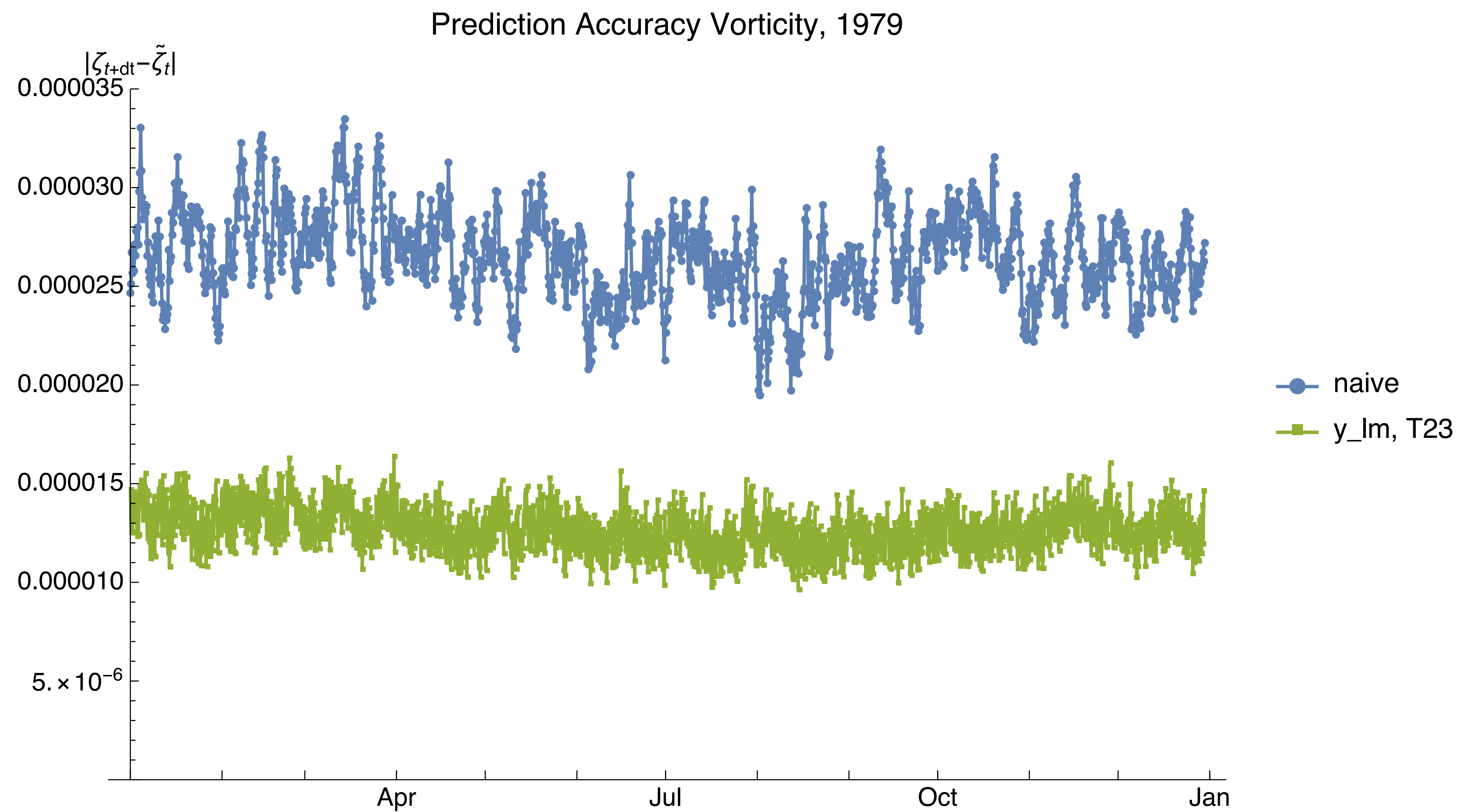
Status

- Shallow water equation
 - Completed most of standard test set
 - Completed basic forecasts experiments
- Primitive equations
 - Currently working on standard test for primitive equation

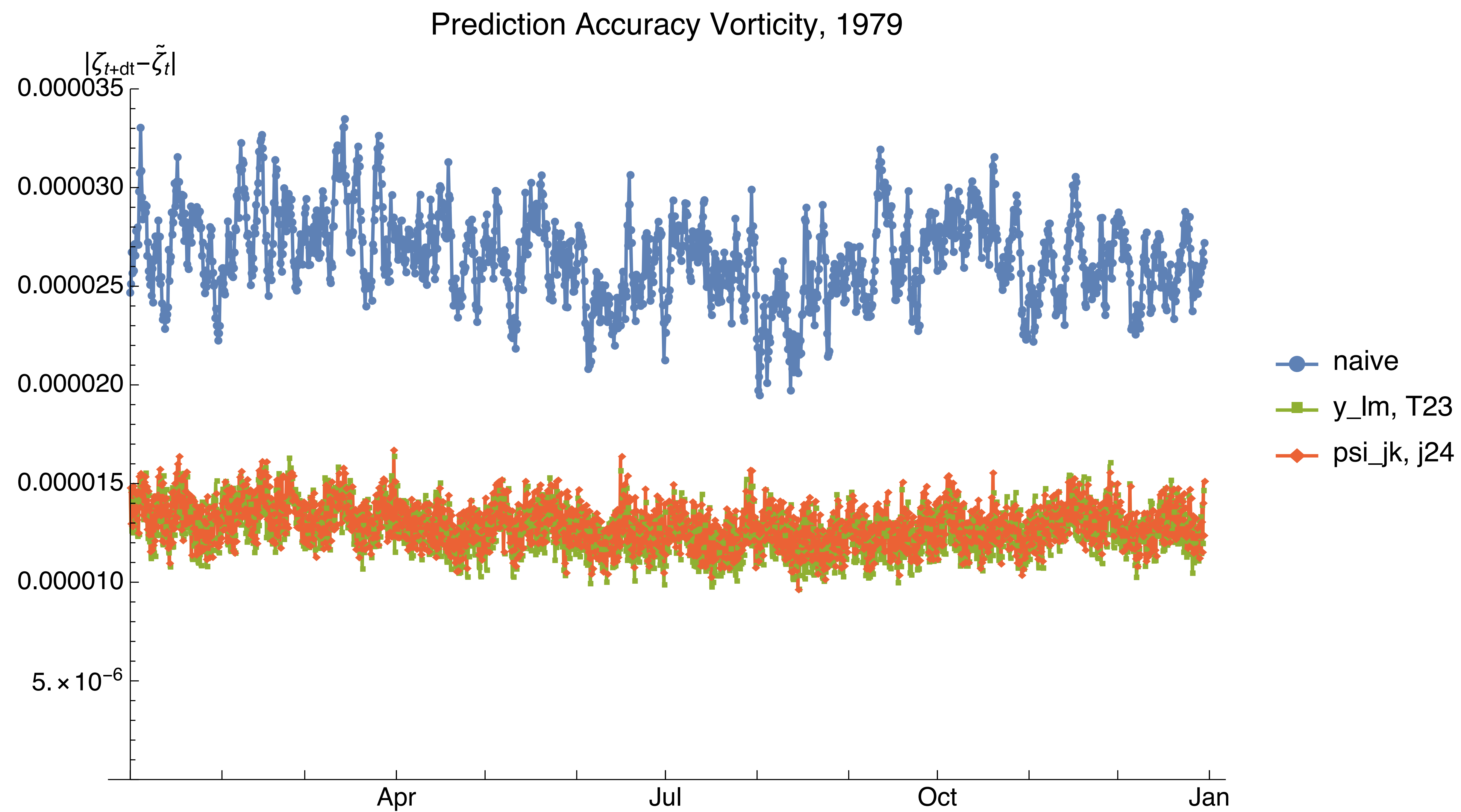
Results: shallow water equation



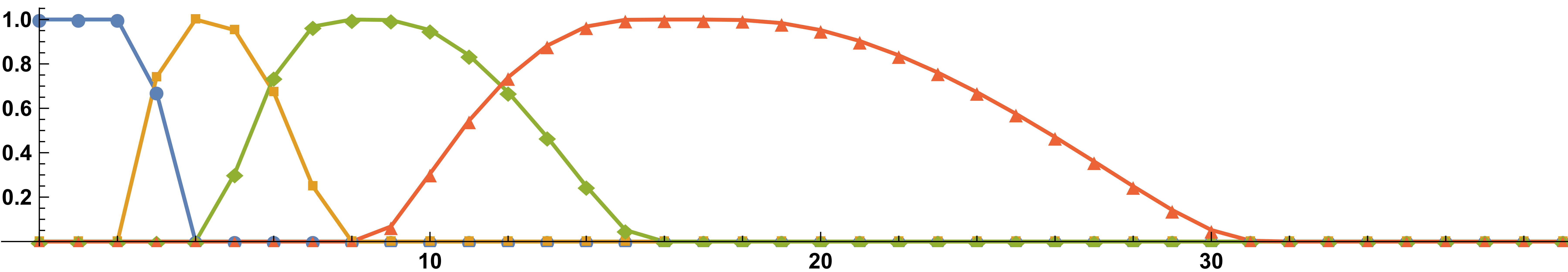
Results: shallow water equation



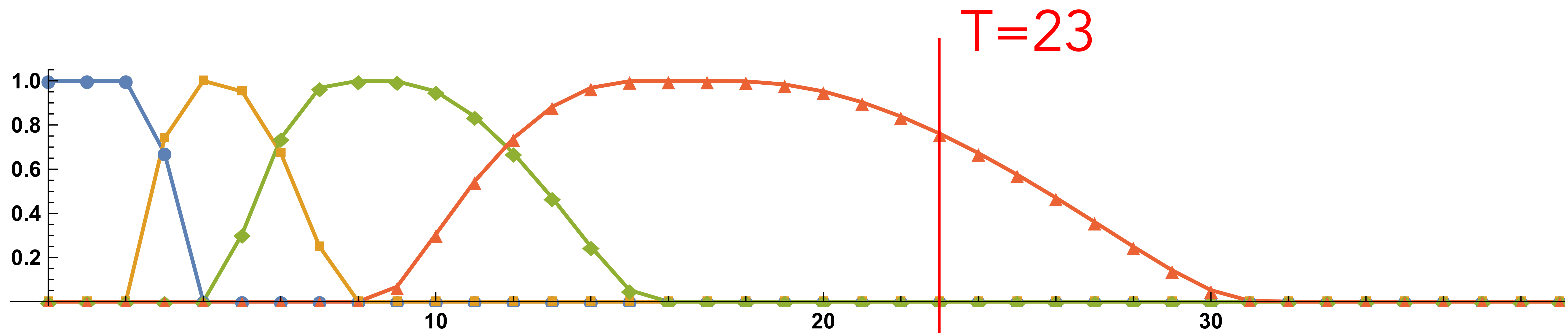
Results: shallow water equation



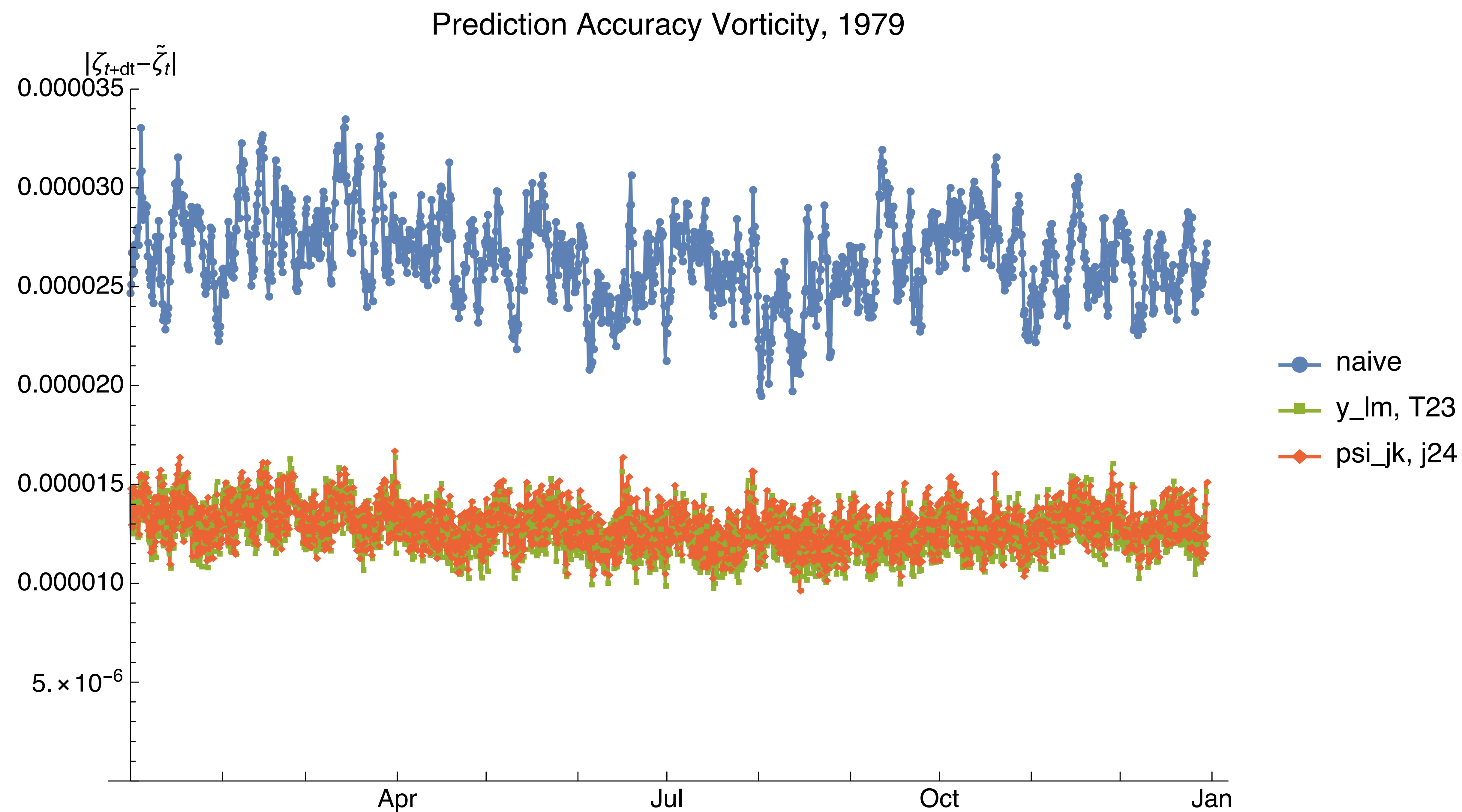
Results: shallow water equation



Results: shallow water equation



Results: shallow water equation



Data assimilation using deep learning

Data assimilation using deep learning

- Neural network:
 - Trained on reanalysis data with error to analytic prediction as objective function
 - Correct for model bias and account for physics and described by partial differential equations

Data assimilation using deep learning

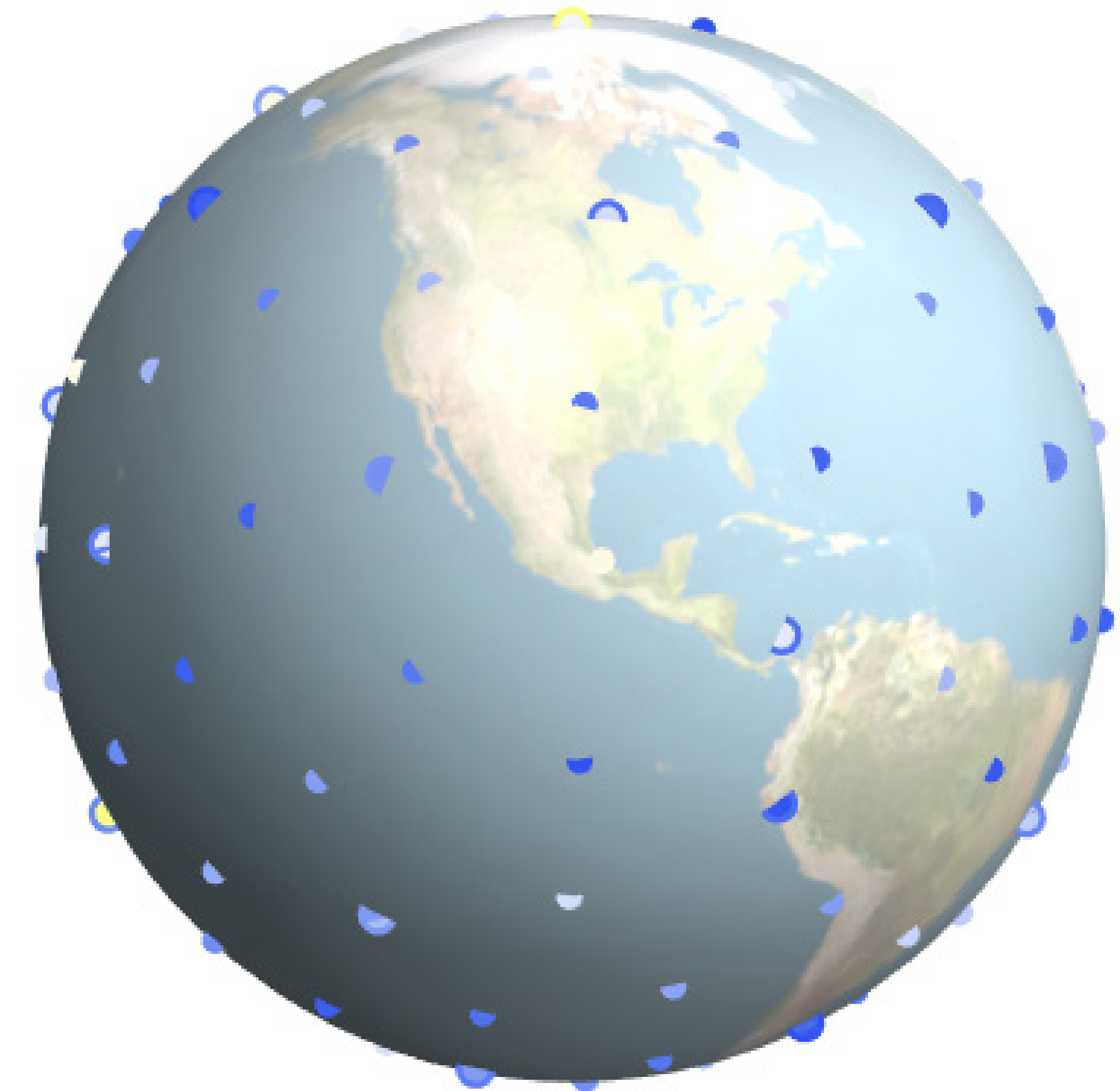
- Neural network:
 - Trained on reanalysis data with error to analytic prediction as objective function
 - Correct for model bias and account for physics and described by partial differential equations

How to make neural network aware of physics?
(and how much physics?)

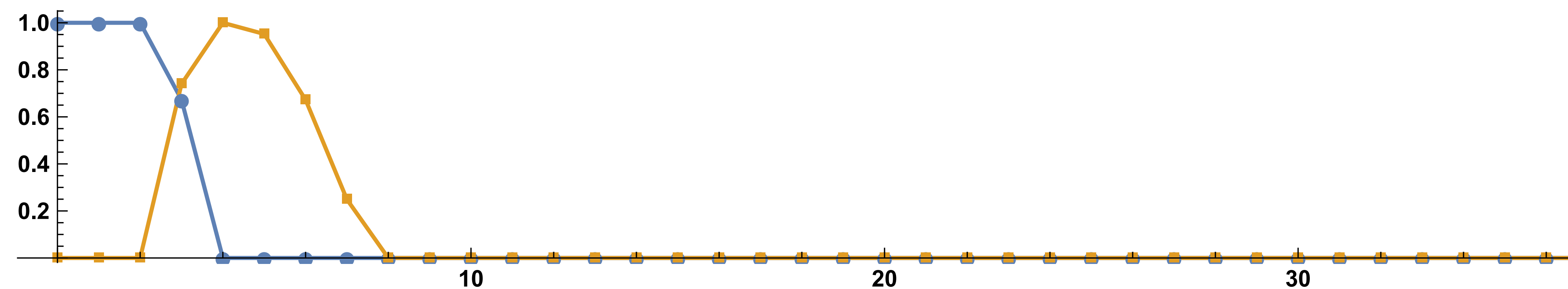
Data assimilation using deep learning

- Representation of vorticity:

$$\zeta = \sum_{jk} \zeta_{jk} \psi_{jk}$$



$j \leq 0$

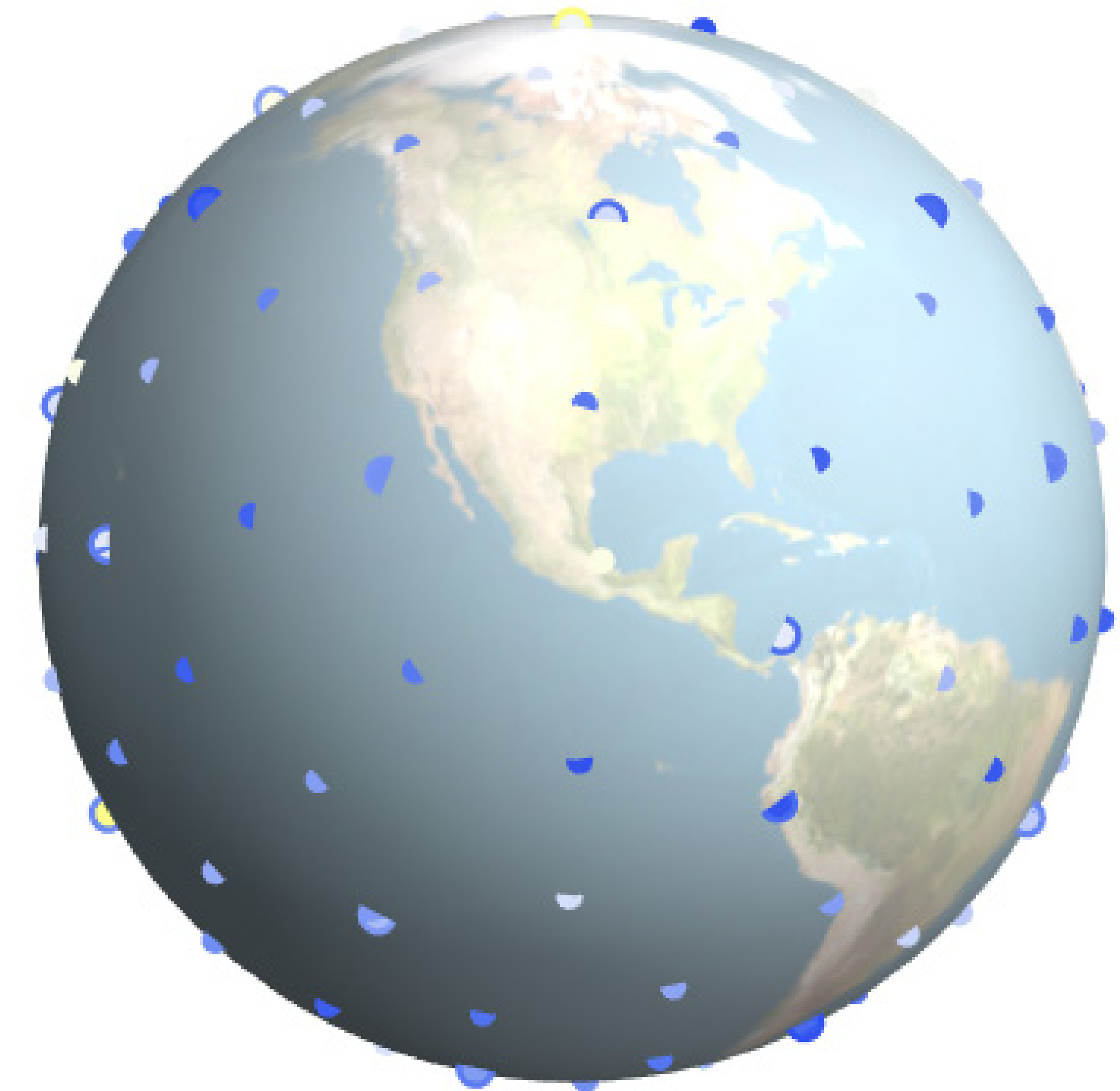
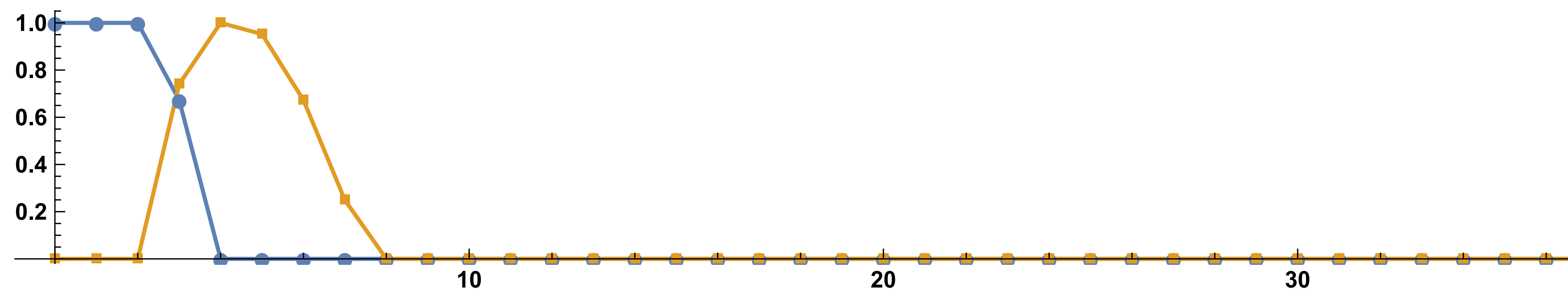


Data assimilation using deep learning

- Representation of vorticity:

$$\zeta = \sum_{jk} \zeta_{jk} \psi_{jk}$$

Train on basis
function
coefficients



$$j \leq 0$$

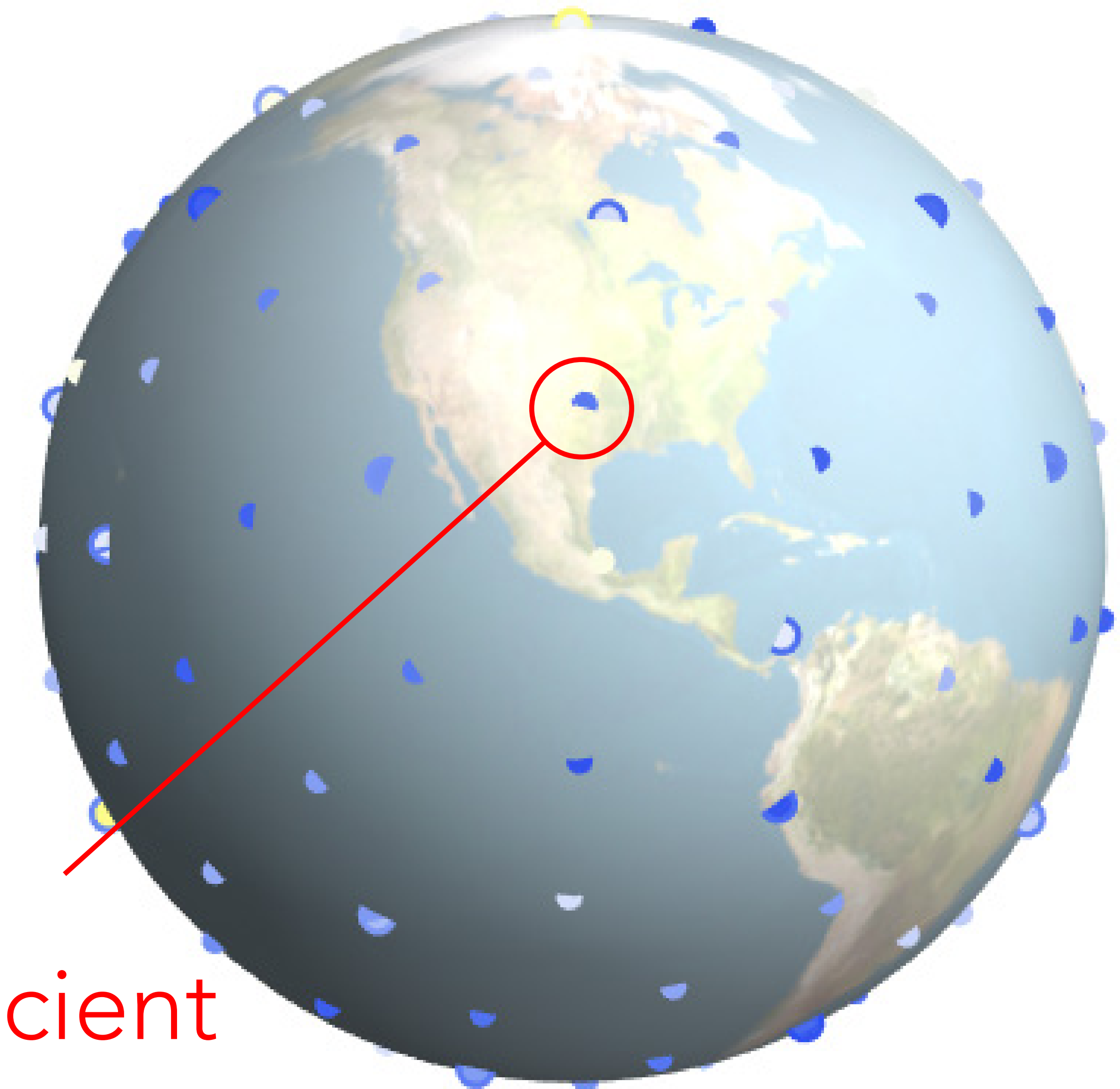
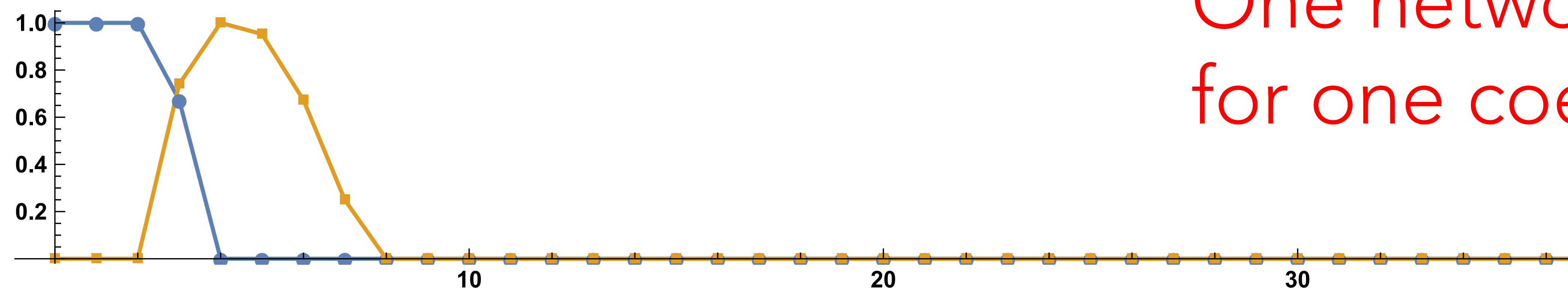
Data assimilation using deep learning

- Representation of vorticity:

$$\zeta = \sum_{jk} \zeta_{jk} \psi_{jk}$$

Train on basis
function
coefficients

One network
for one coefficient



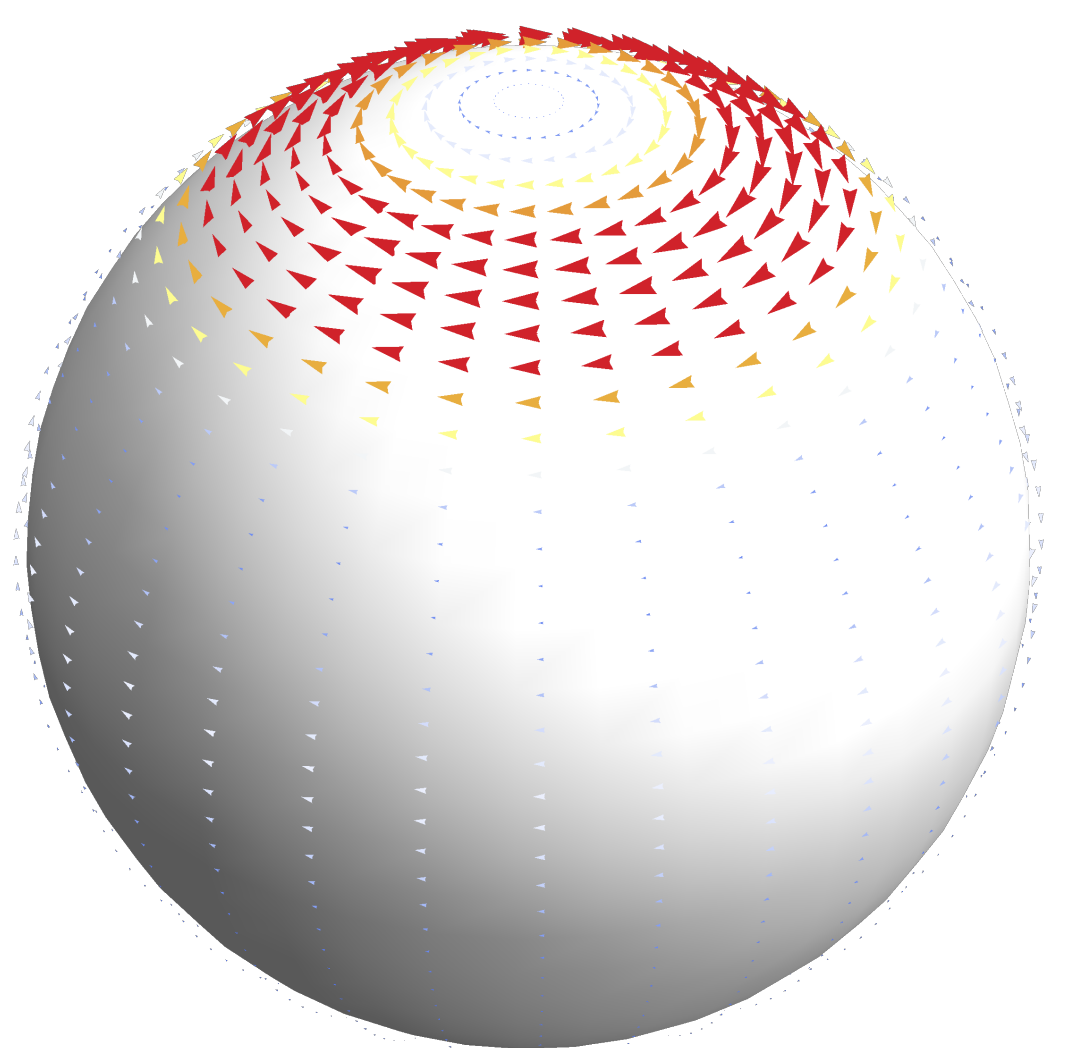
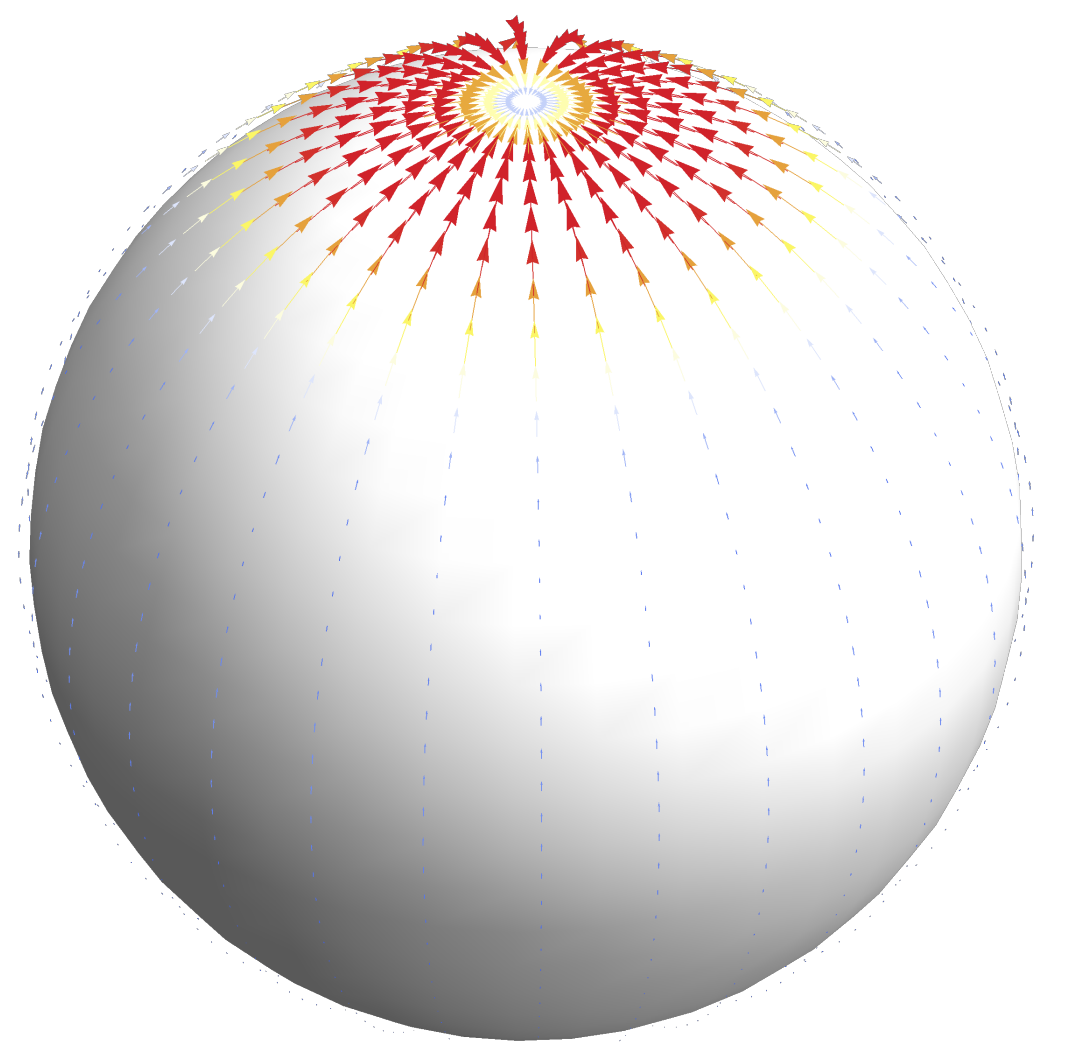
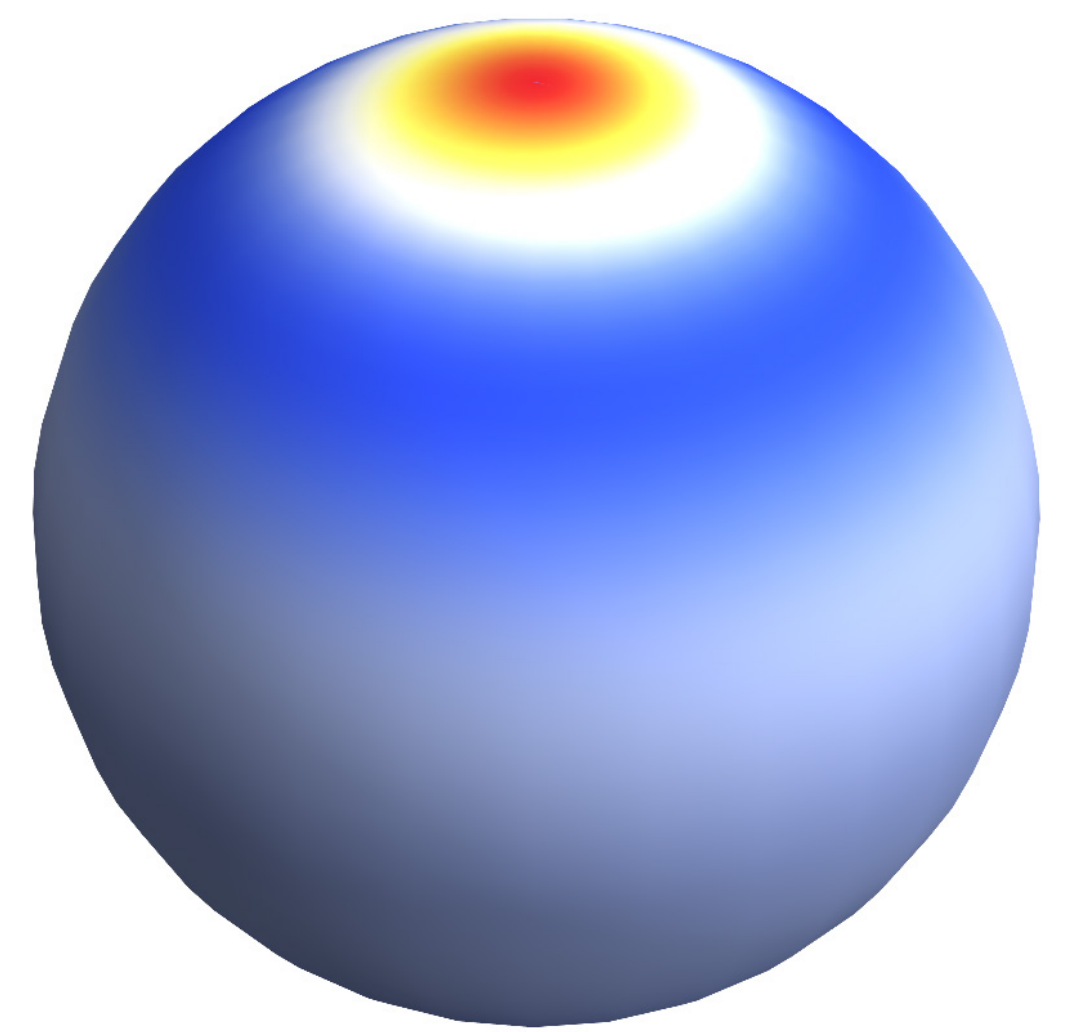
$j \leq 0$

Future work

- More and more thorough experiments
 - Long time forecasts, analysis of multiscale structure
- Performance improvement
 - E.g. fast wavelet transform, parallelization
- Anisotropic wavelets
- Bandlimited vs. compactly supported wavelets
- Structure preserving time integrator

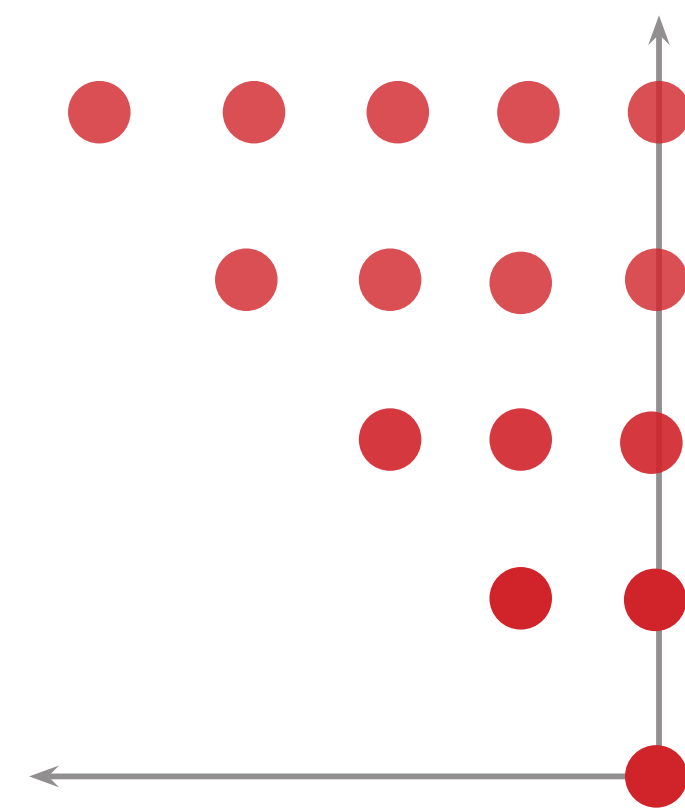
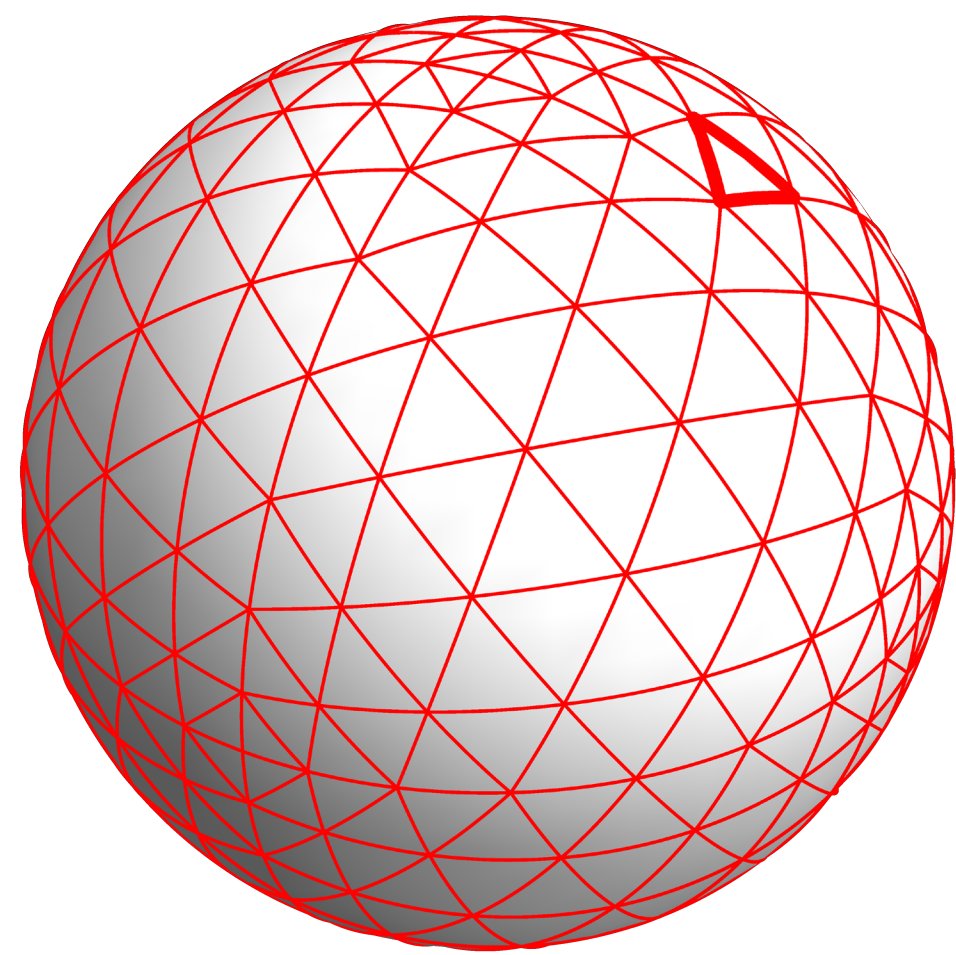
Conclusion

- Wavelet representation for prognostic fields

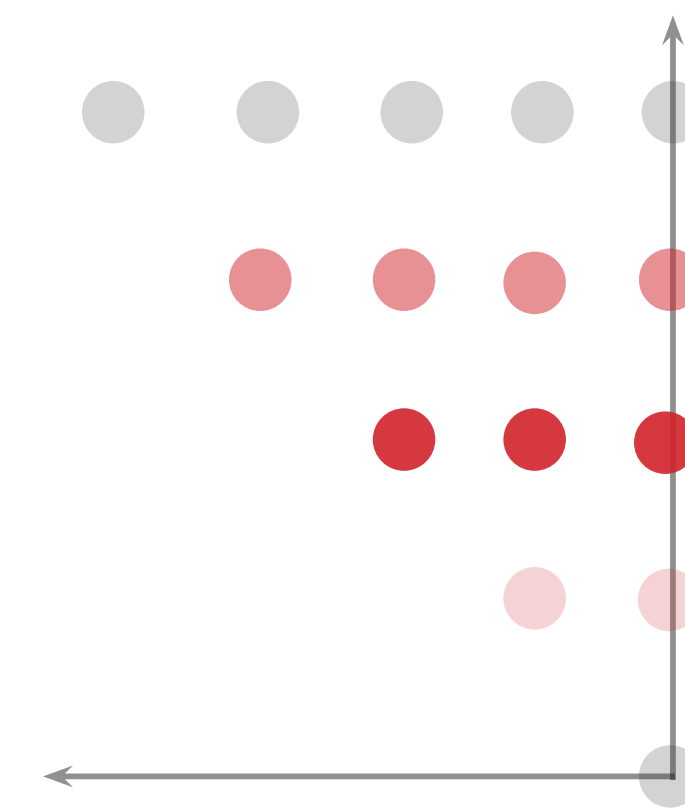
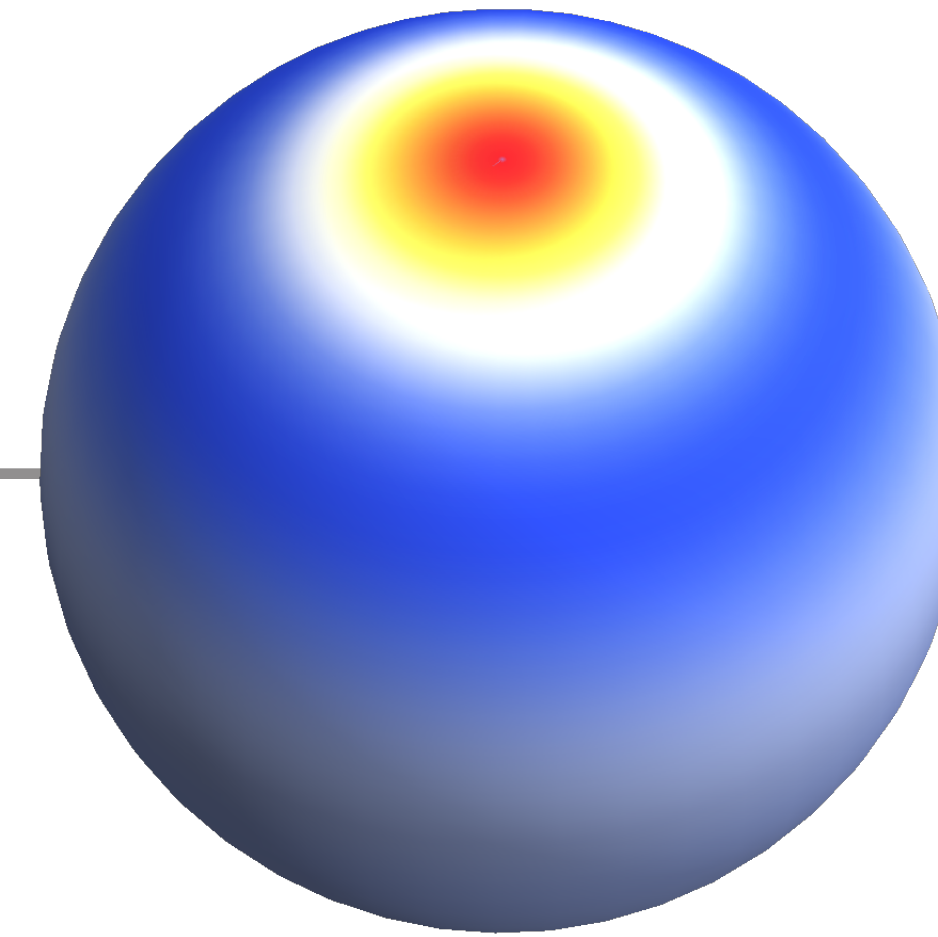


Conclusion

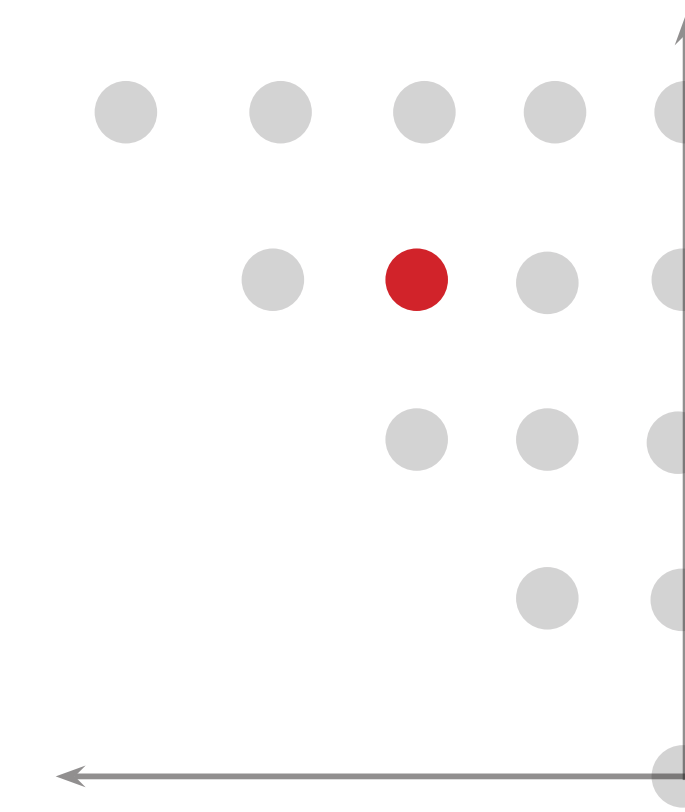
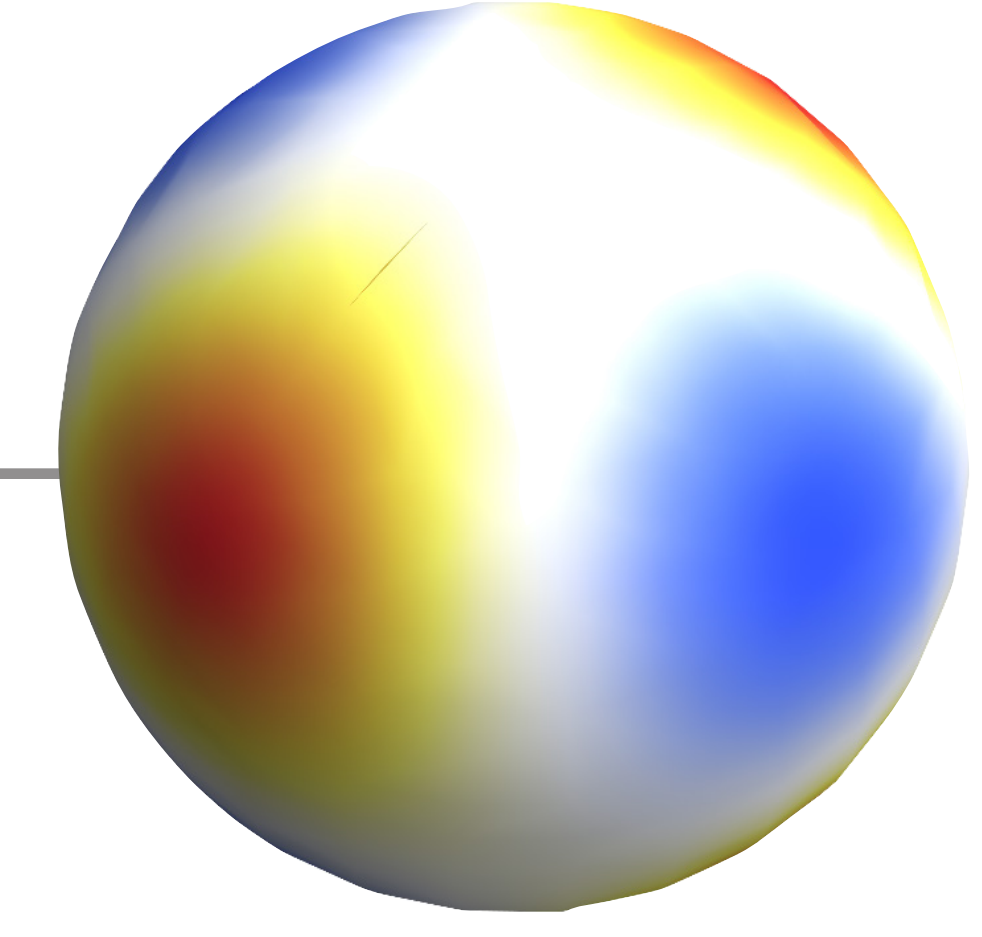
Finite element
type models



Local spectral
model

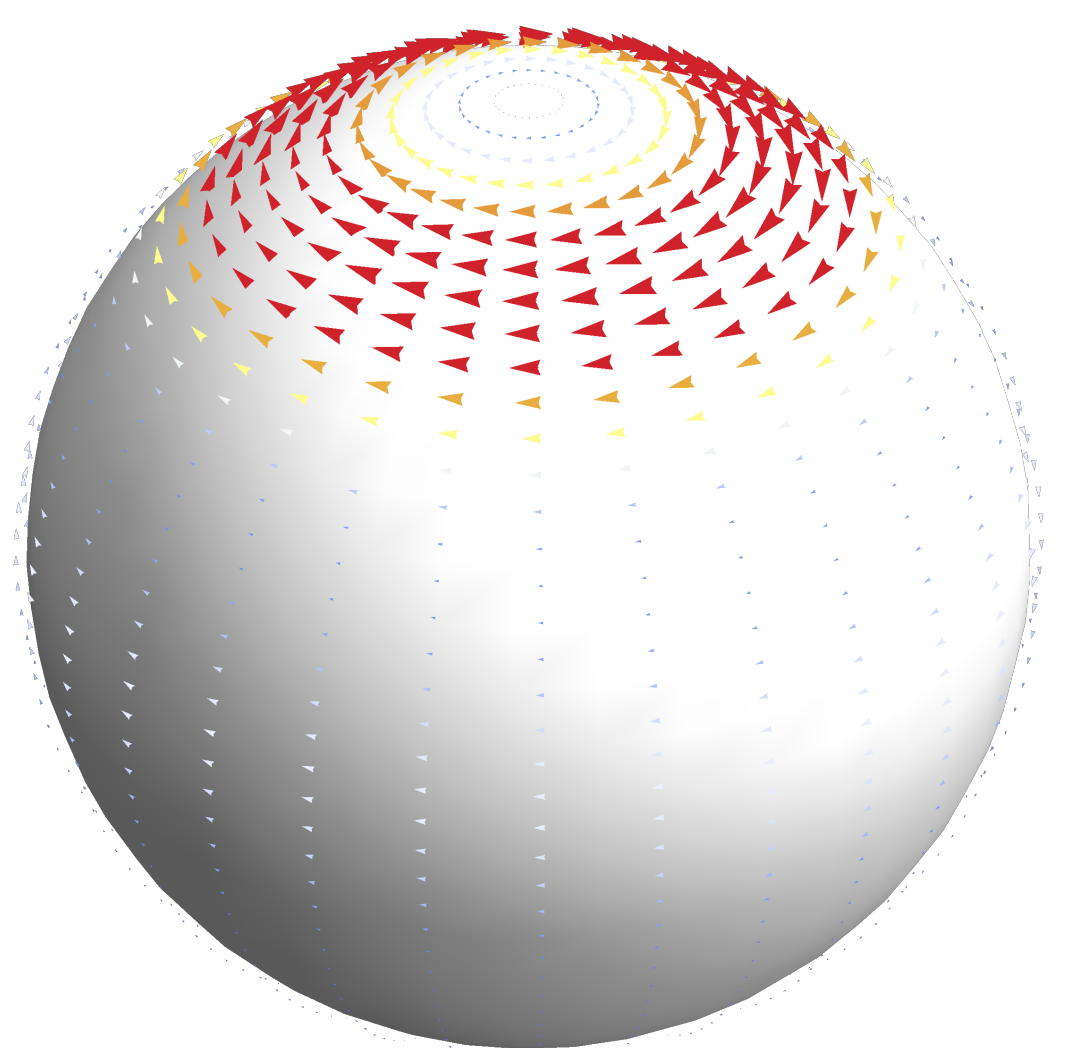
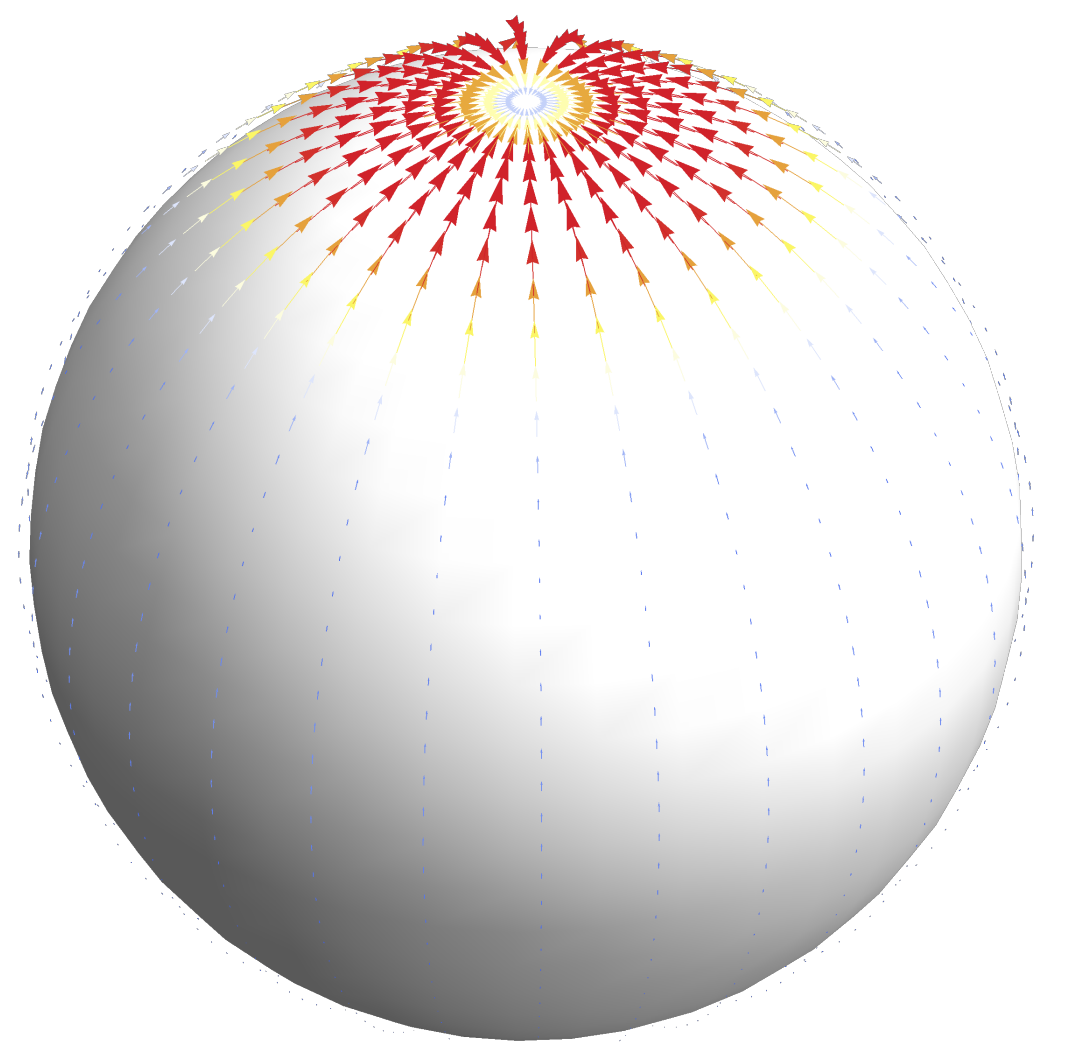
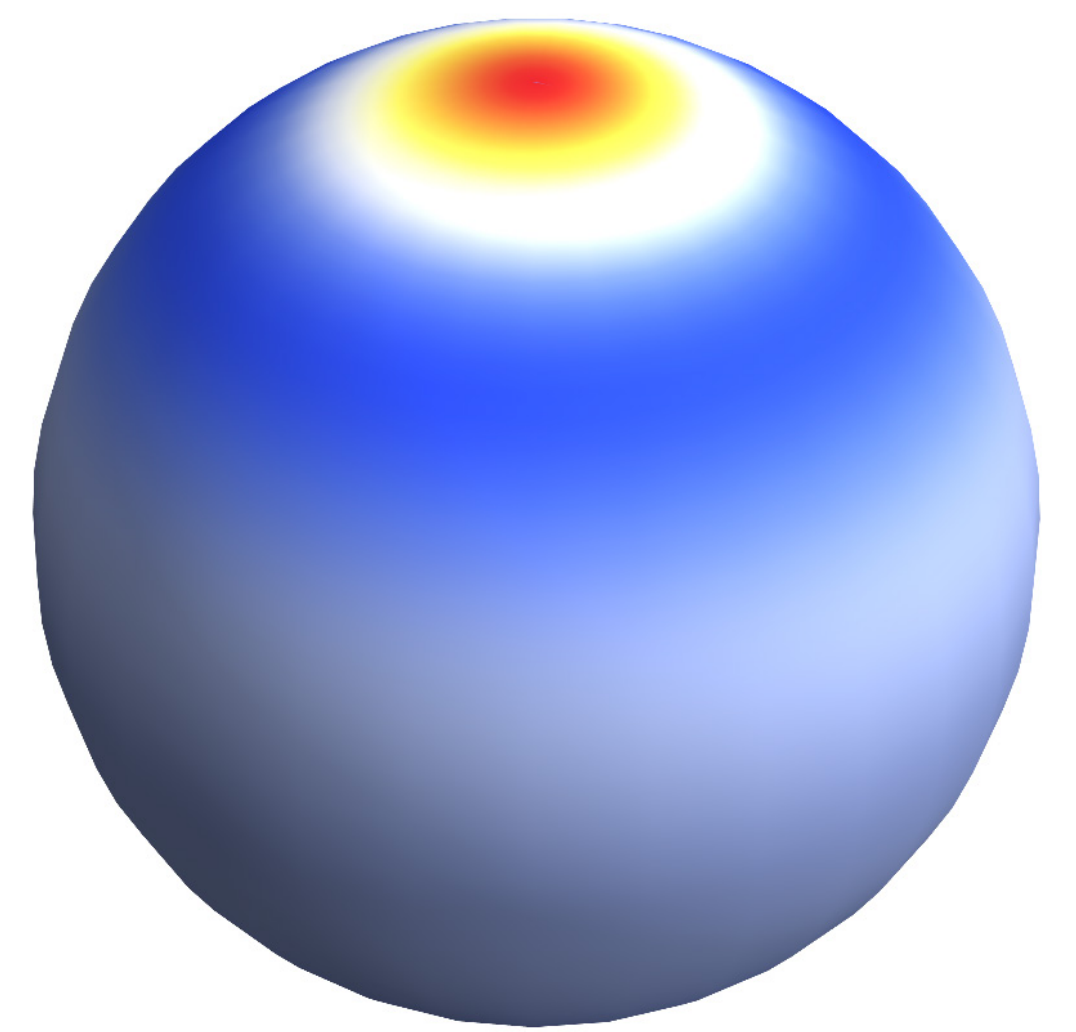


Spectral
models



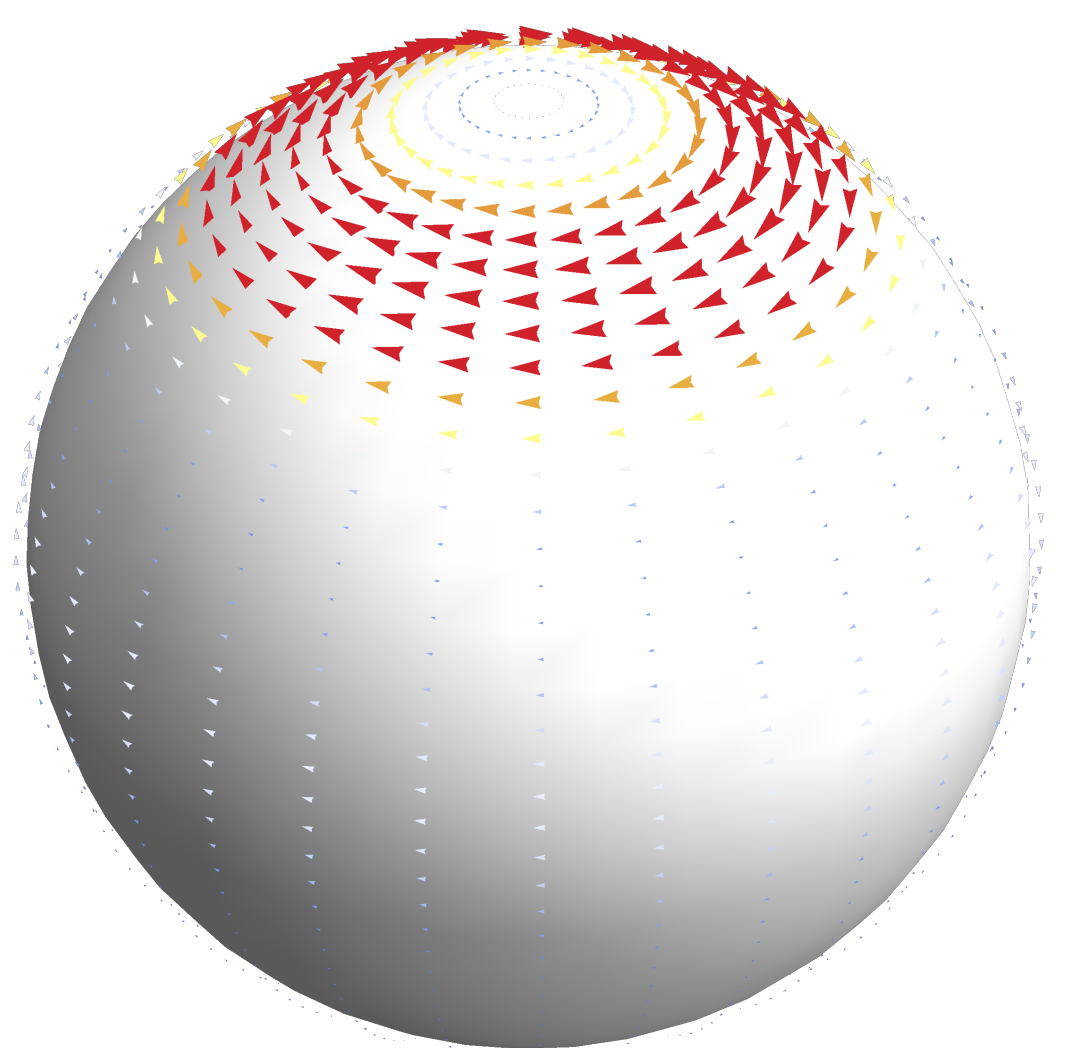
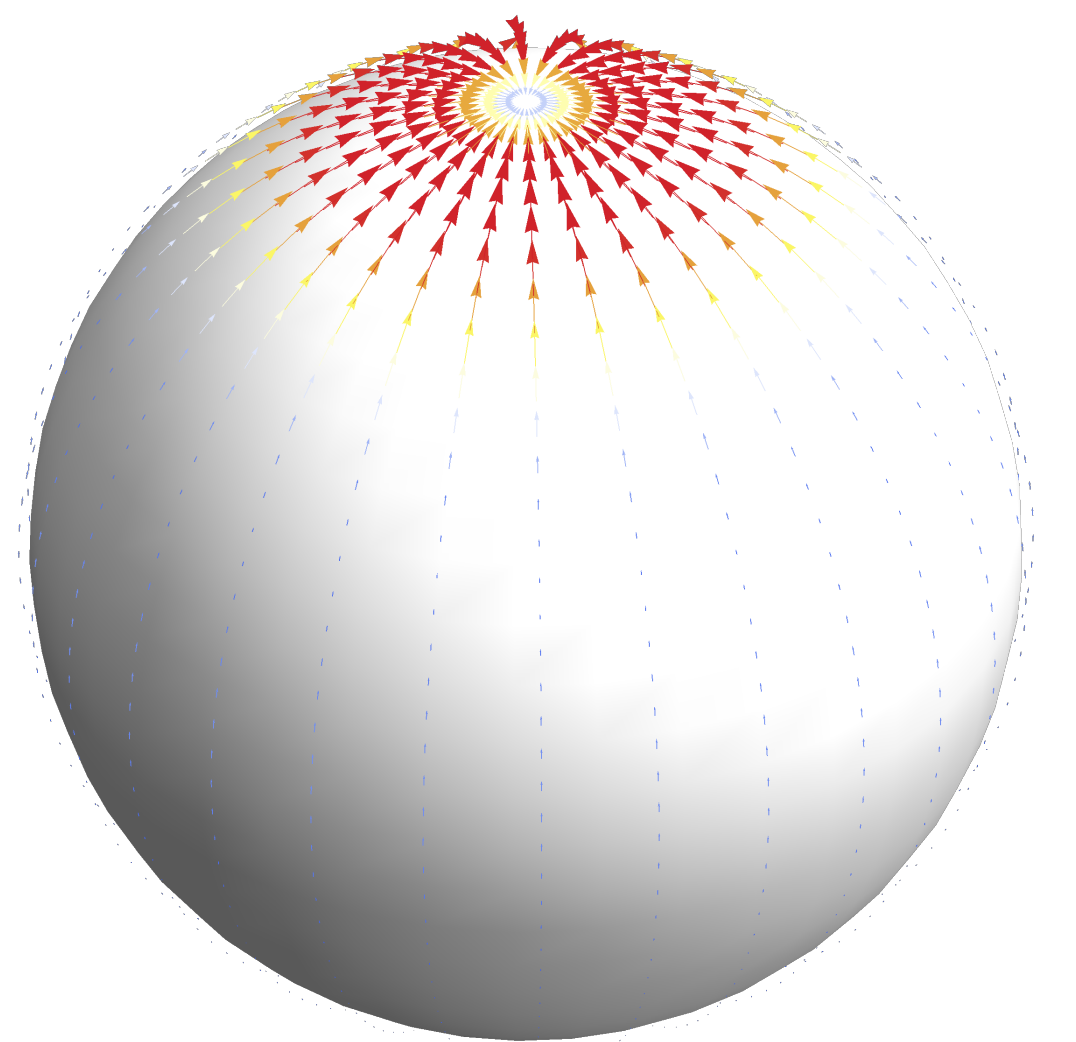
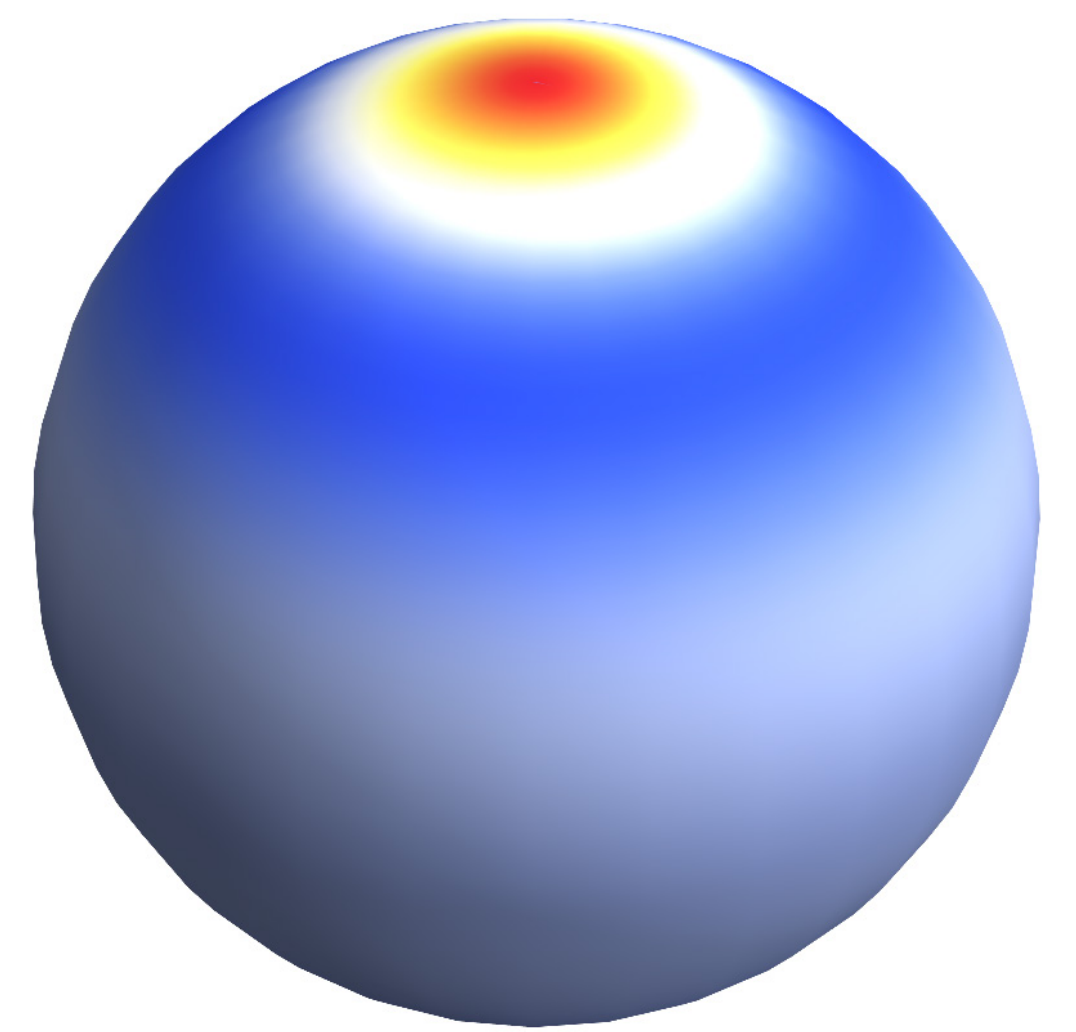
Conclusion

- Wavelet representation for prognostic fields
 - Well localized in space and frequency
 - Vector-valued wavelets for velocity
 - Discretization of exterior calculus



Conclusion

- Wavelet representation for prognostic fields
 - Well localized in space and frequency
 - Vector-valued wavelets for velocity
 - Discretization of exterior calculus
- Well-adapted for data assimilation using deep learning



Slides available:
[http://graphics.cs.uni-magdeburg.de/
talks/noaa.pdf](http://graphics.cs.uni-magdeburg.de/talks/noaa.pdf)

