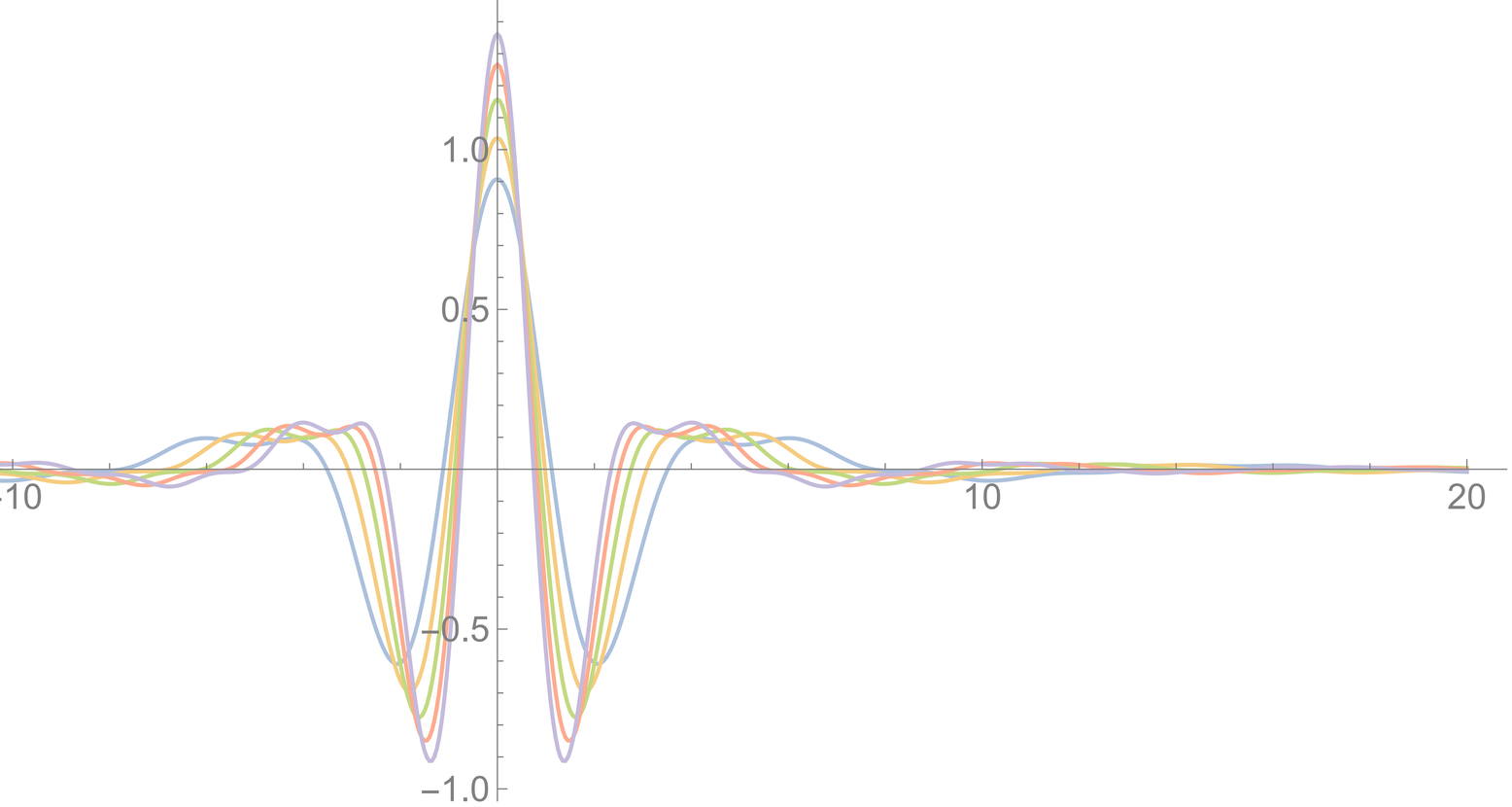
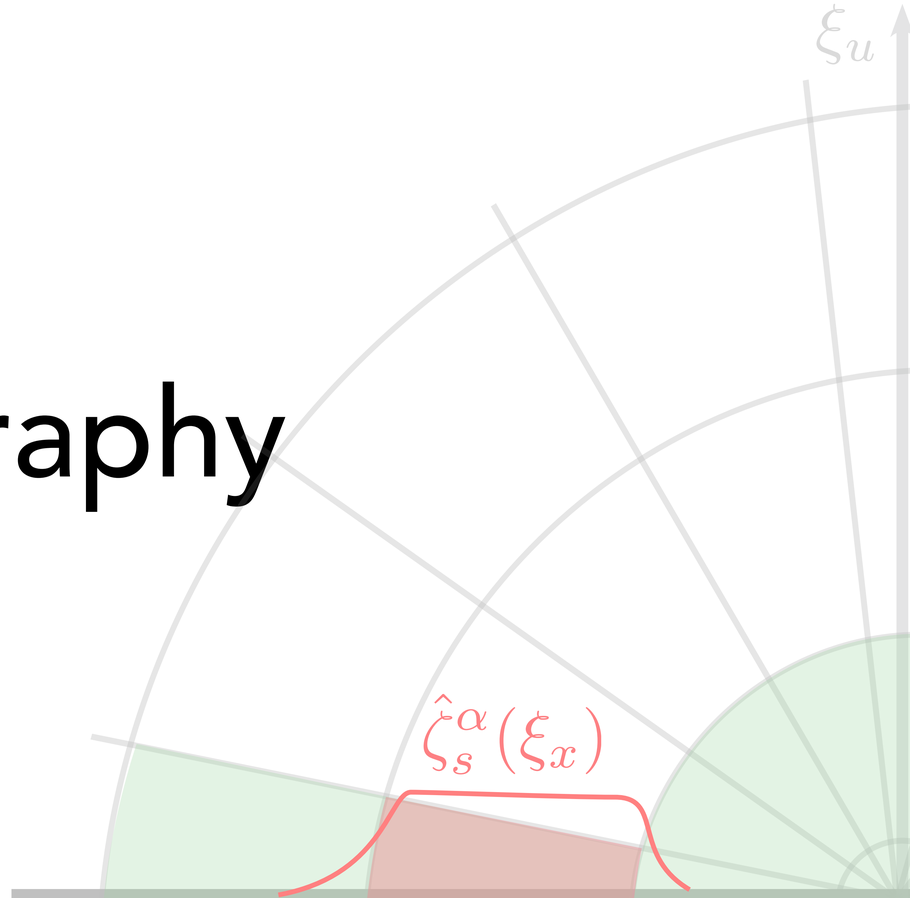


Local Fourier Slice Photography

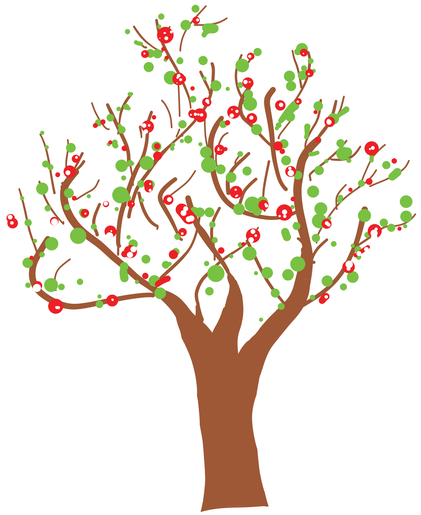
Christian Lessig

Institute for Simulation and Graphics, Otto-von-Guericke-Universität Magdeburg

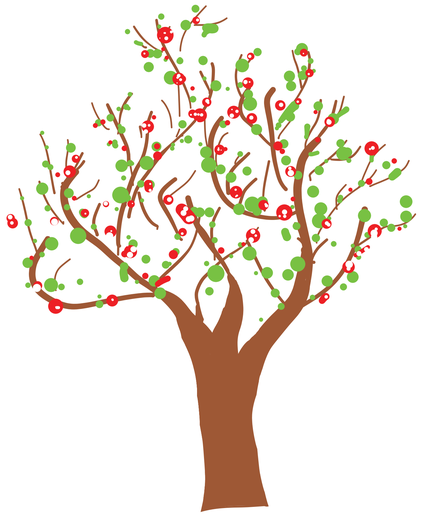


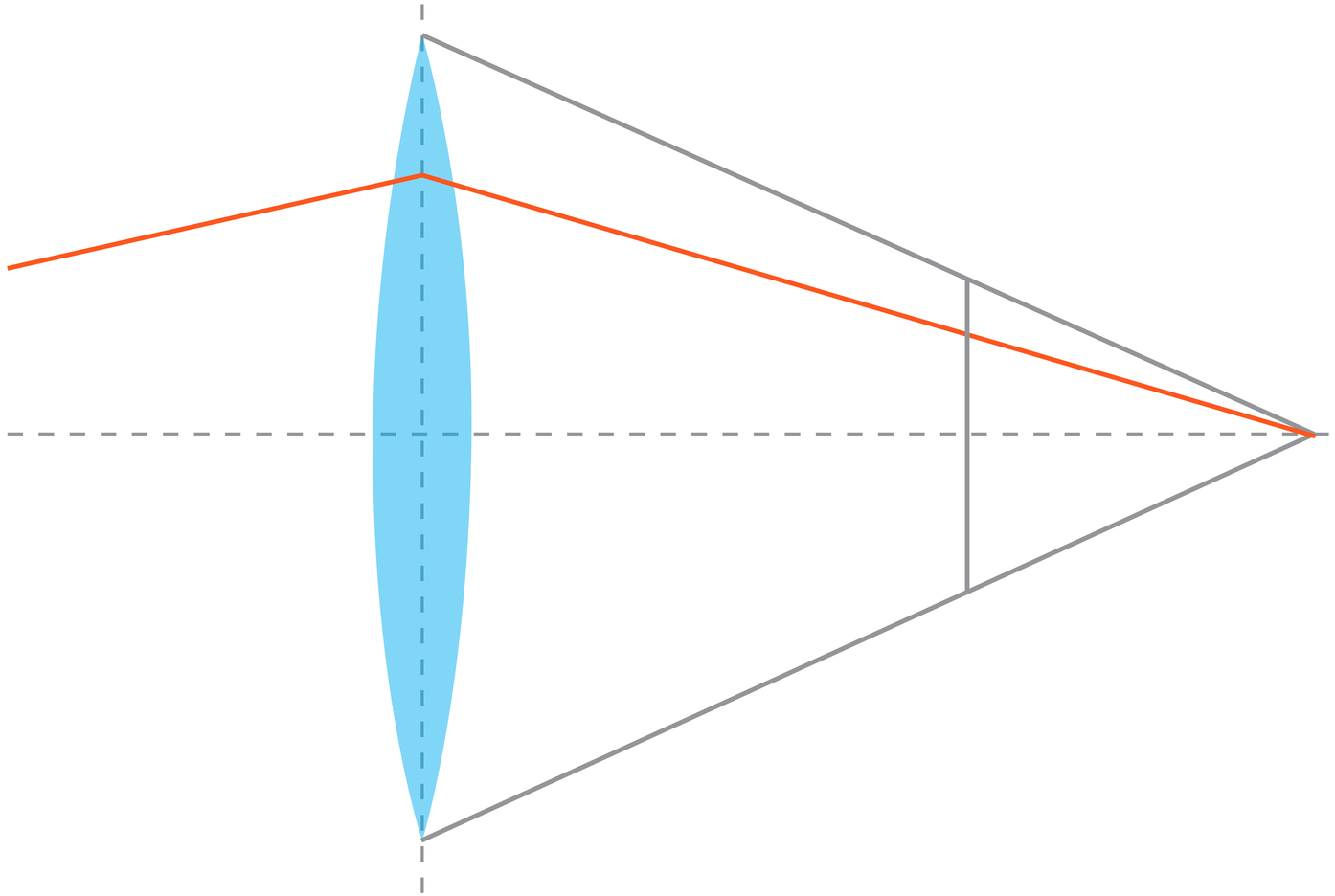
C. Lessig. Local Fourier Slice Photography. ACM Trans. Graph., 39(3), Apr. 2020

Light field refocusing

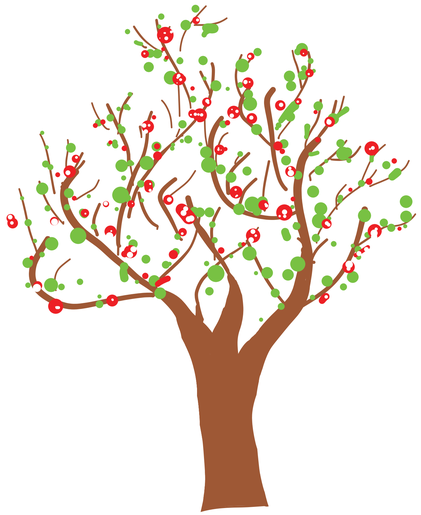


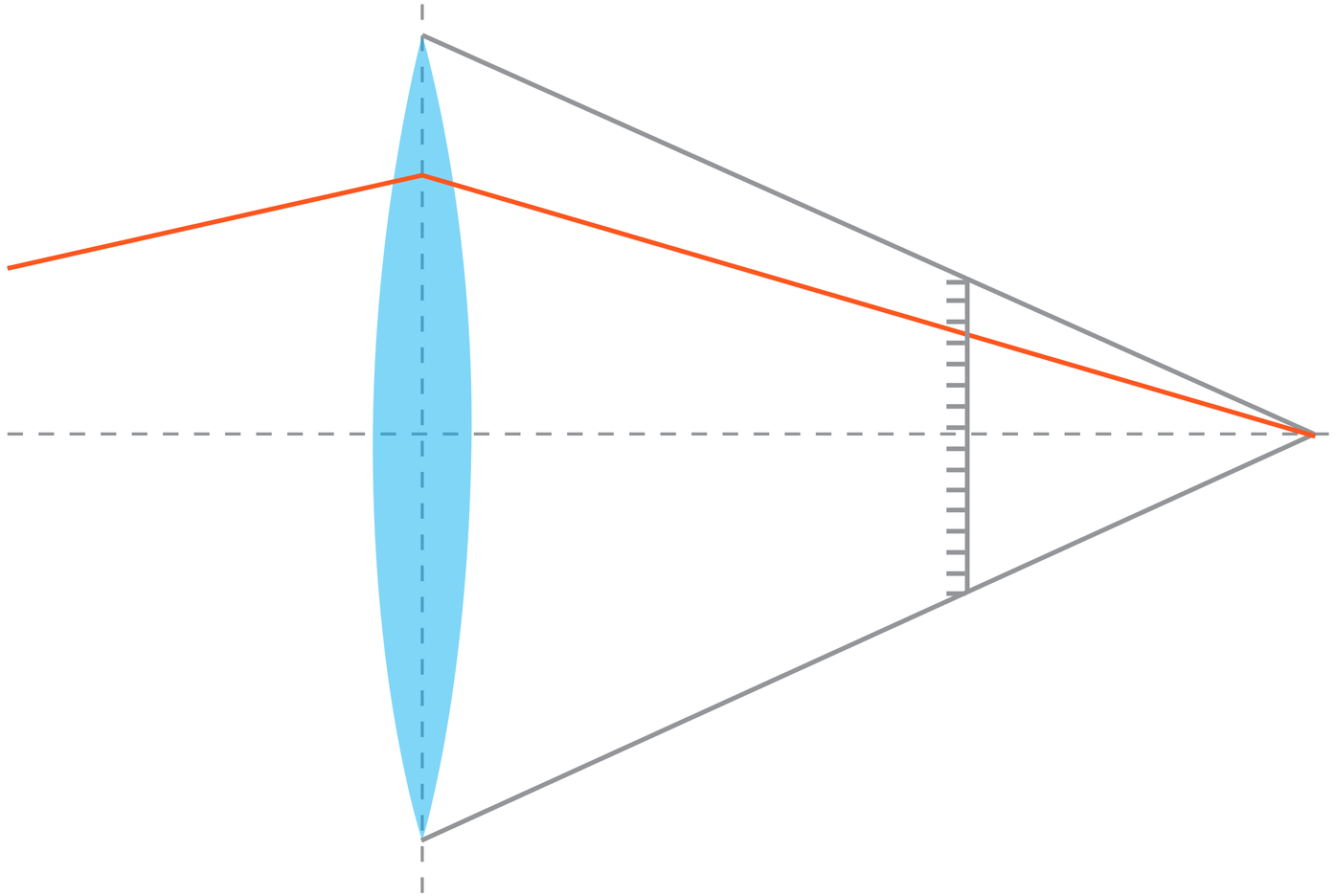
Light field refocusing





Light field refocusing





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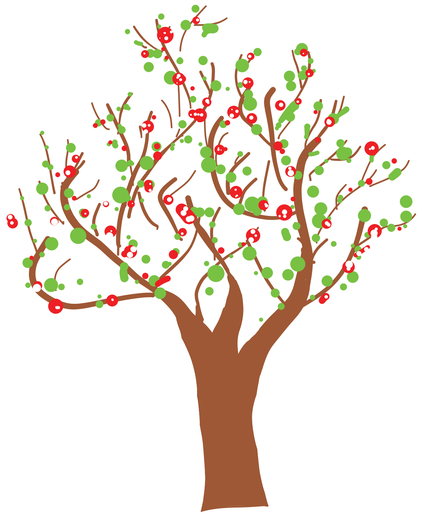
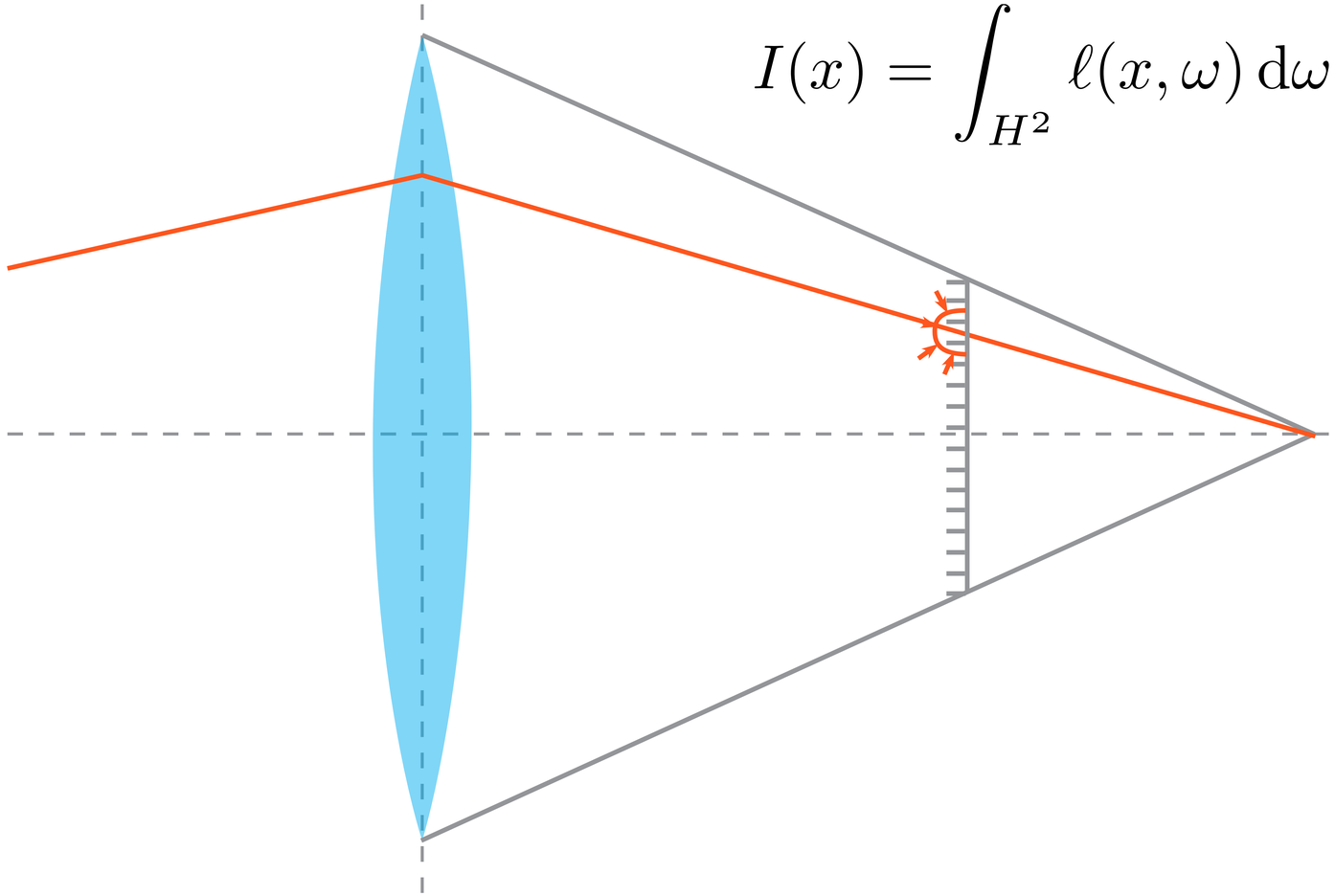
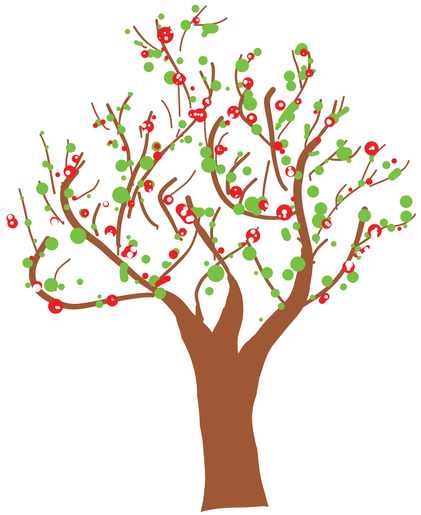
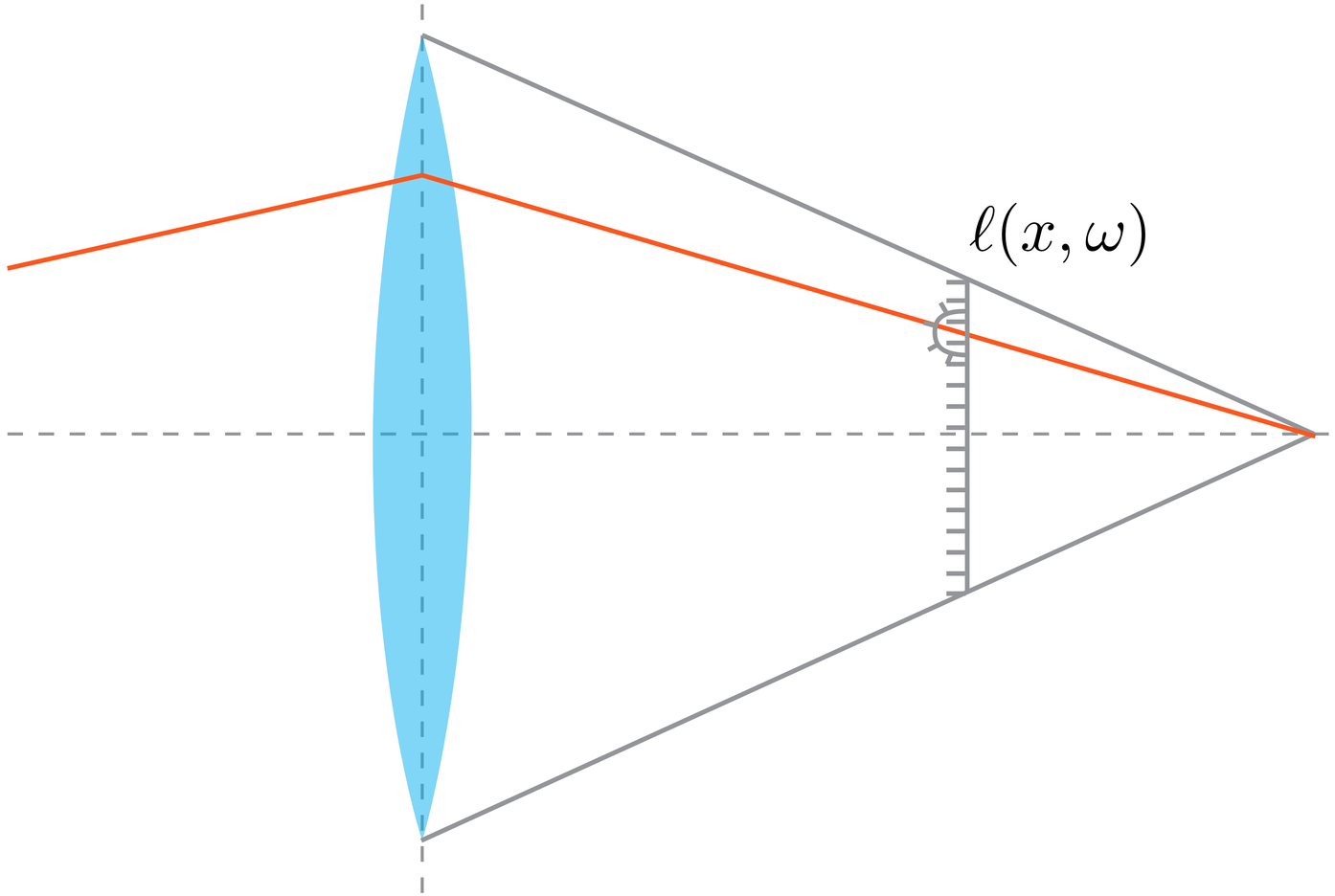




$$I(x) = \int_{H^2} \ell(x, \omega)\, \mathrm{d}\omega$$

Light field refocusing





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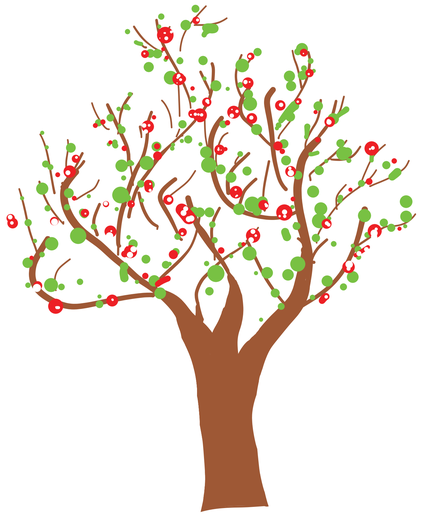
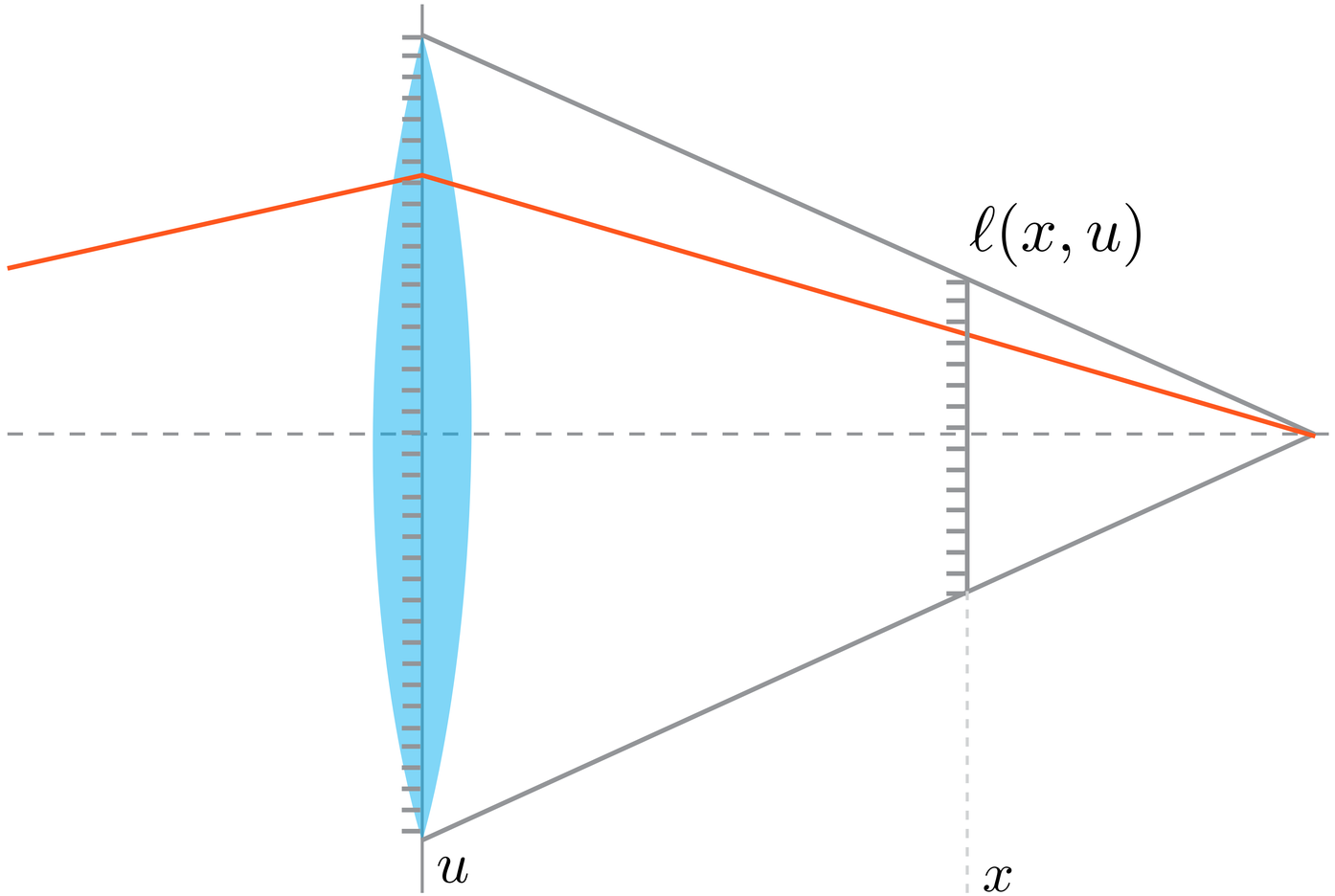
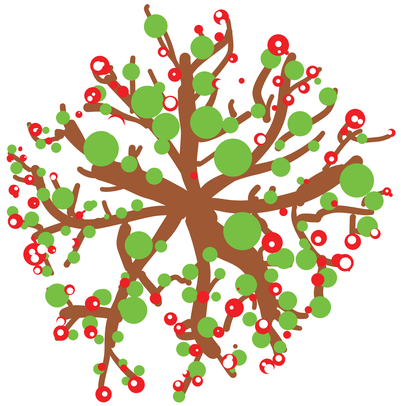
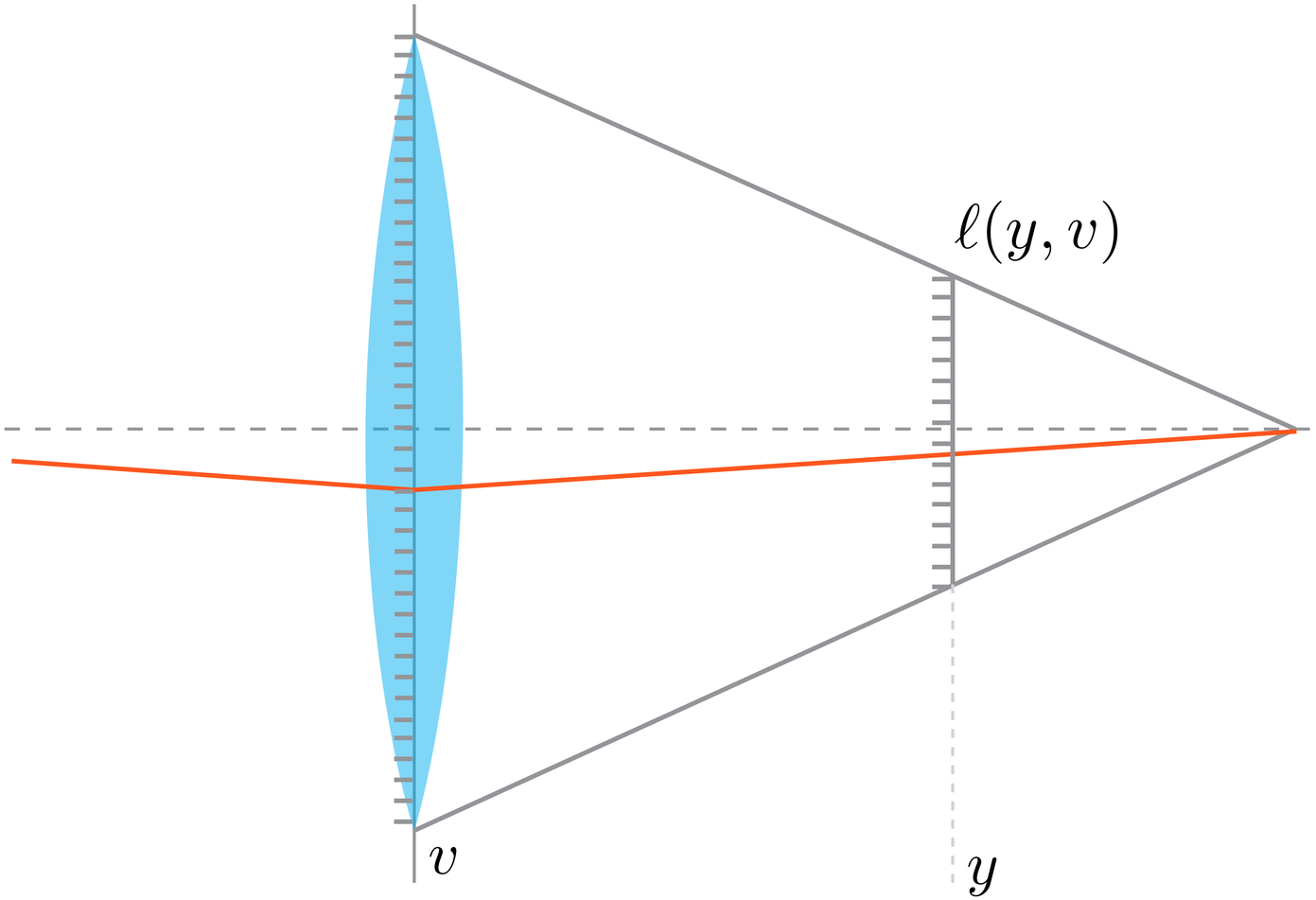


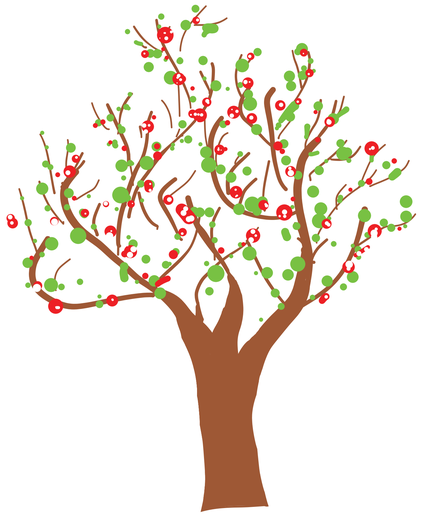


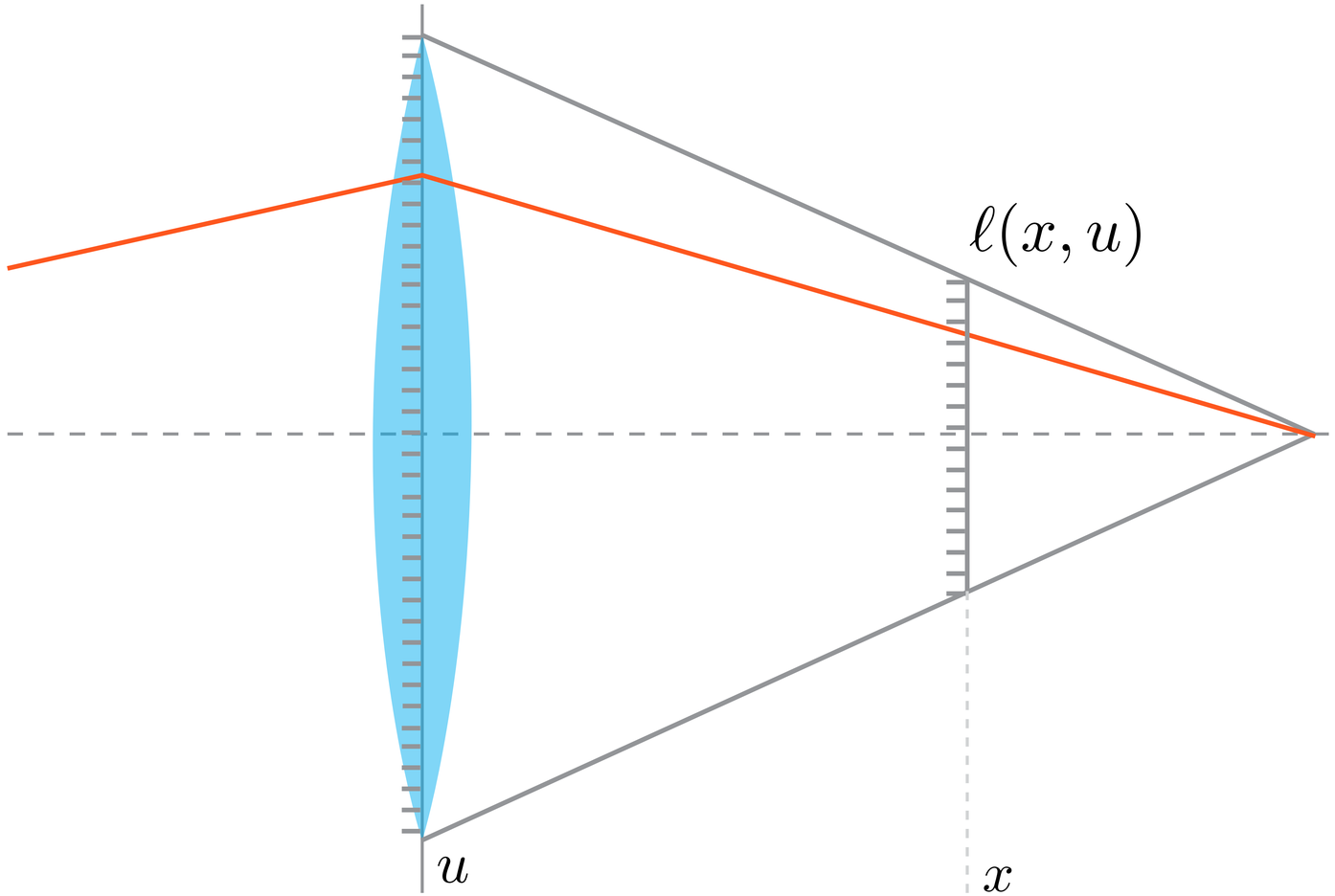
Light field refocusing



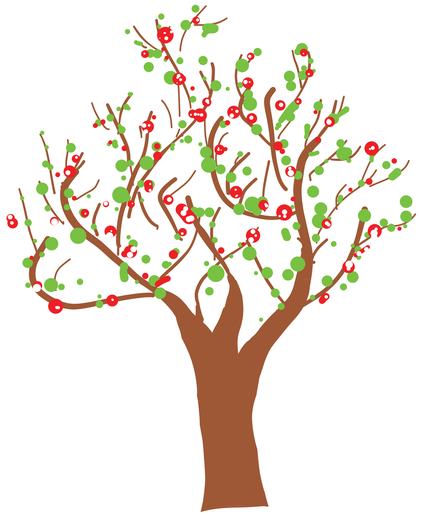


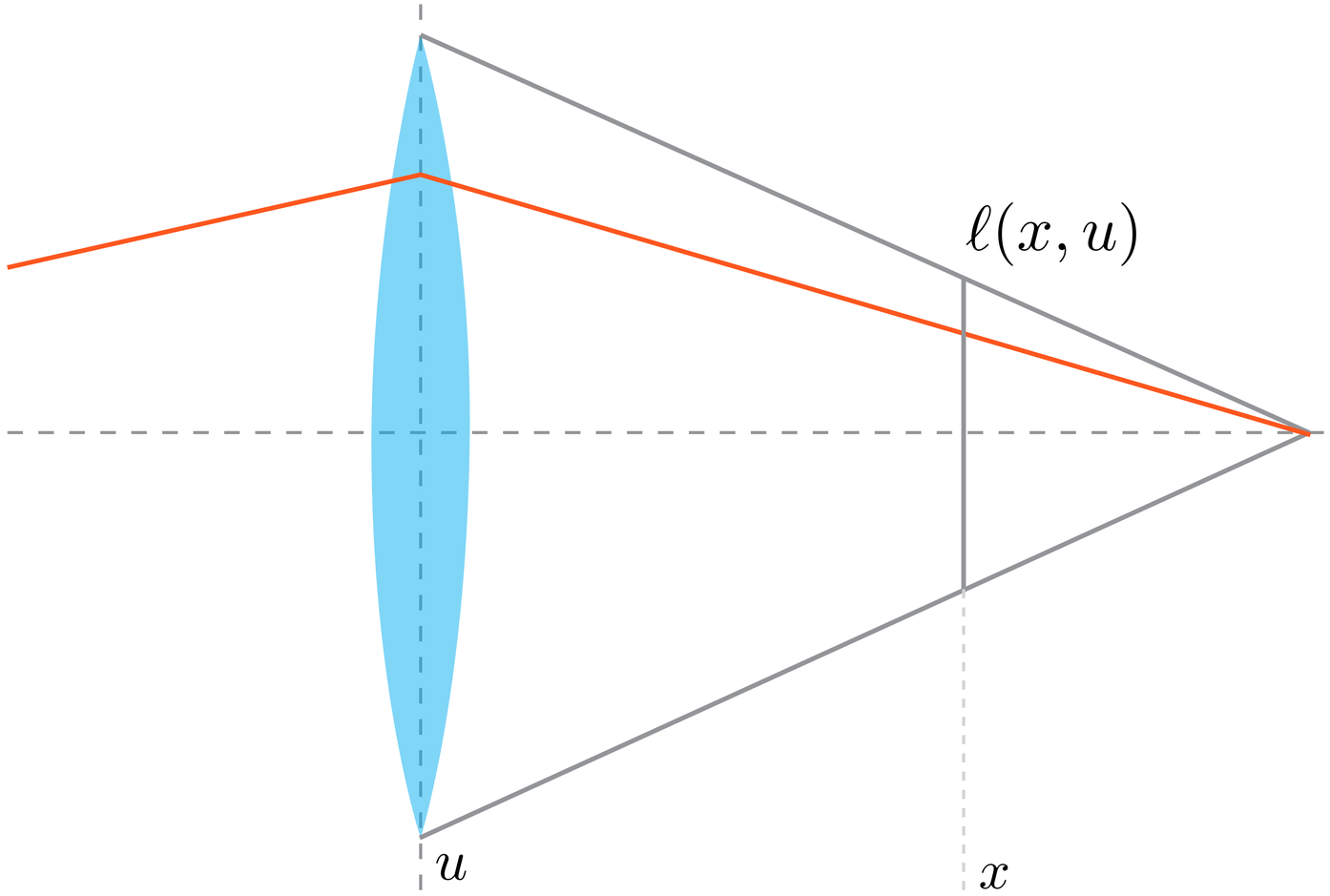
Light field refocusing



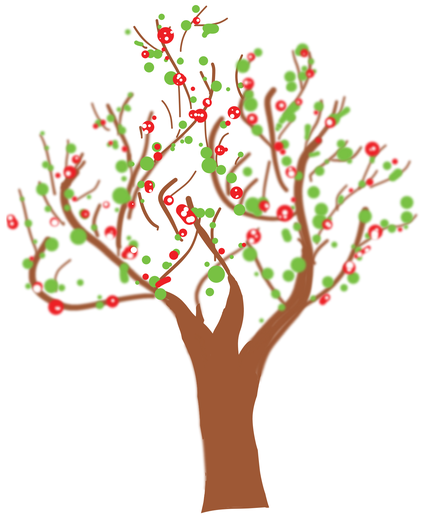


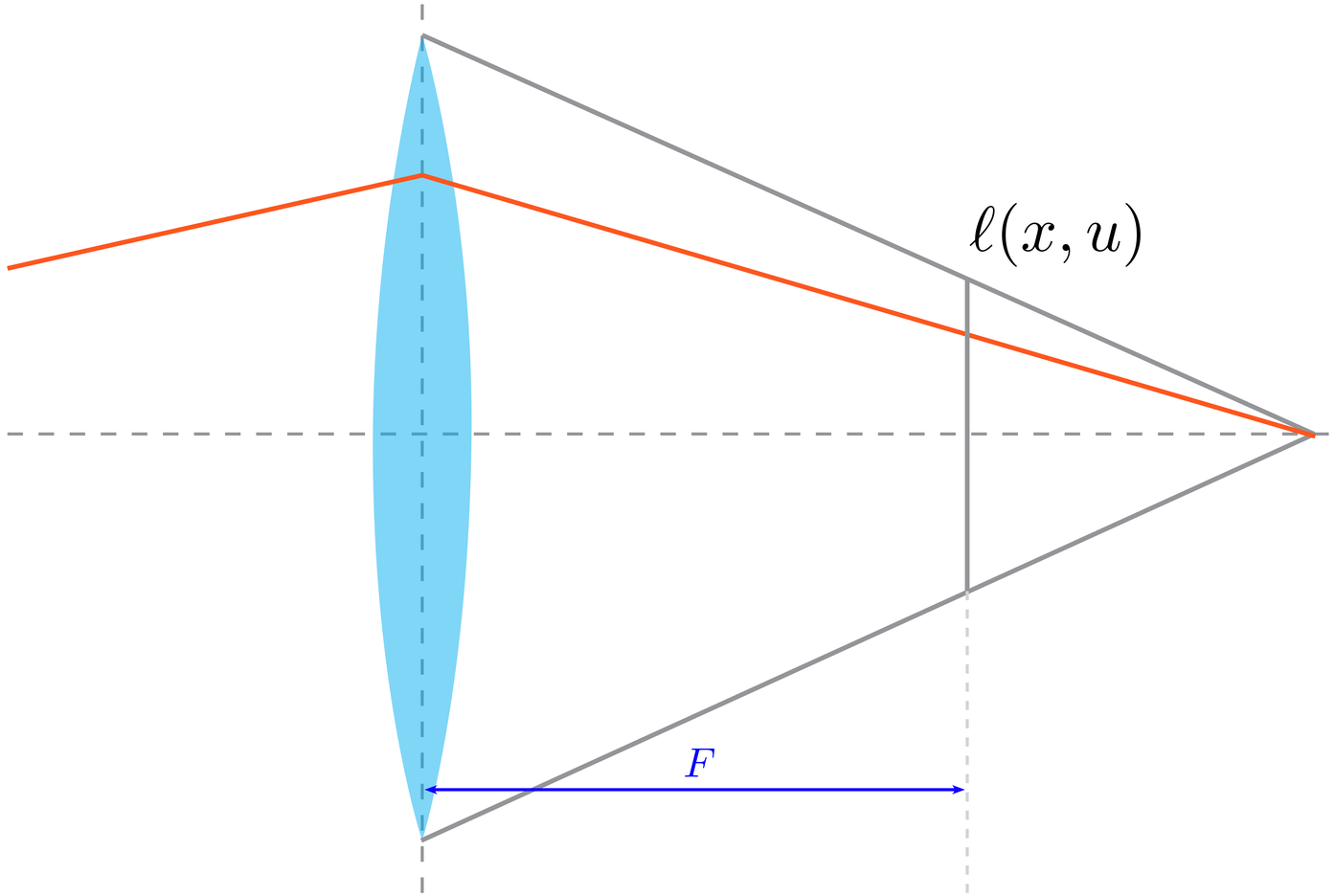
Light field refocusing





Light field refocusing





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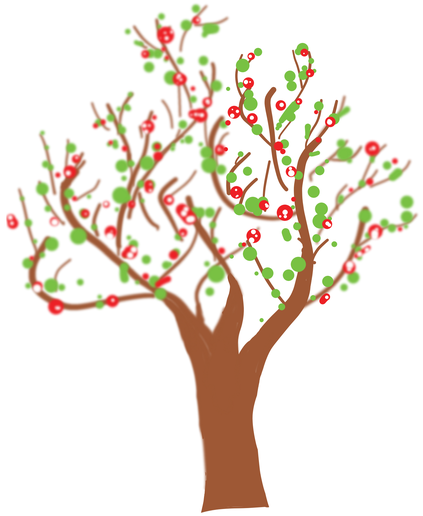

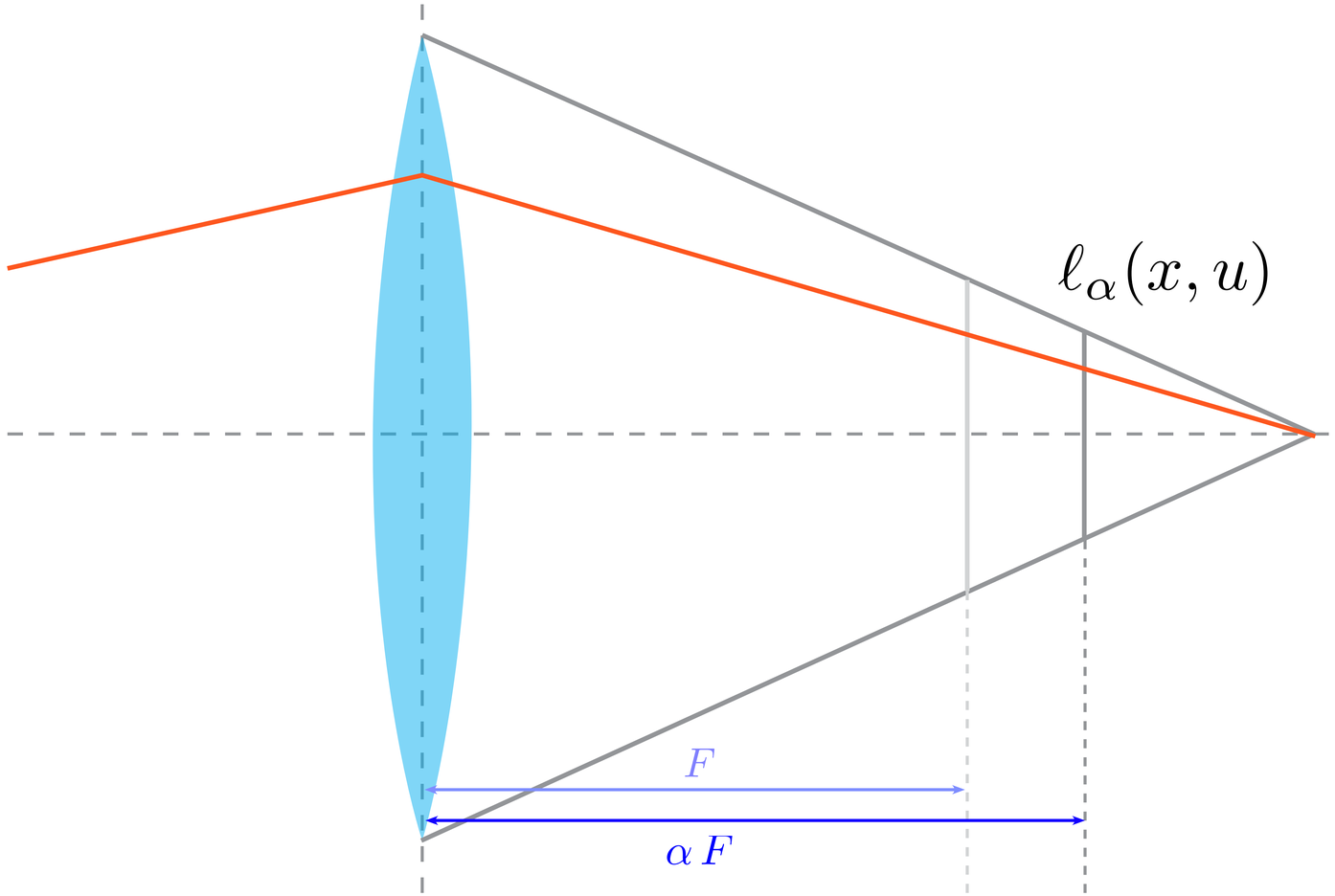

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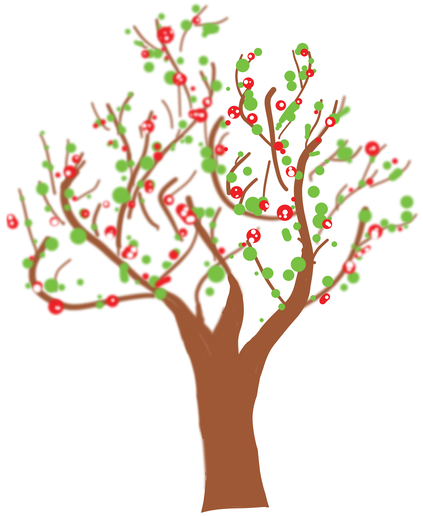

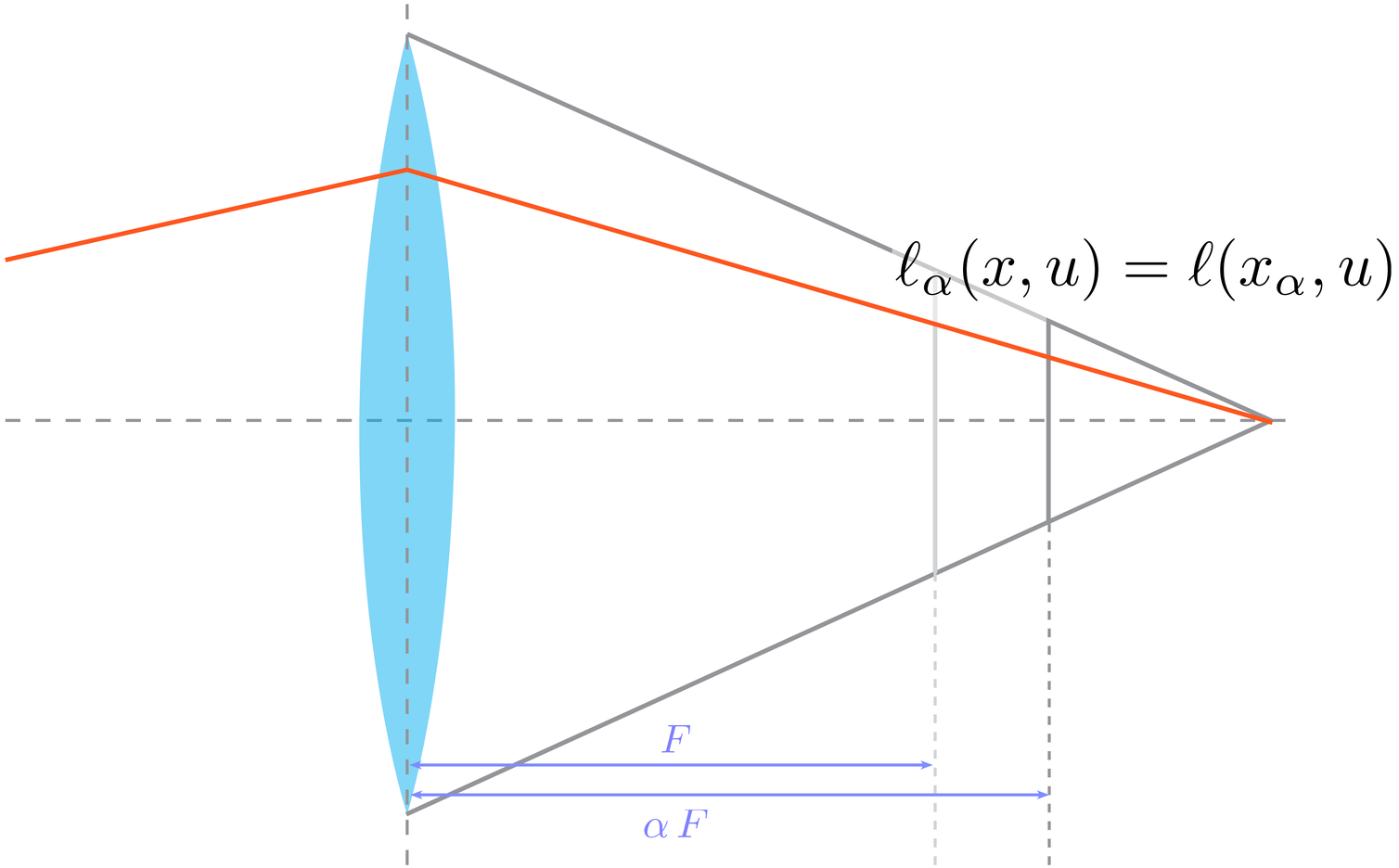

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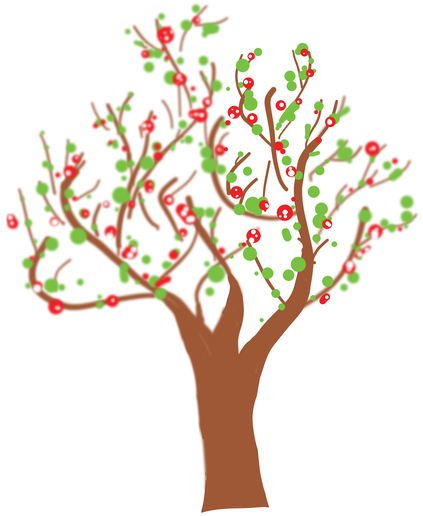
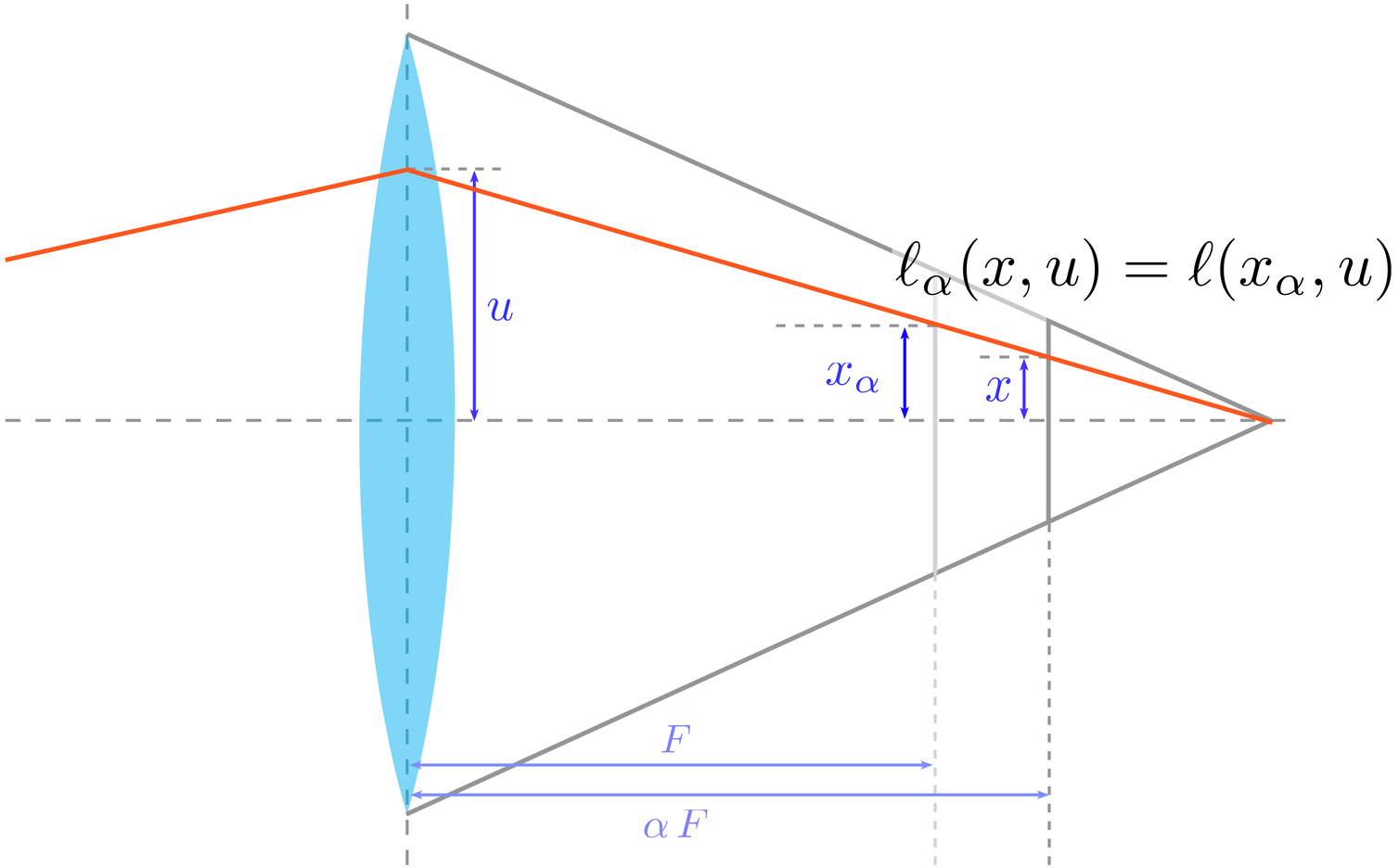
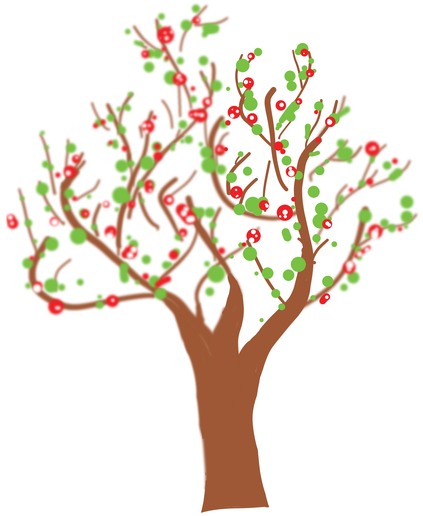
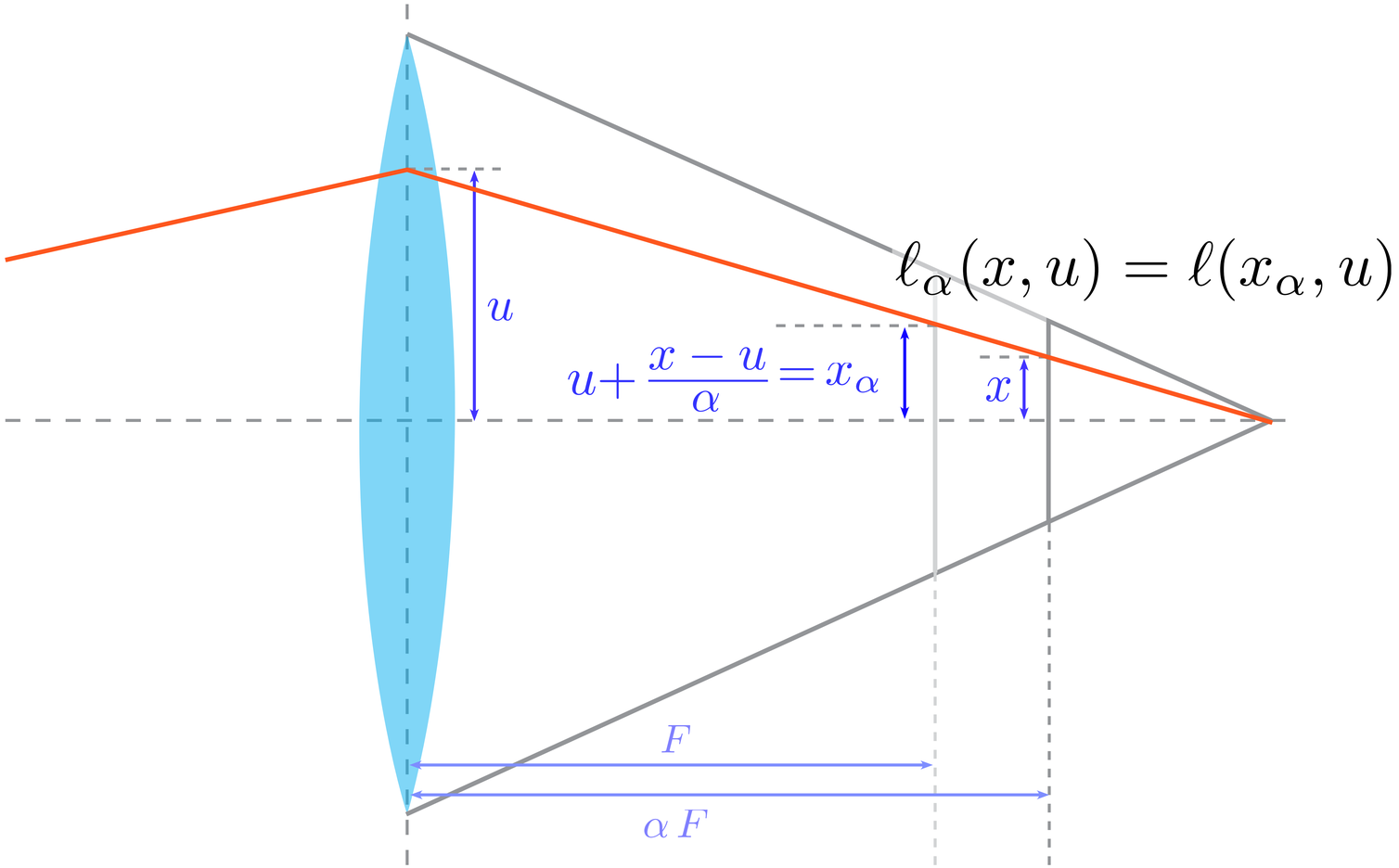




Light field refocusing





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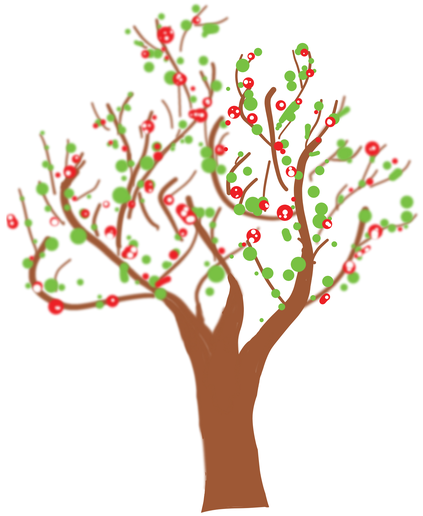
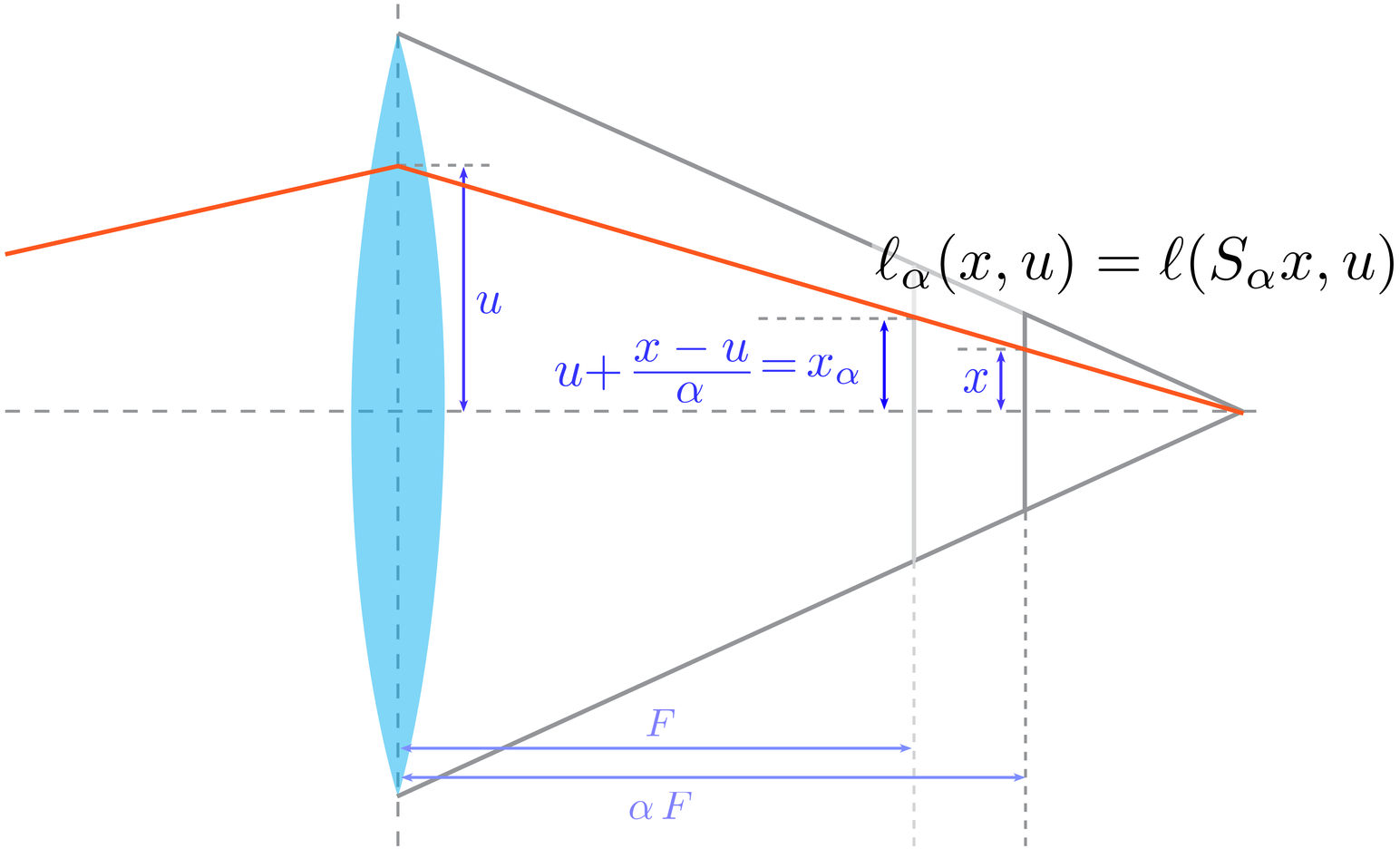
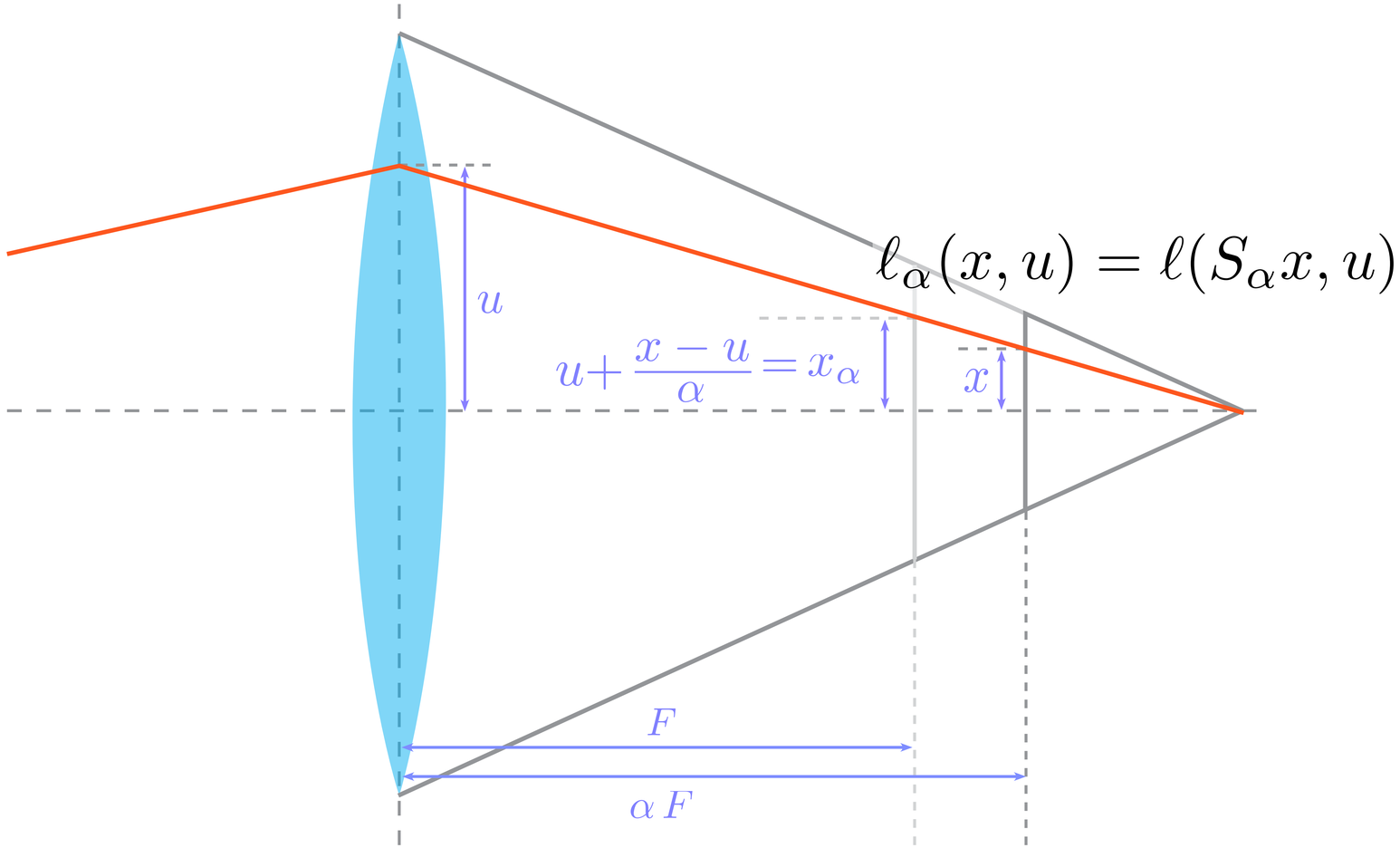




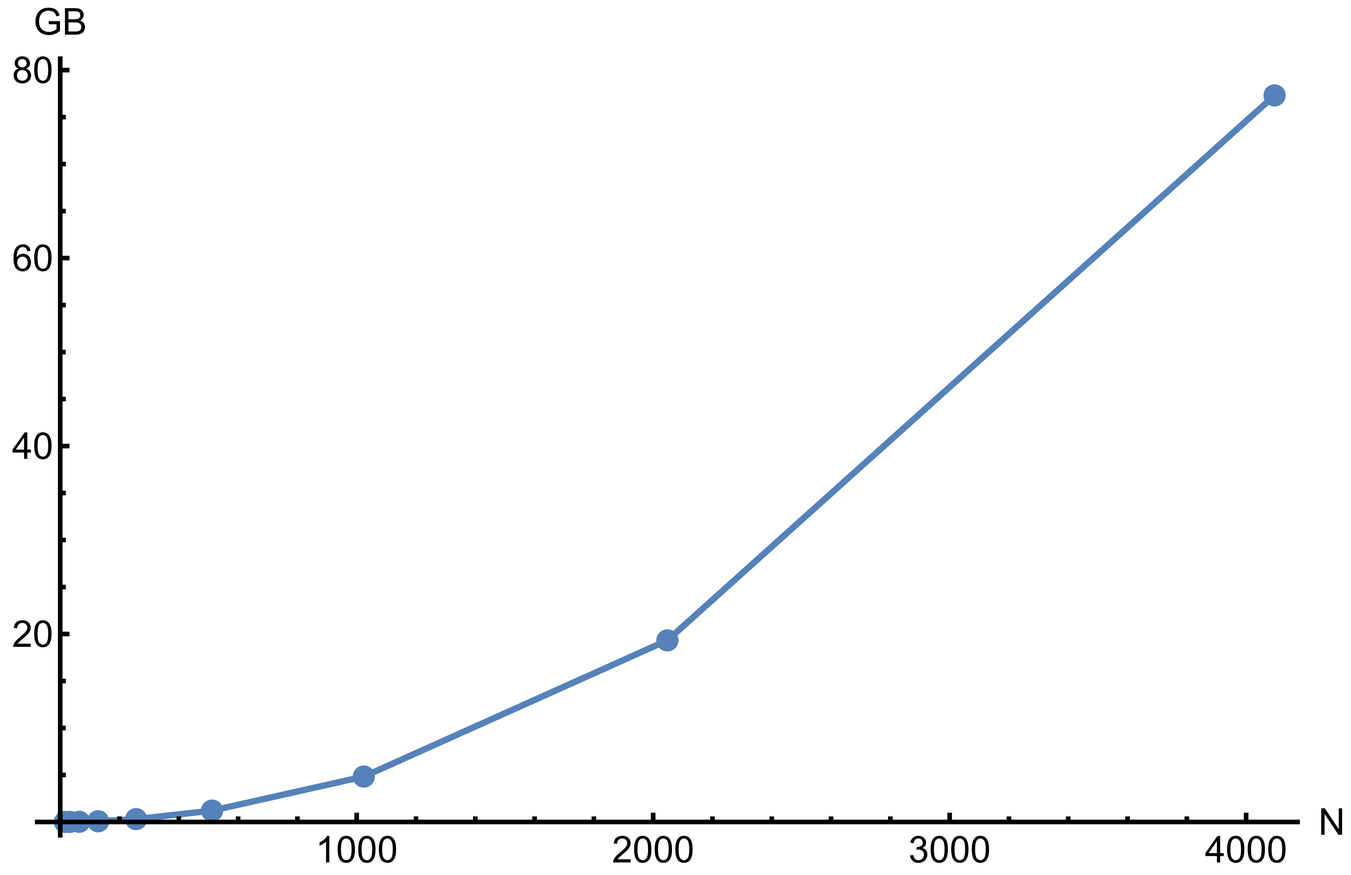
Light field refocusing

$$I_\alpha(x) = \frac{1}{\alpha^2} \int \ell_\alpha(x,u)\,\mathrm{d}u$$

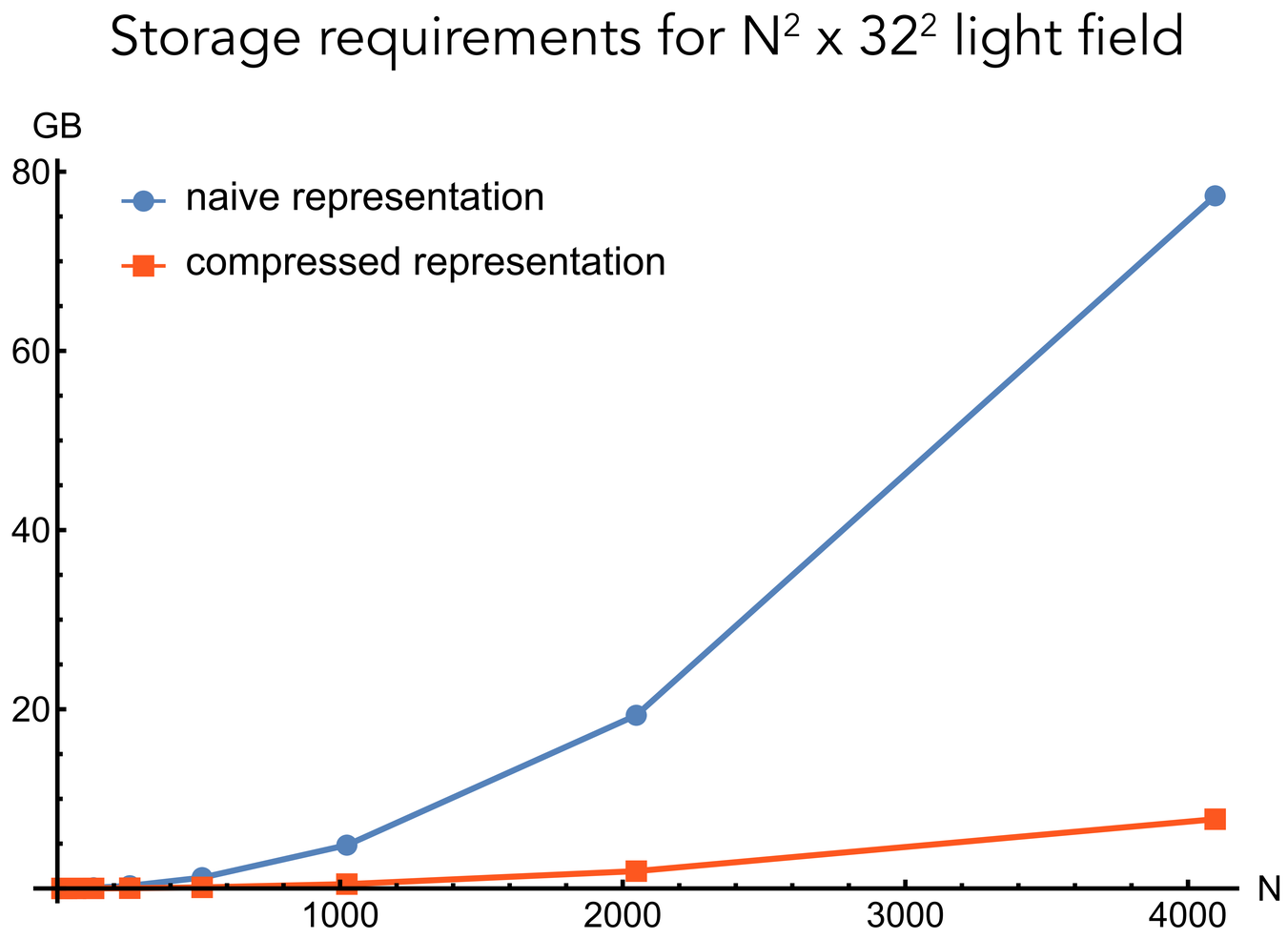


Light field representation

Storage requirements for $N^2 \times 32^2$ light field

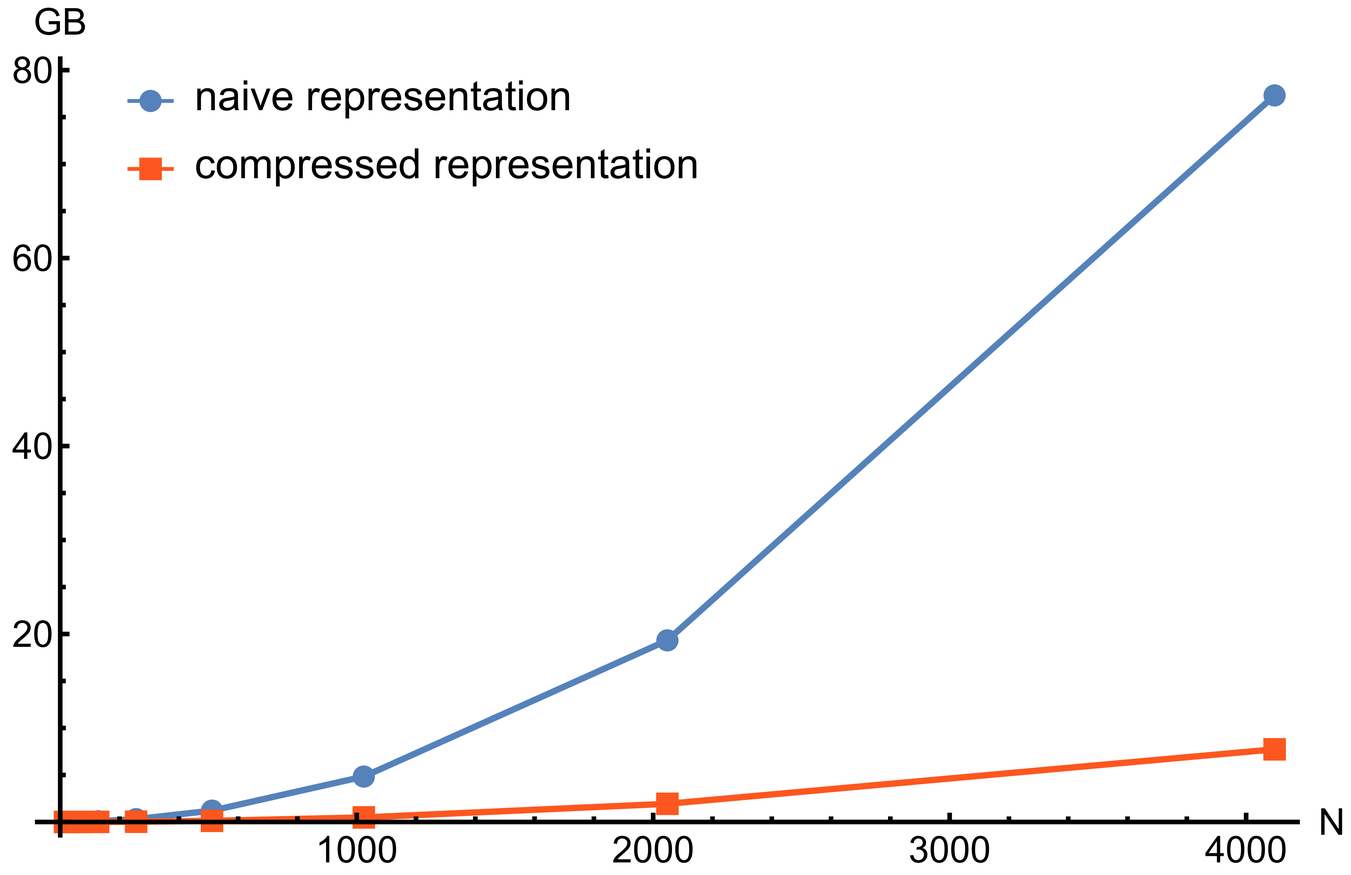


Light field representation



Light field representation

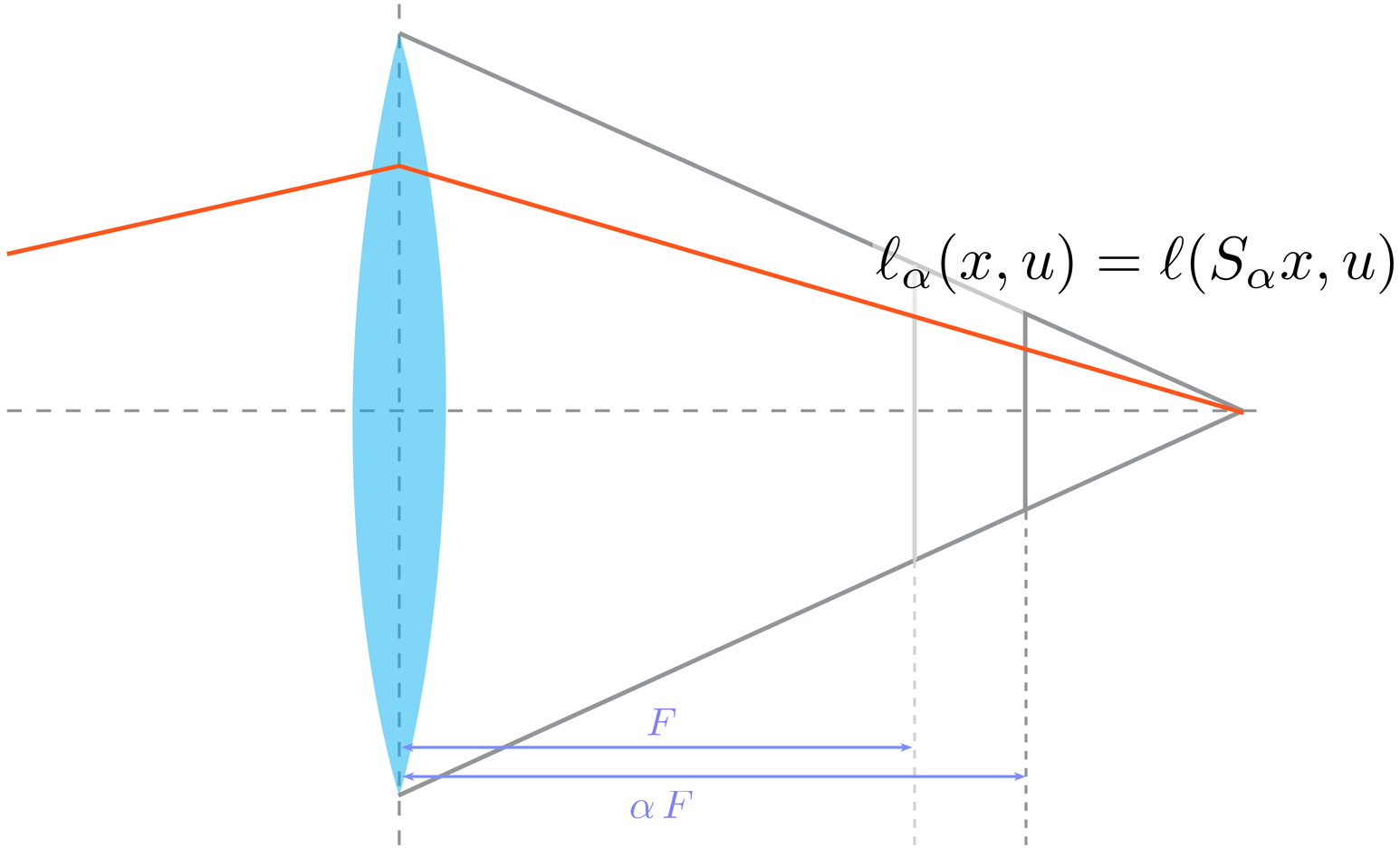
Storage requirements for $N^2 \times 32^2$ light field



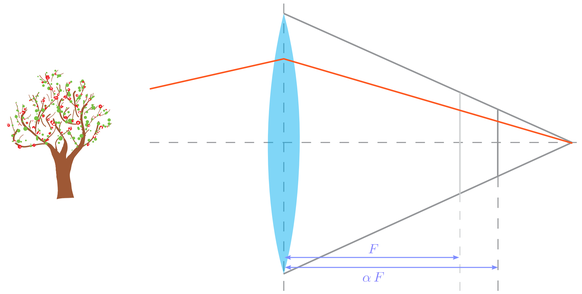
But refocusing?

Light field refocusing

$$I_\alpha(x) = \frac{1}{\alpha^2} \int \ell_\alpha(x, u)\, \mathrm{d}u$$

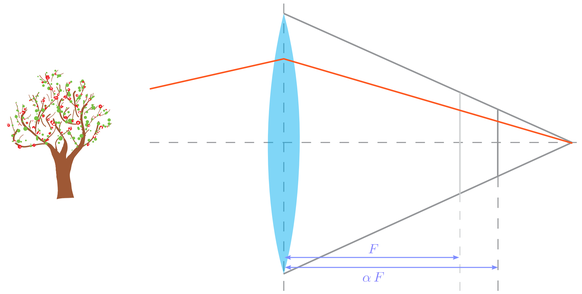


Light field refocusing

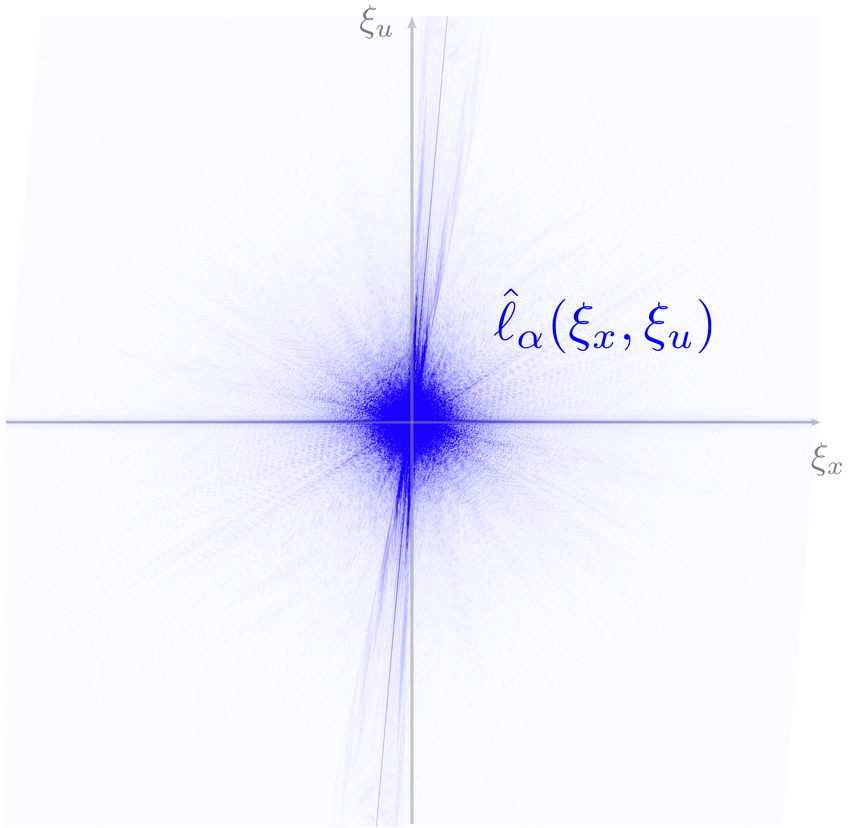


$$I(x) = \frac{1}{\alpha^2} \int_{\mathbb{R}_u} \ell_\alpha(x, u) \, \mathrm{d}u$$

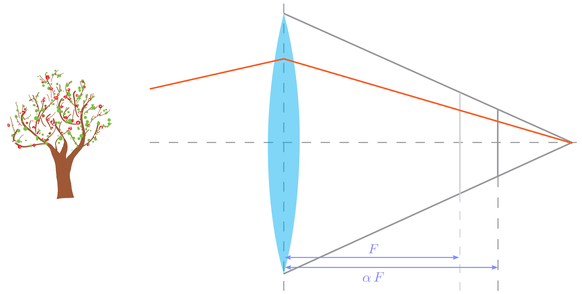
Light field refocusing



$$I(x) = \frac{1}{\alpha^2} \int_{\mathbb{R}_u} \ell_\alpha(x, u)\, \mathrm{d}u$$

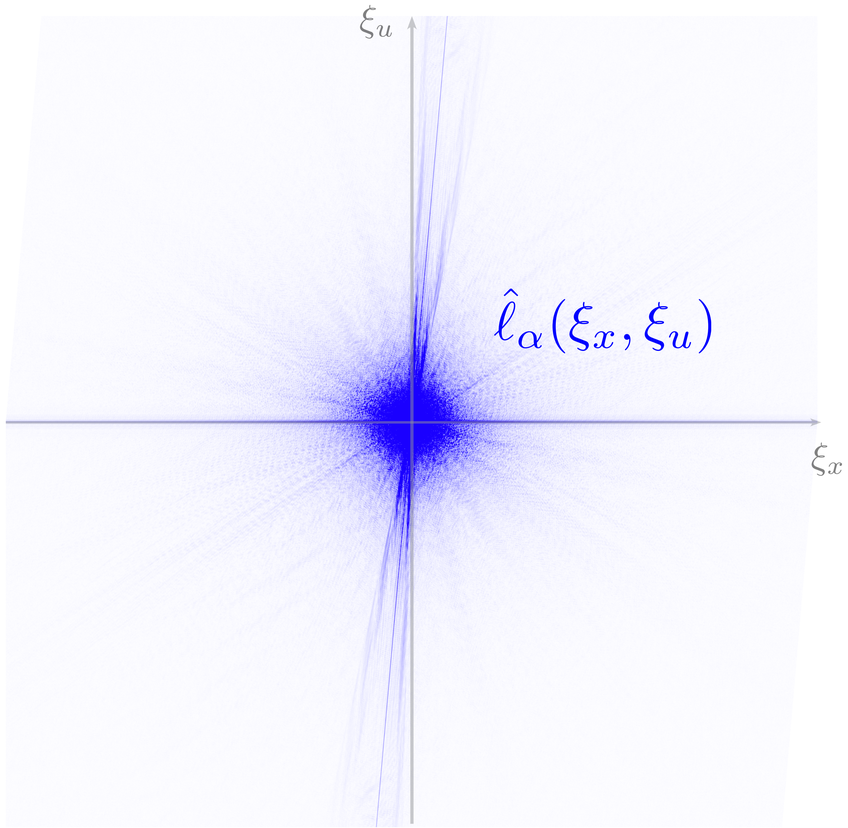


Light field refocusing

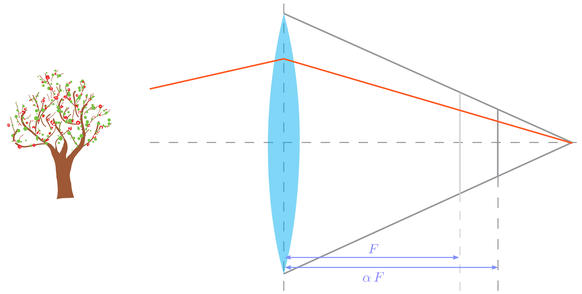


$$I(x) = \frac{1}{\alpha^2} \int_{\mathbb{R}_u} \ell_\alpha(x, u)\, \mathrm{d}u$$

$$= \frac{1}{\alpha^2} \int_{\mathbb{R}_u} \mathcal{F}_{2\mathrm{d}}^{-1}\big(\hat{\ell}_\alpha(\xi_x, \xi_u)\big)\, \mathrm{d}u$$

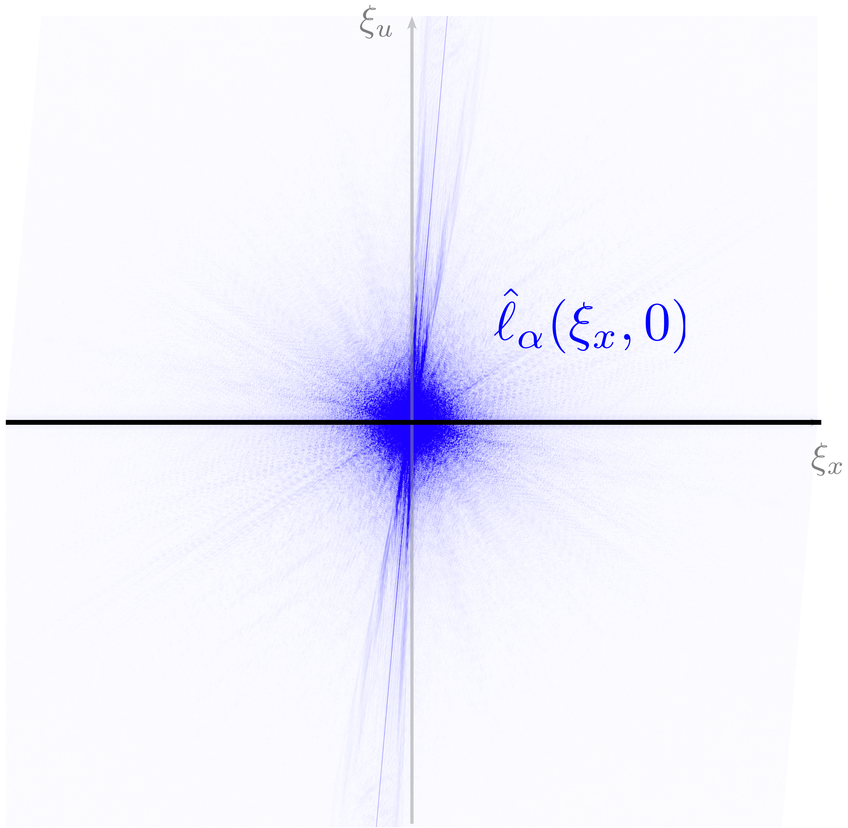


Light field refocusing

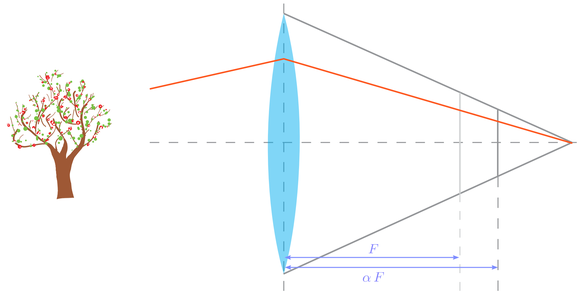


$$I(x) = \frac{1}{\alpha^2} \int_{\mathbb{R}_u} \ell_\alpha(x, u)\, \mathrm{d}u$$

$$= \frac{1}{\alpha^2} \int_{\mathbb{R}_u} \mathcal{F}_{2\mathrm{d}}^{-1}\big(\hat{\ell}_\alpha(\xi_x, \xi_u)\big)\, \mathrm{d}u$$

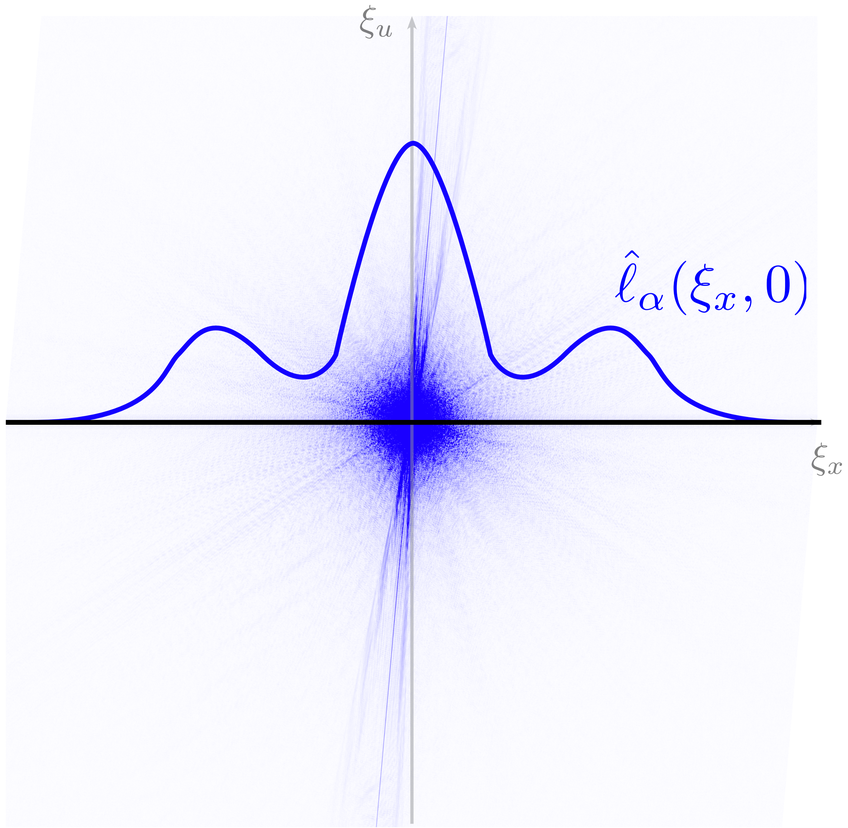


Light field refocusing

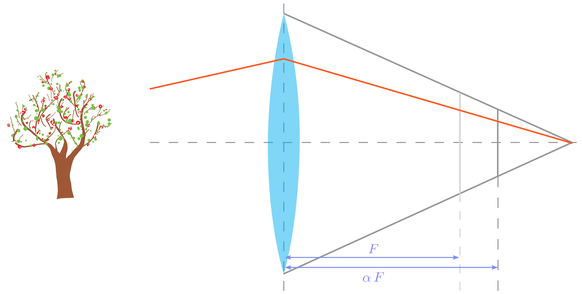


$$I(x) = \frac{1}{\alpha^2} \int_{\mathbb{R}_u} \ell_\alpha(x, u)\, \mathrm{d}u$$

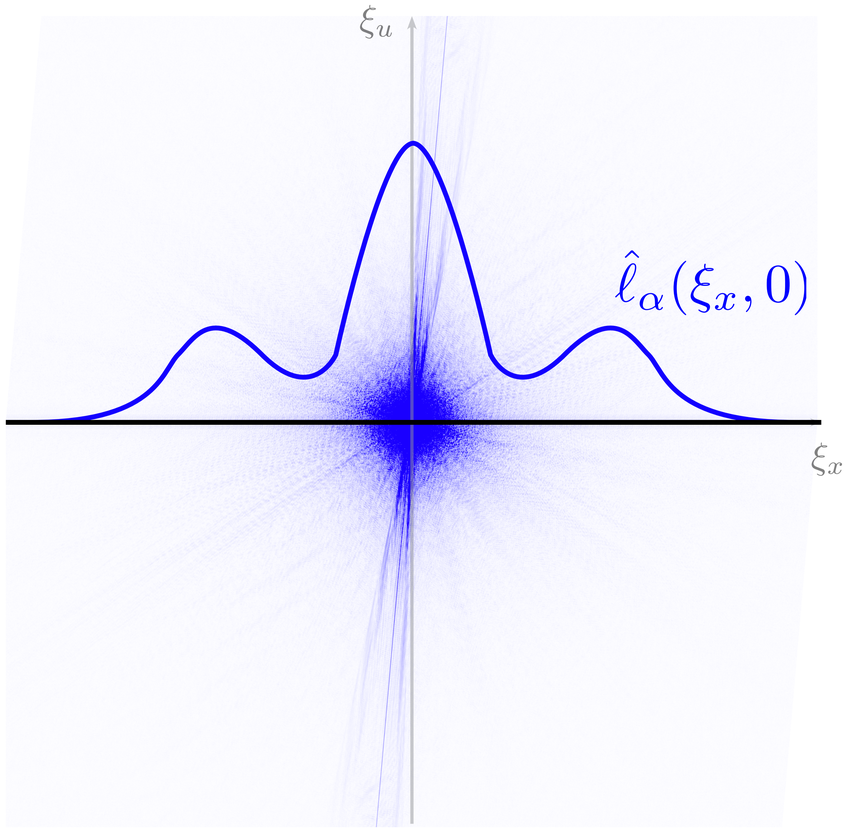
$$= \frac{1}{\alpha^2} \int_{\mathbb{R}_u} \mathcal{F}_{2\mathrm{d}}^{-1}\big(\hat{\ell}_\alpha(\xi_x, \xi_u)\big)\, \mathrm{d}u$$



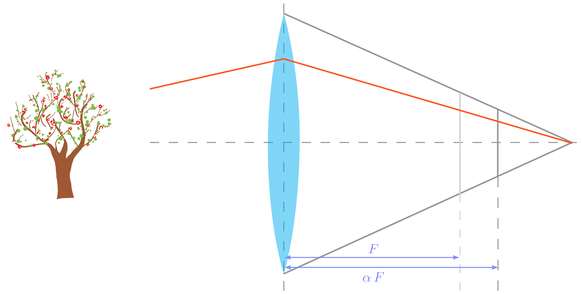
Light field refocusing



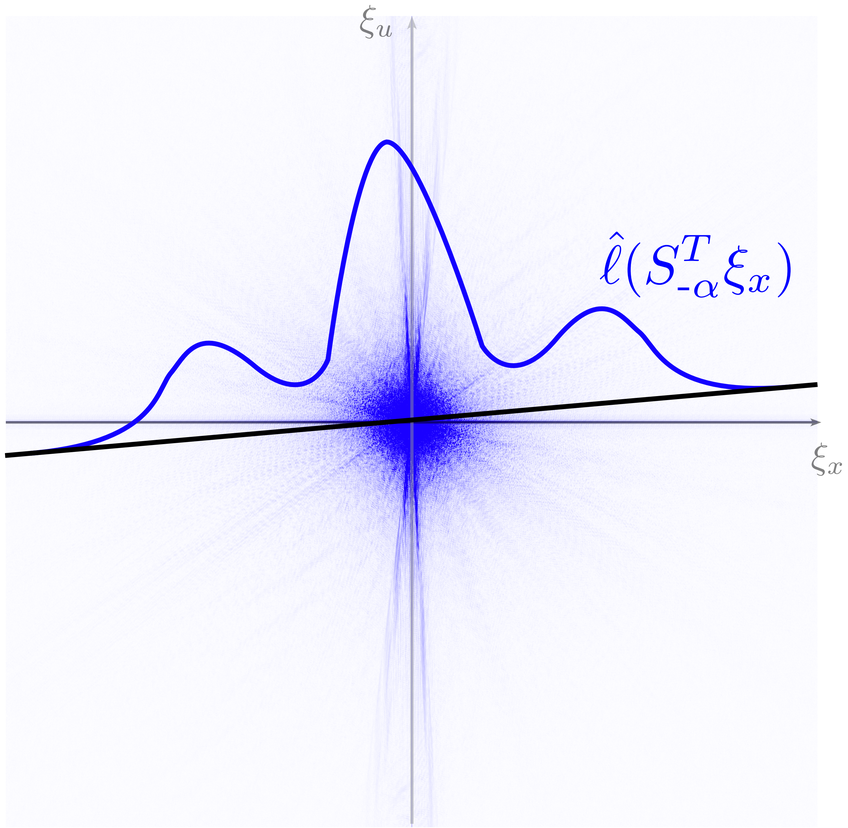
$$
\begin{aligned}
I(x) &= \frac{1}{\alpha^2} \int_{\mathbb{R}_u} \ell_\alpha(x,u)\,\mathrm{d}u \\
&= \frac{1}{\alpha^2} \int_{\mathbb{R}_u} \mathcal{F}_{2\mathrm{d}}^{-1}\big(\hat{\ell}_\alpha(\xi_x,\xi_u)\big)\,\mathrm{d}u \\
&= \frac{1}{\alpha^2} \mathcal{F}_{1\mathrm{d}}^{-1}\big(\hat{\ell}_\alpha(\xi_x,0)\big)
\end{aligned}
$$



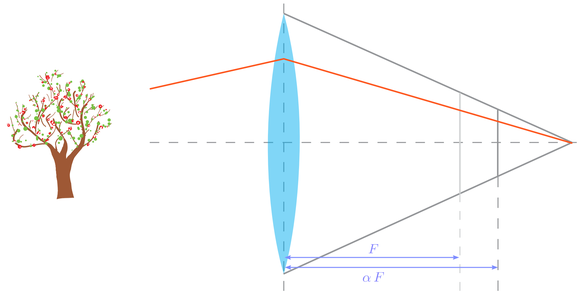
Light field refocusing



$$
\begin{aligned}
I(x) &= \frac{1}{\alpha^2} \int_{\mathbb{R}_u} \ell_\alpha(x,u)\,\mathrm{d}u \\
&= \frac{1}{\alpha^2} \int_{\mathbb{R}_u} \mathcal{F}_{2\mathrm{d}}^{-1}\big(\hat{\ell}_\alpha(\xi_x,\xi_u)\big)\,\mathrm{d}u \\
&= \frac{1}{\alpha^2} \mathcal{F}_{1\mathrm{d}}^{-1}\big(\hat{\ell}_\alpha(\xi_x,0)\big) \\
&= \frac{1}{\alpha^2} \mathcal{F}_{1\mathrm{d}}^{-1}\big(\hat{\ell}(S_{-\alpha}^T\xi_x)\big)
\end{aligned}
$$

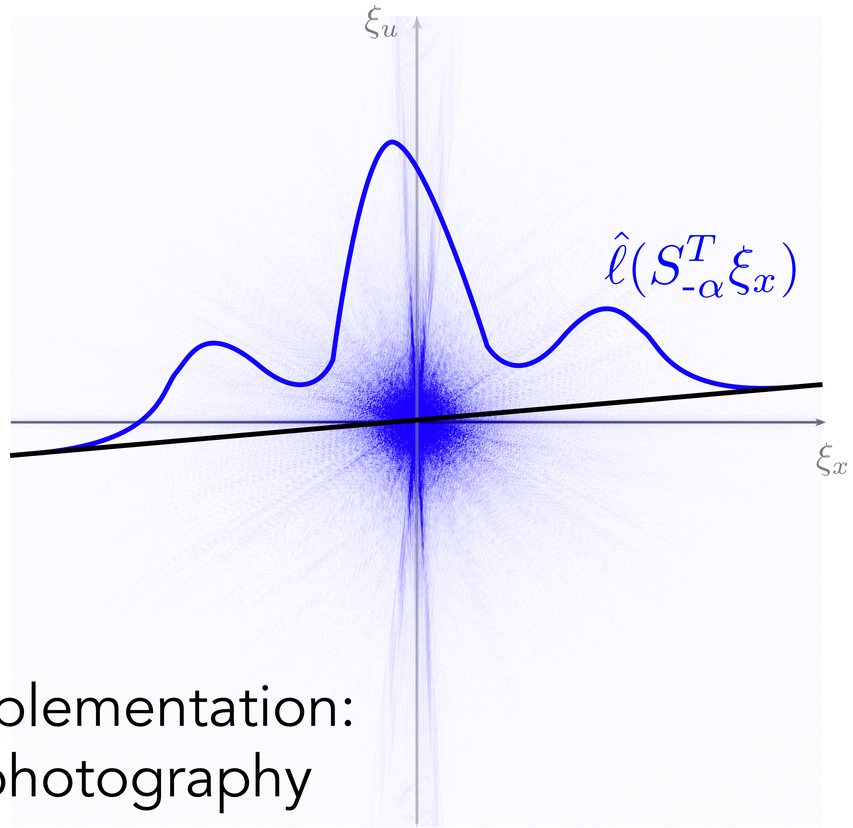


Light field refocusing

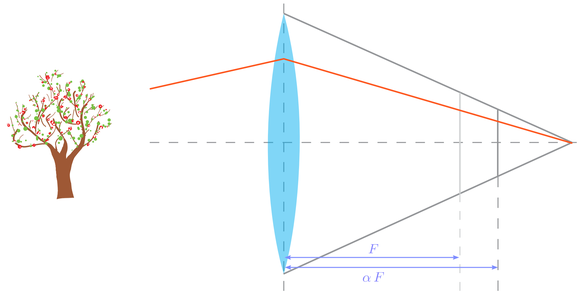


$$
\begin{aligned}
I(x) &= \frac{1}{\alpha^2} \int_{\mathbb{R}_u} \ell_\alpha(x,u)\,\mathrm{d}u \\
&= \frac{1}{\alpha^2} \int_{\mathbb{R}_u} \mathcal{F}_{2\mathrm{d}}^{-1}\big(\hat{\ell}_\alpha(\xi_x,\xi_u)\big)\,\mathrm{d}u \\
&= \frac{1}{\alpha^2} \mathcal{F}_{1\mathrm{d}}^{-1}\big(\hat{\ell}_\alpha(\xi_x,0)\big) \\
&= \frac{1}{\alpha^2} \mathcal{F}_{1\mathrm{d}}^{-1}\big(\hat{\ell}(S_{-\alpha}^T\xi_x)\big)
\end{aligned}
$$

FFT-based implementation:
Fourier slice photography
(Ng, 2005)

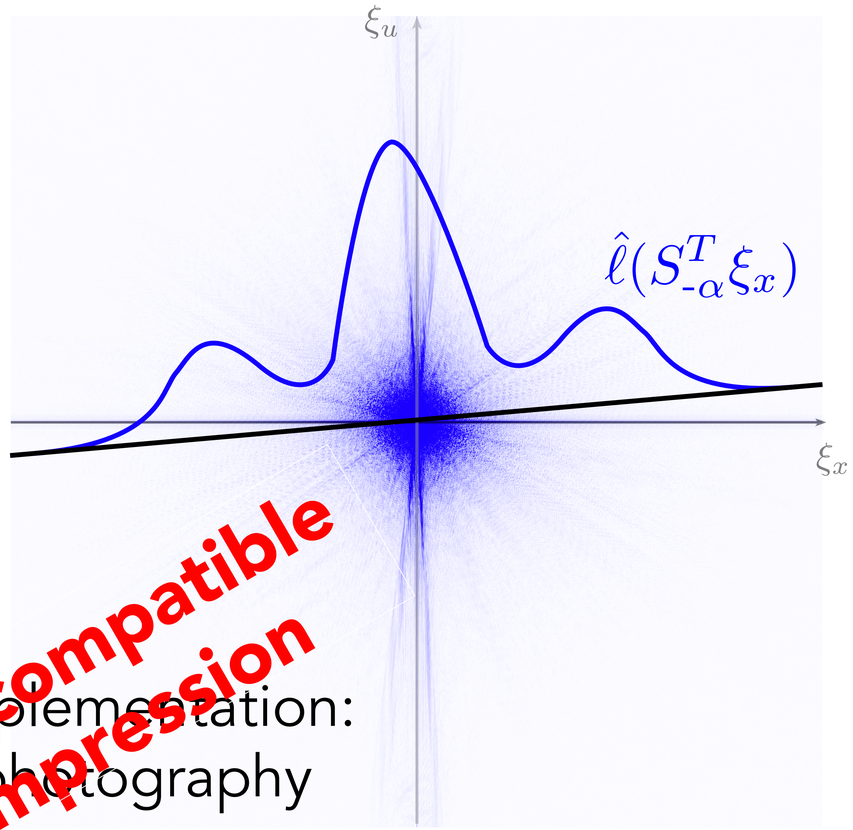


Light field refocusing



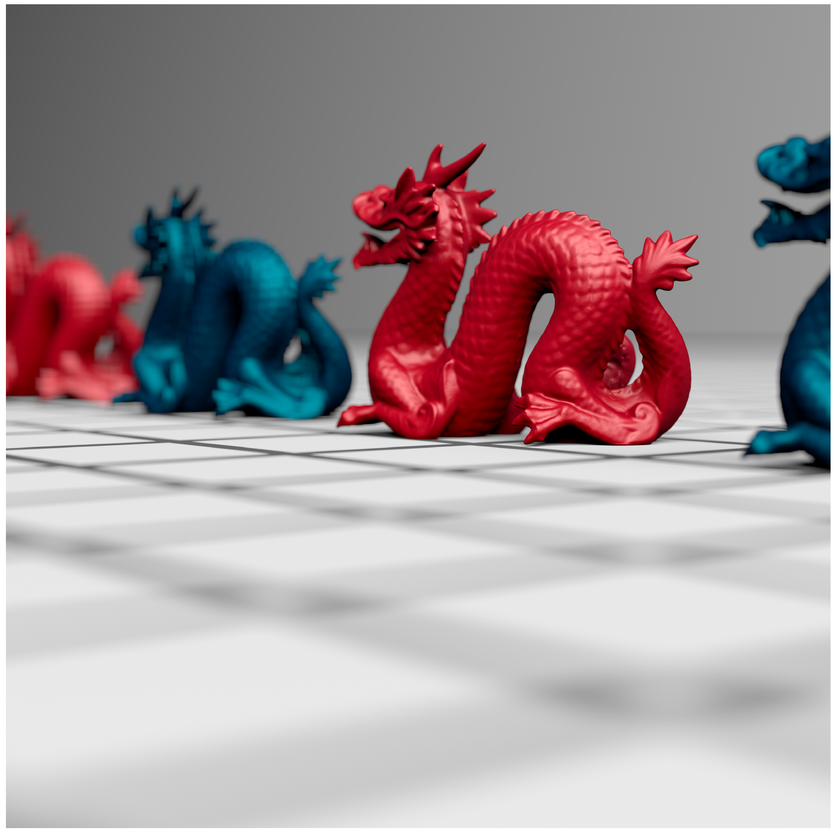
$$
\begin{aligned}
I(x) &= \frac{1}{\alpha^2} \int_{\mathbb{R}_u} \ell_\alpha(x,u)\,\mathrm{d}u \\
&= \frac{1}{\alpha^2} \int_{\mathbb{R}_u} \mathcal{F}_{2\mathrm{d}}^{-1}\big(\hat{\ell}_\alpha(\xi_x,\xi_u)\big)\,\mathrm{d}u \\
&= \frac{1}{\alpha^2} \mathcal{F}_{1\mathrm{d}}^{-1}\big(\hat{\ell}_\alpha(\xi_x,0)\big) \\
&= \frac{1}{\alpha^2} \mathcal{F}_{1\mathrm{d}}^{-1}\big(\hat{\ell}(S_{-\alpha}^T\xi_x)\big)
\end{aligned}
$$

FFT-based implementation: Fourier slice photography (Ng, 2005)



Not easily compatible with compression

Image compression



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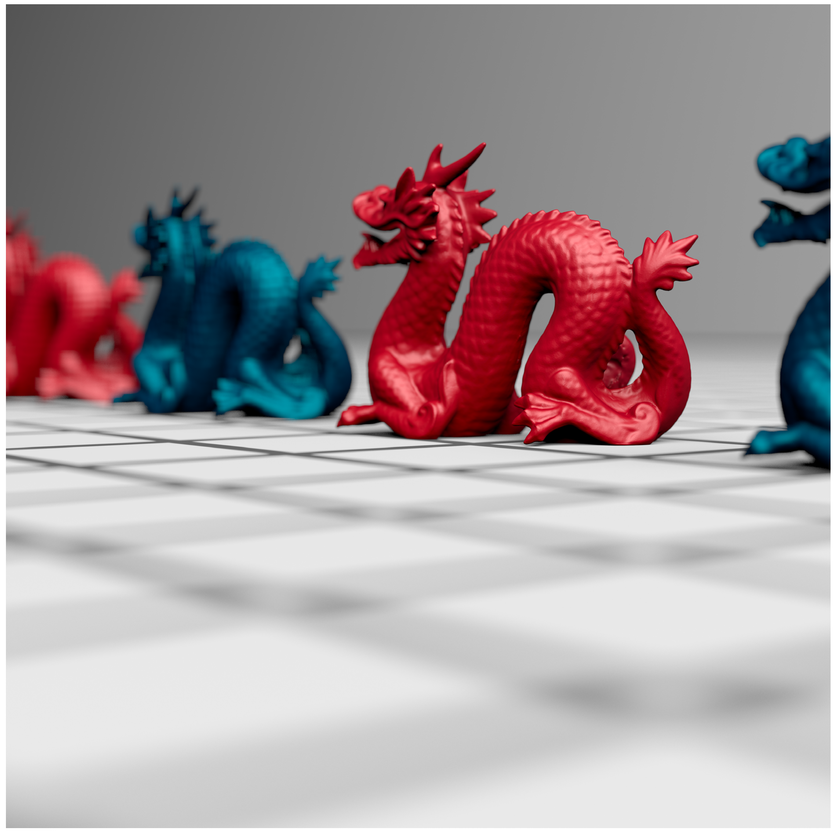

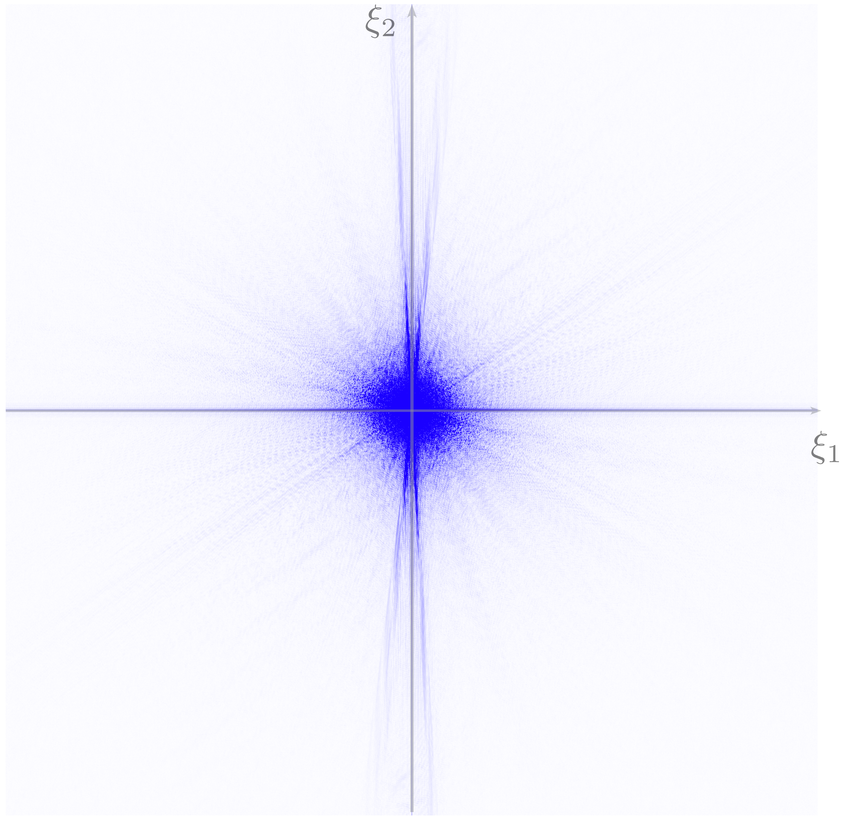

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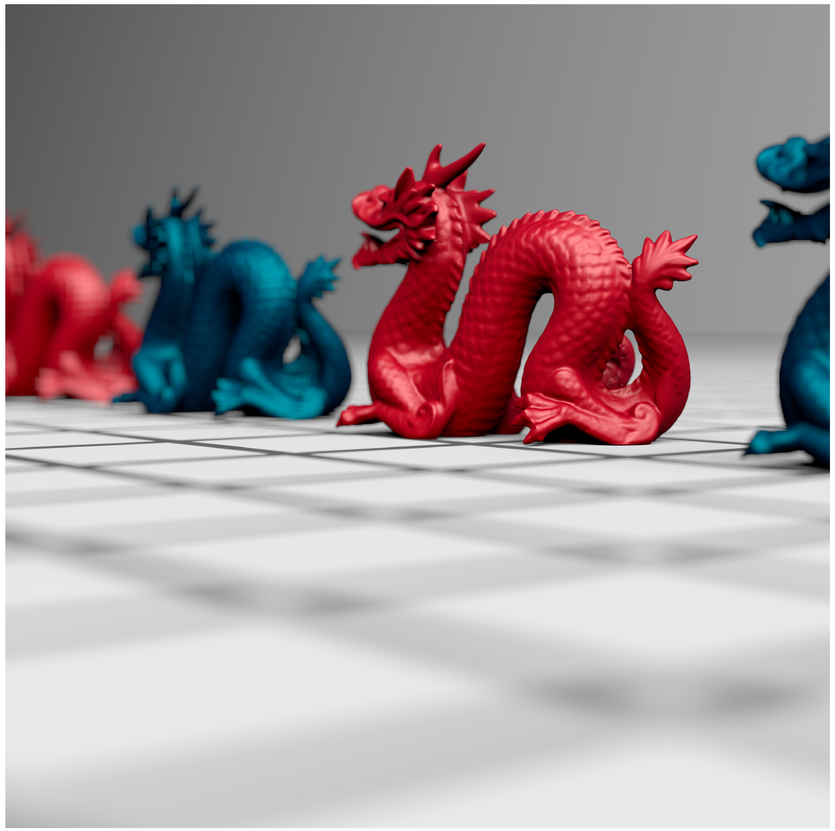
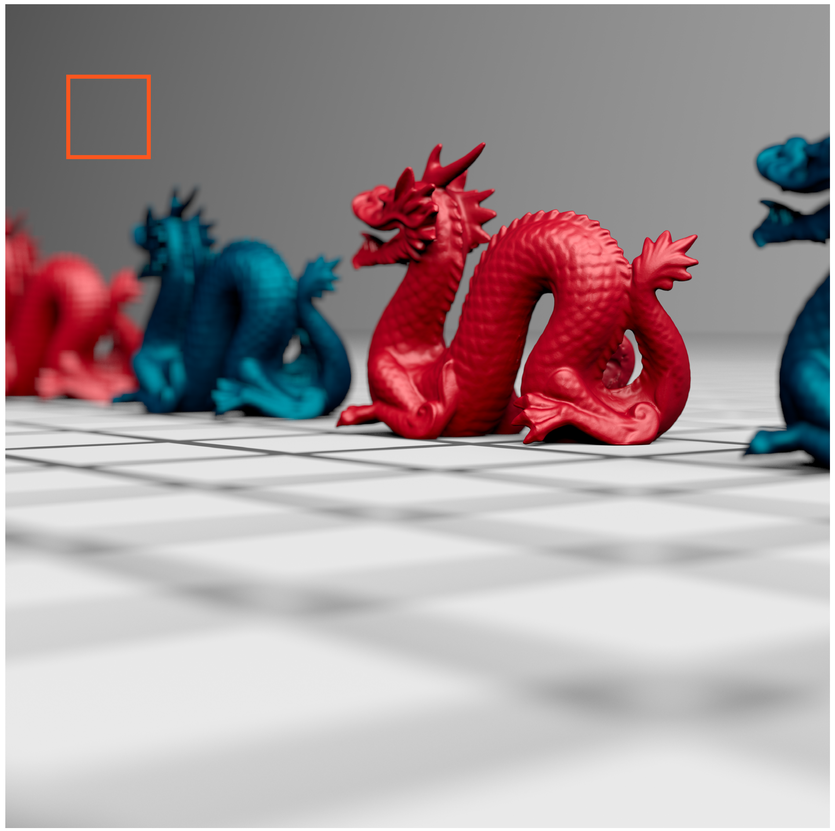


Image compression



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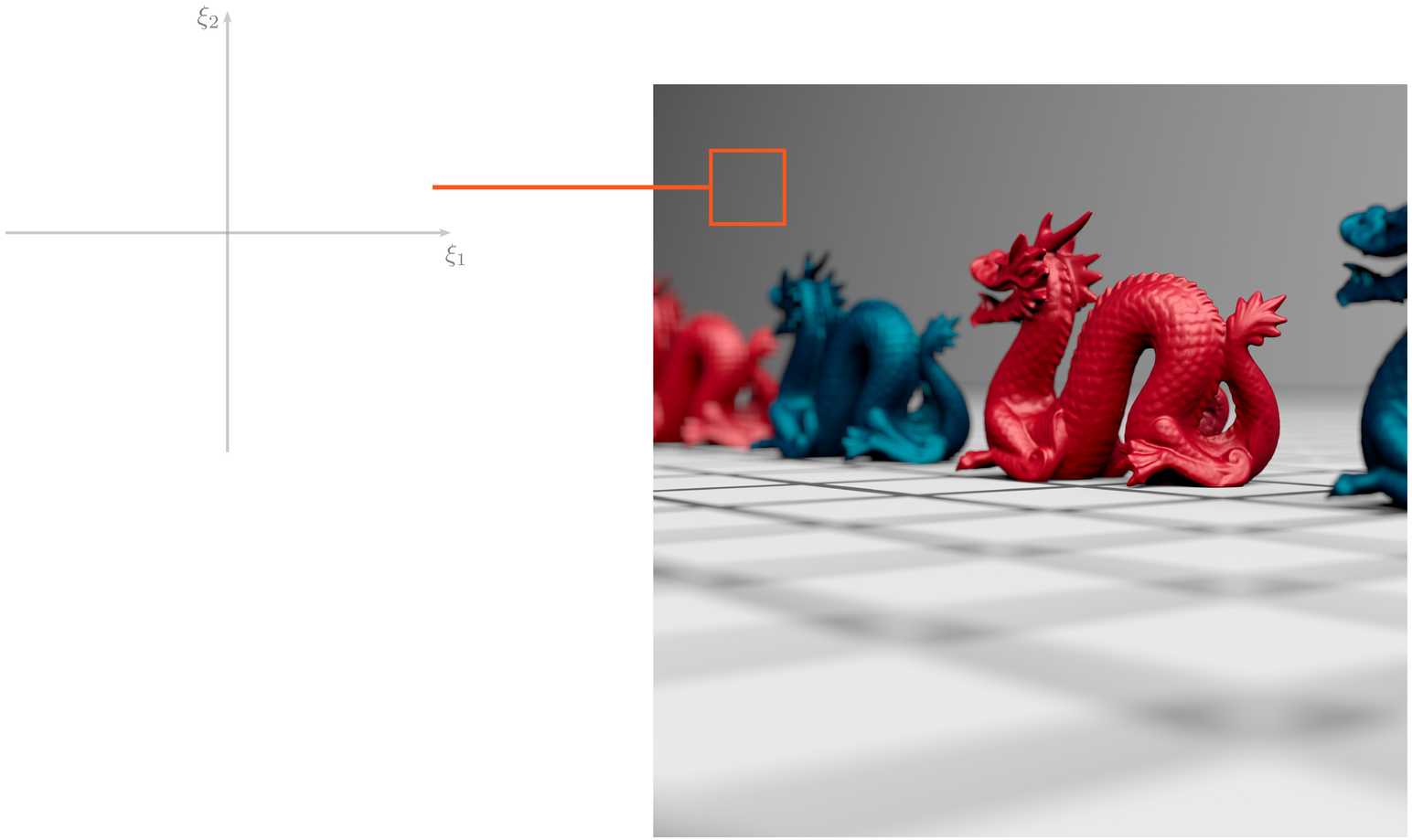

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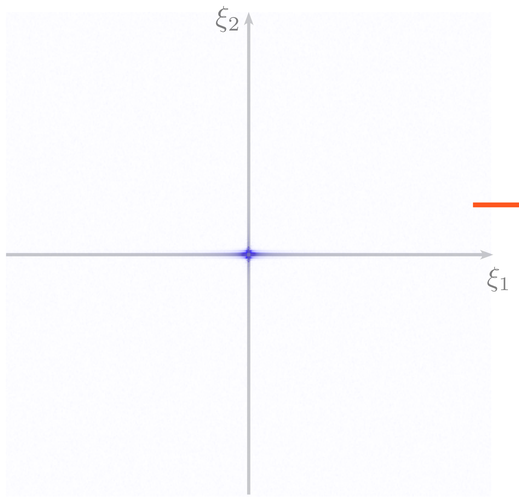

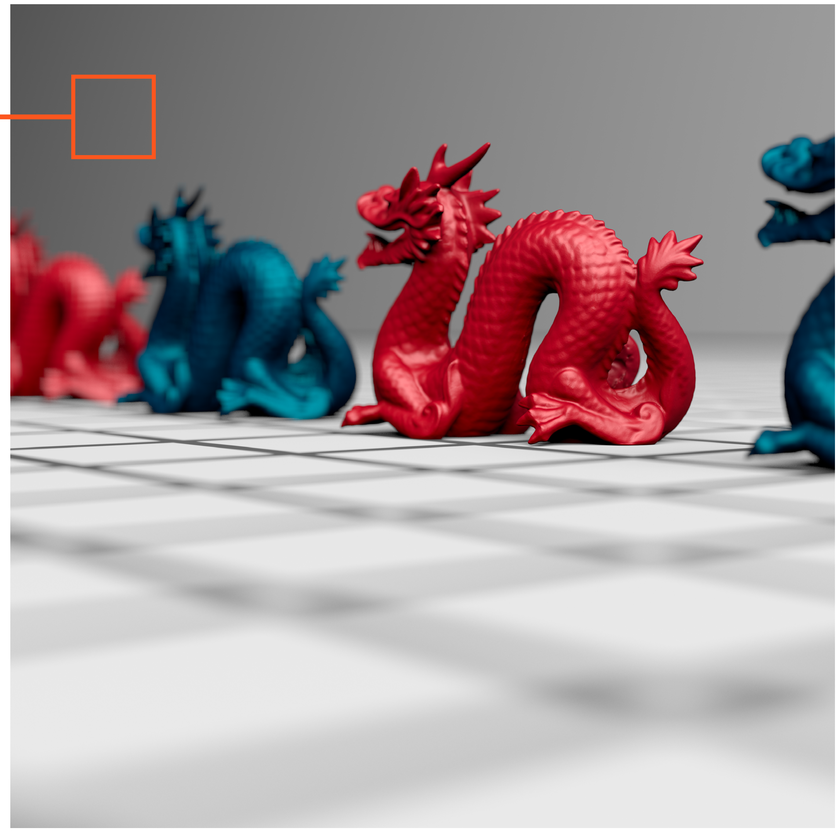

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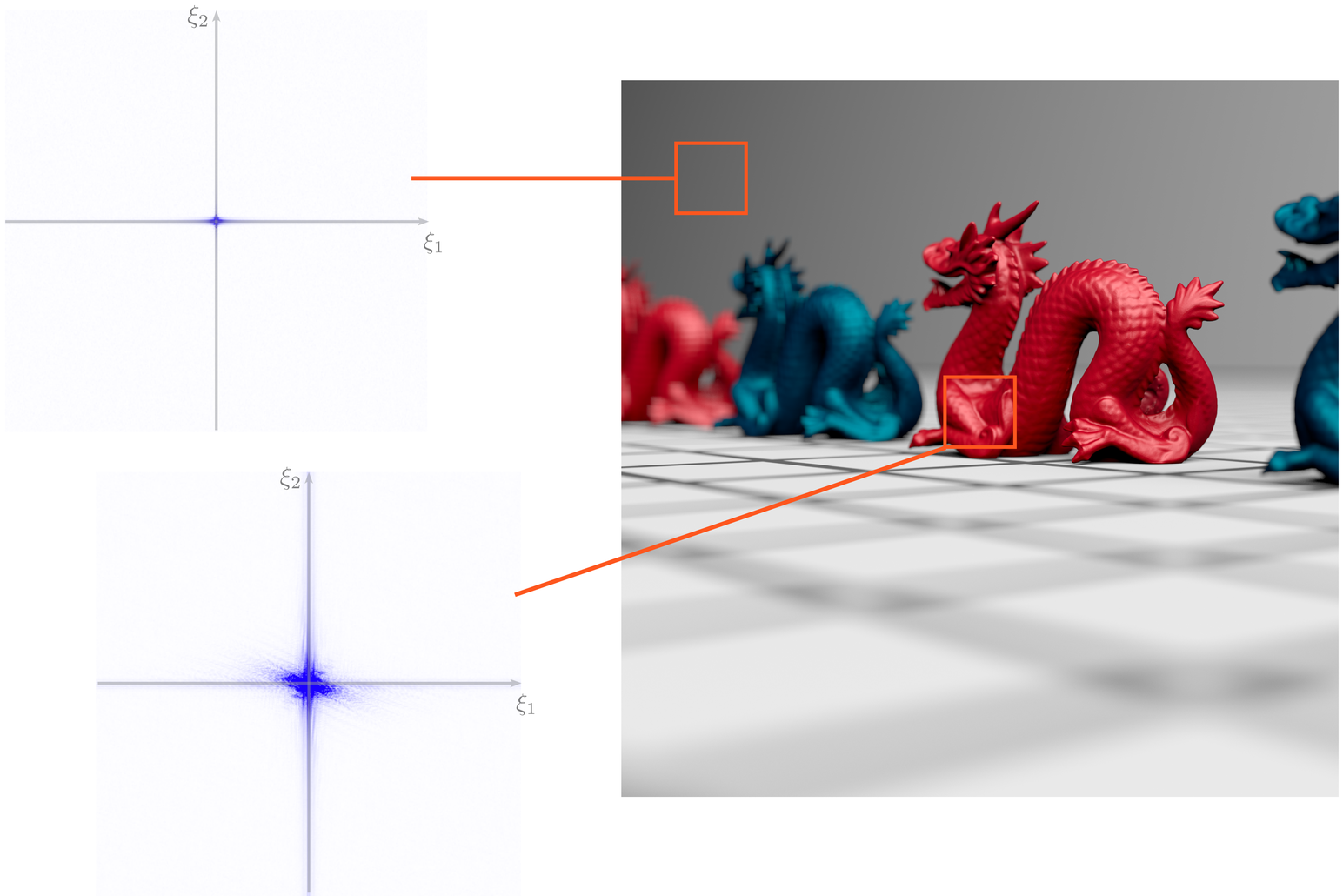

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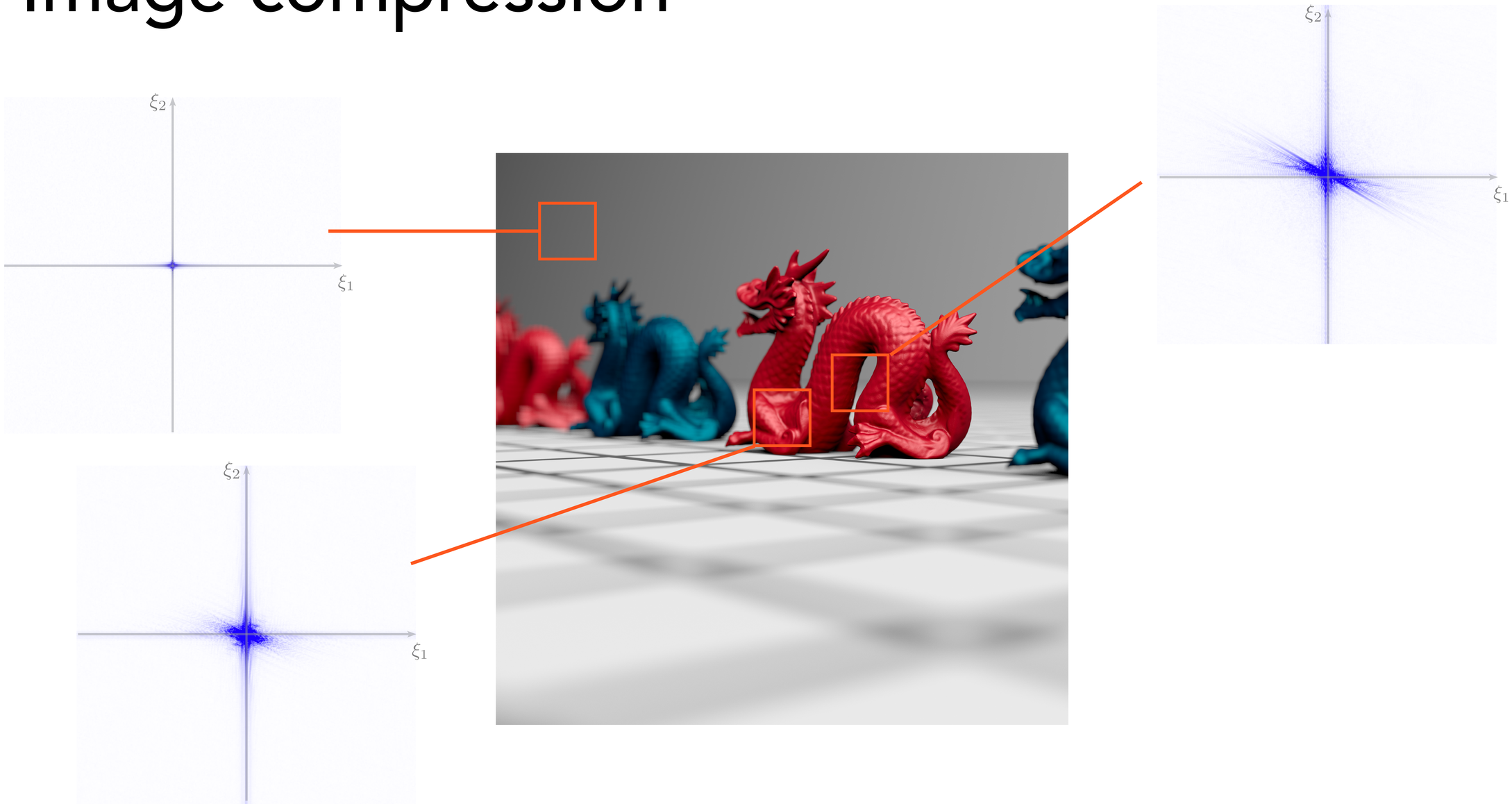

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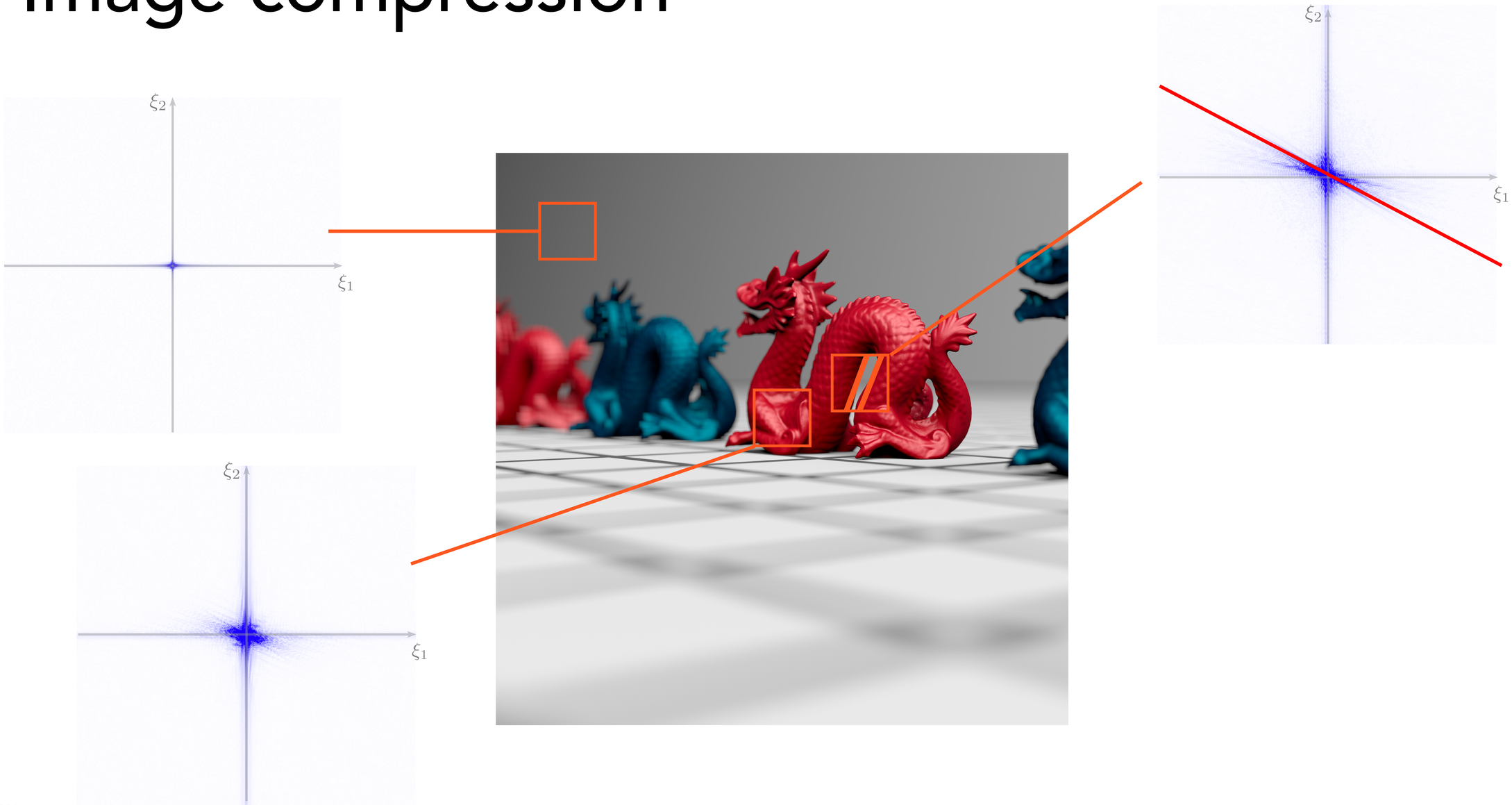

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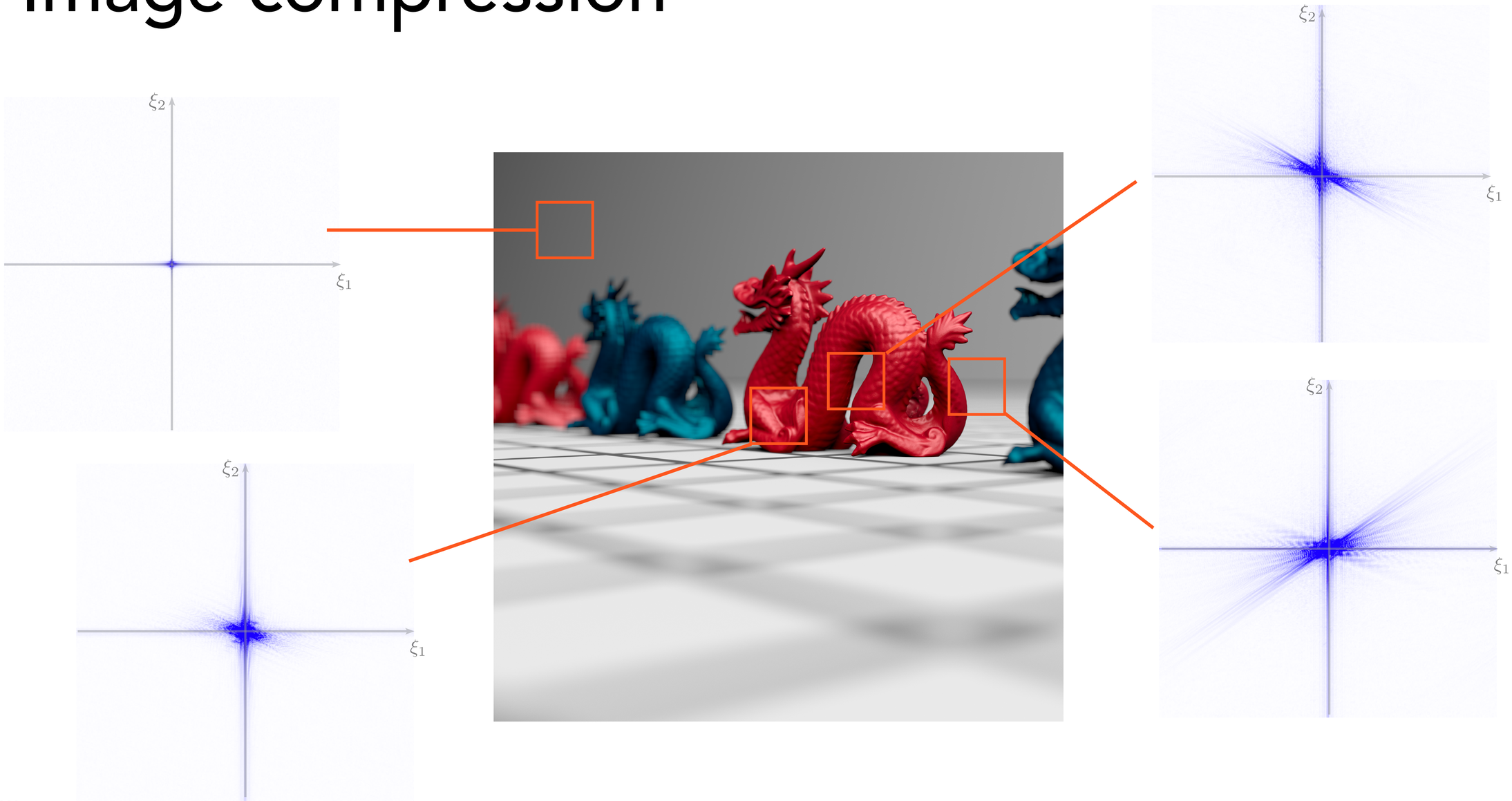

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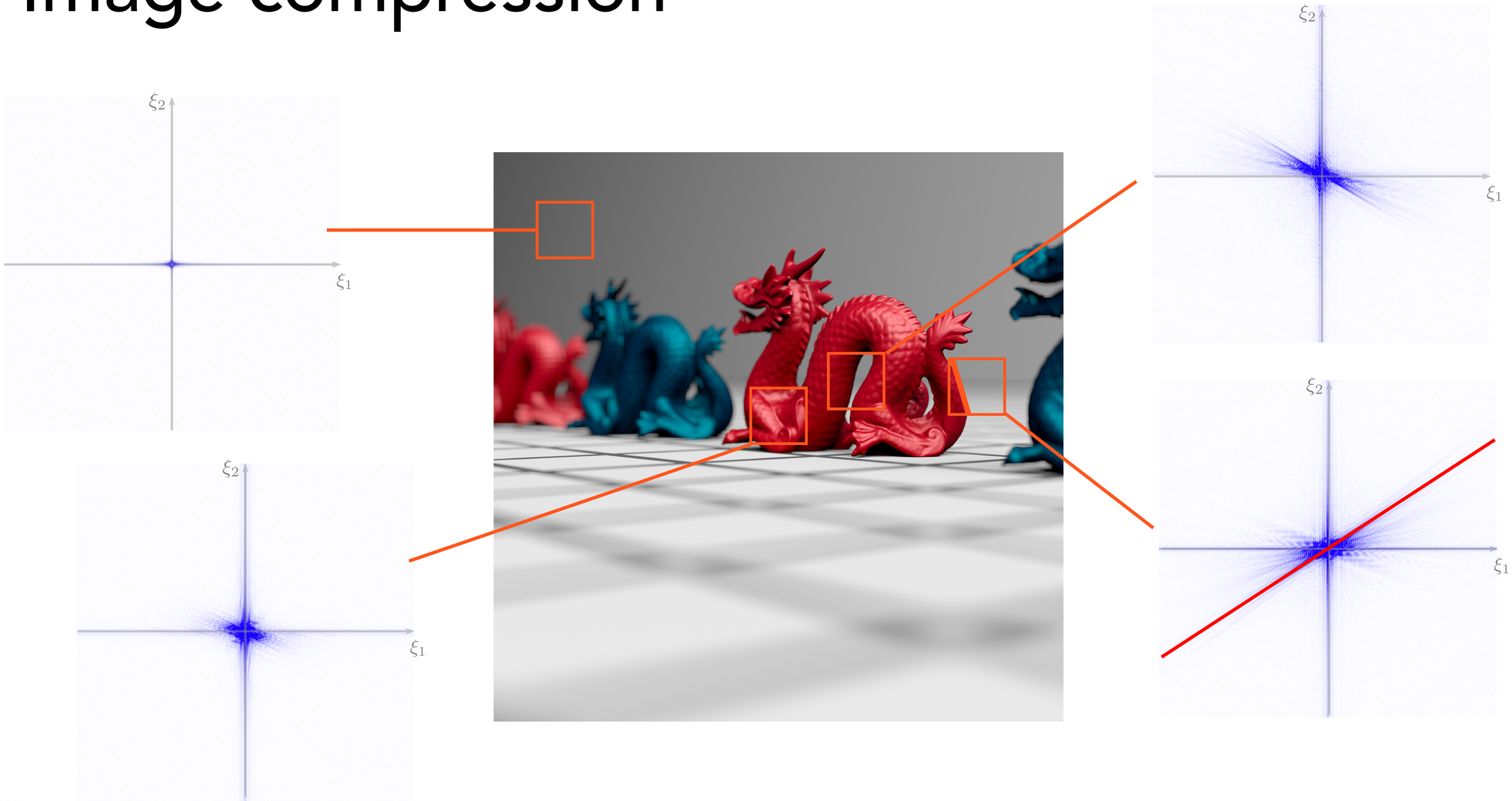

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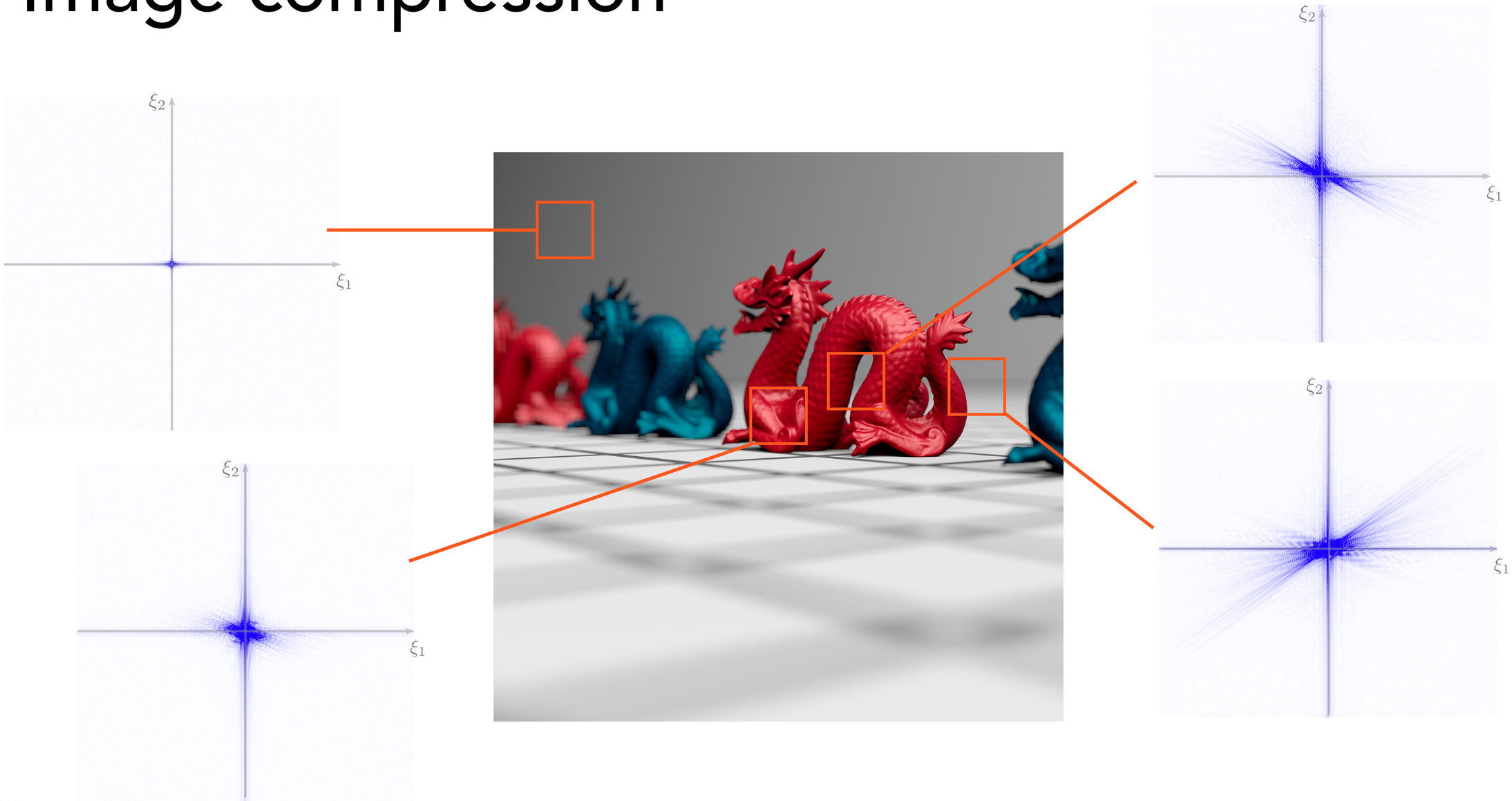

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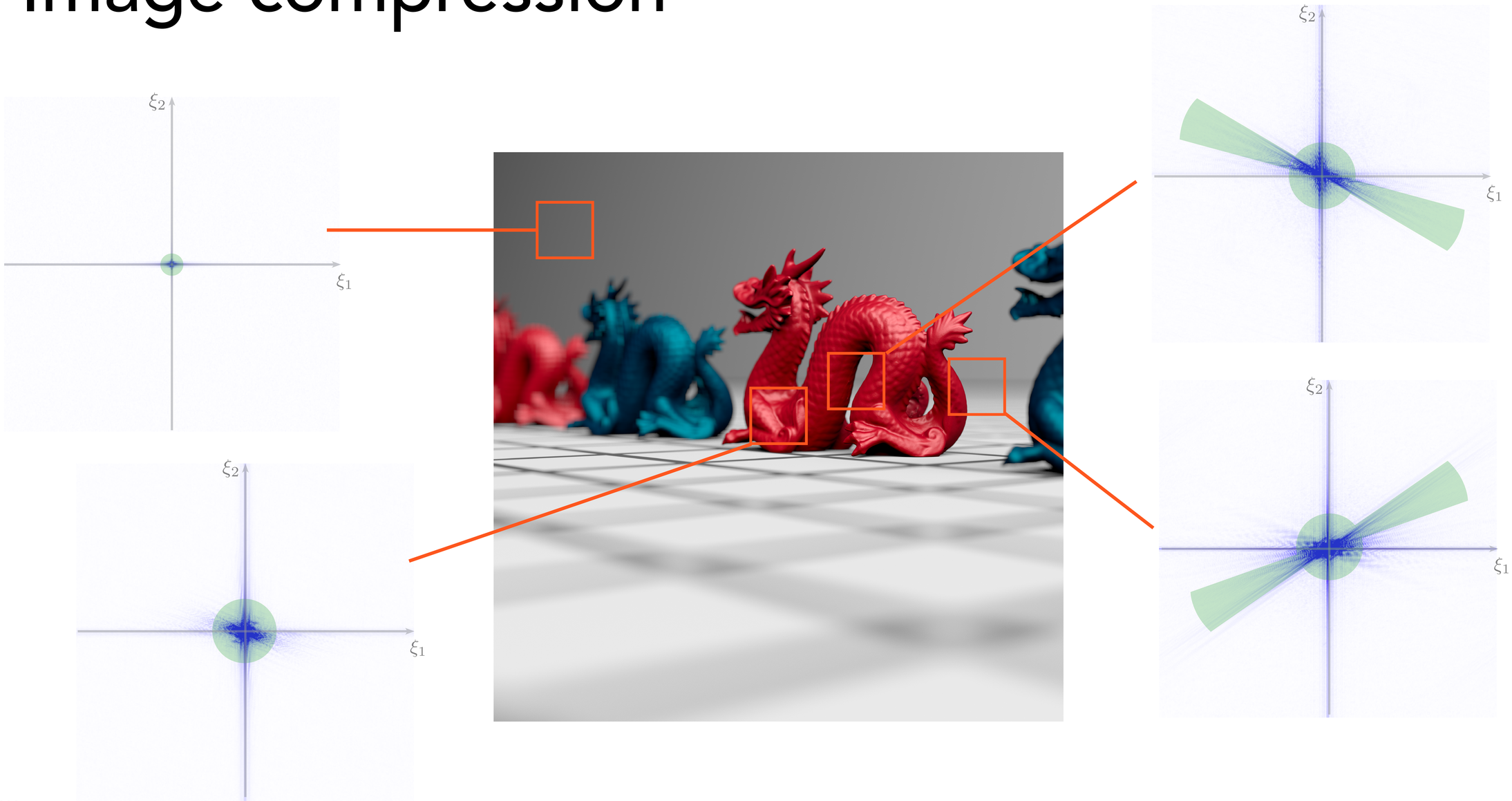

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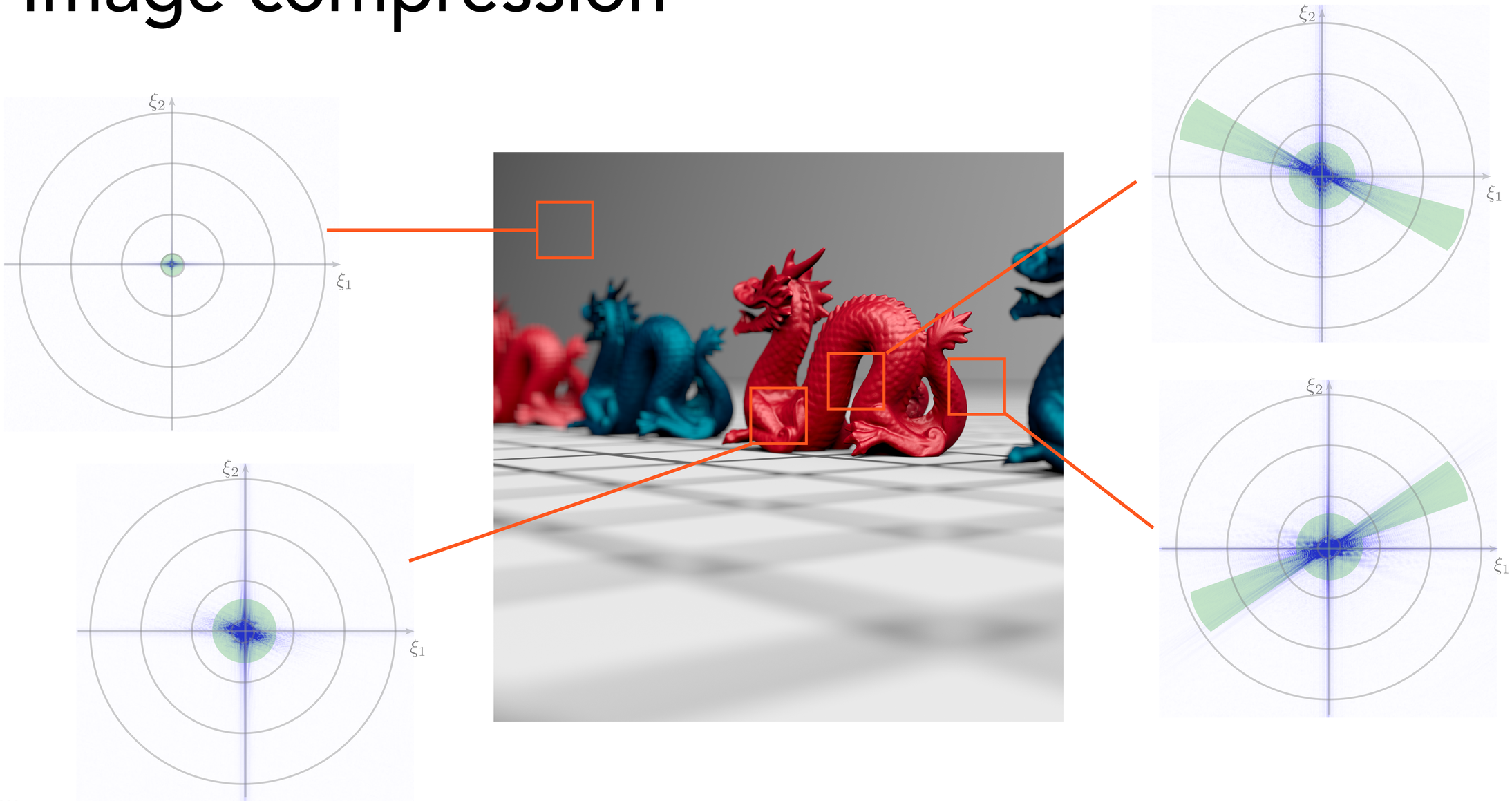

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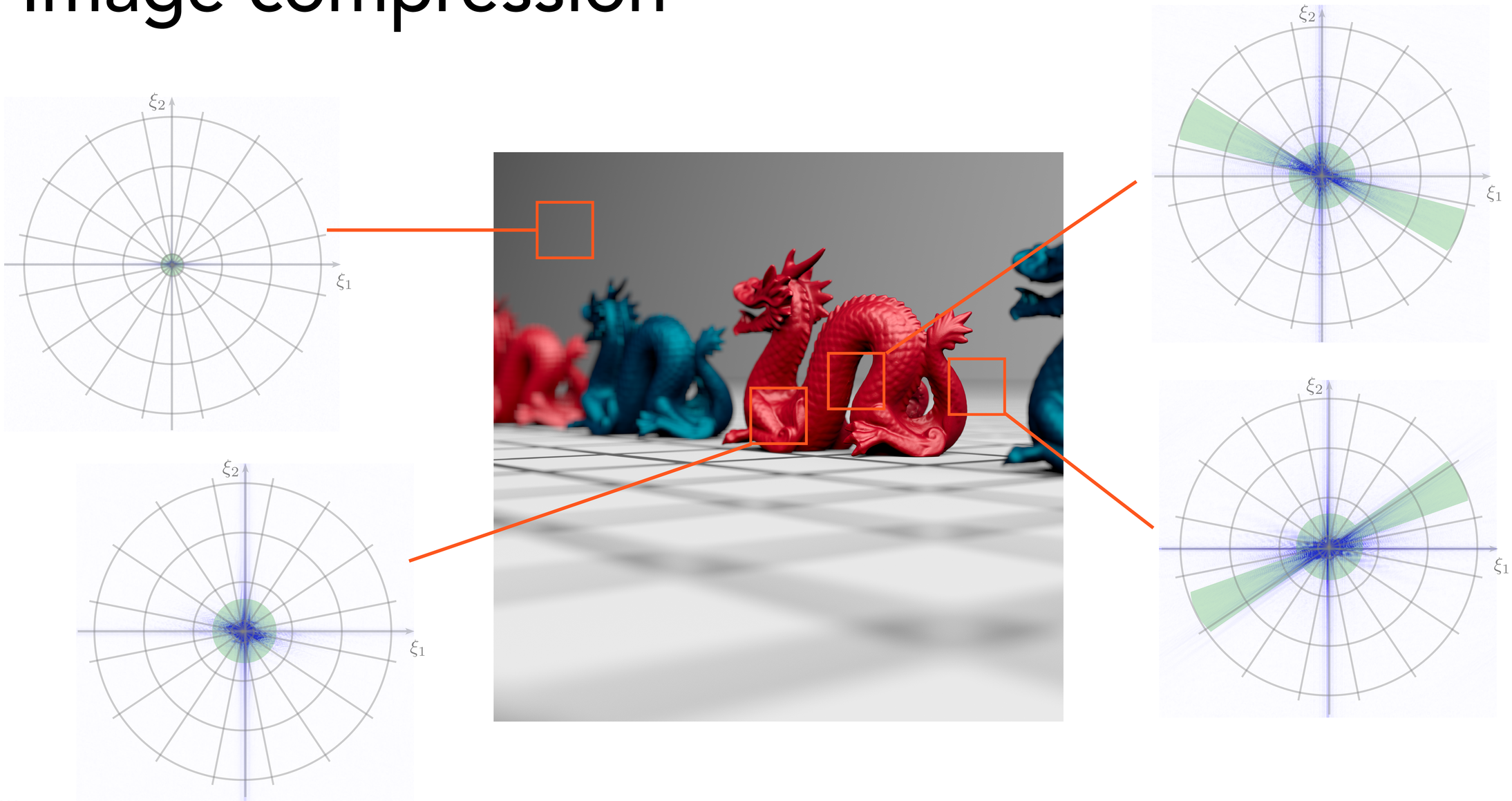

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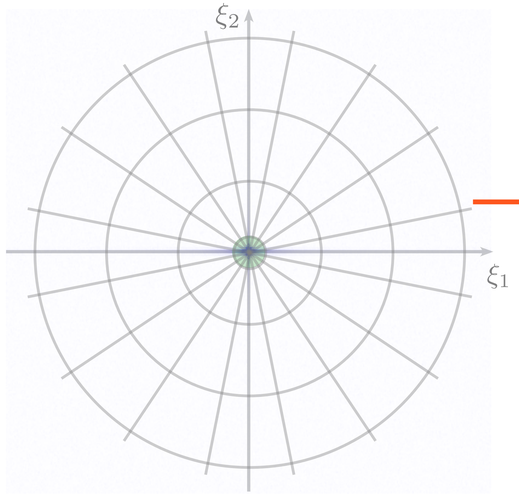
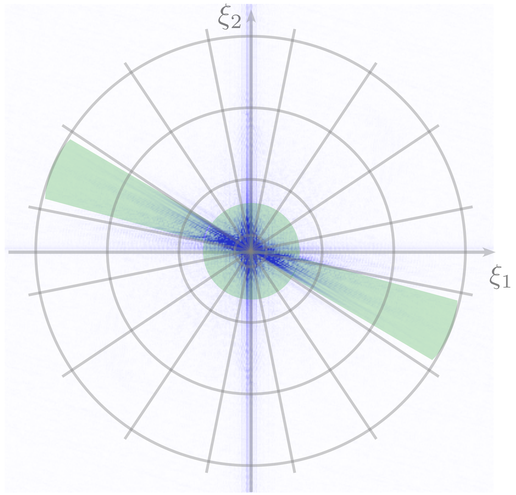
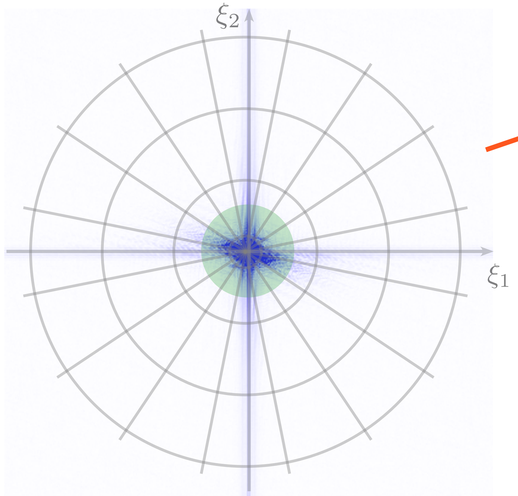


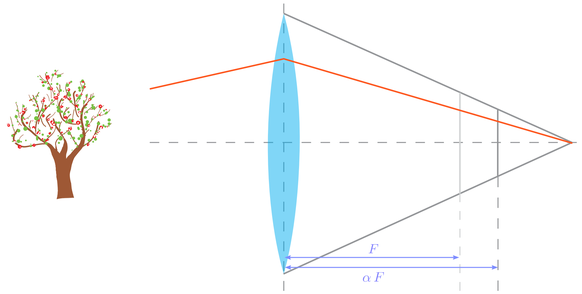




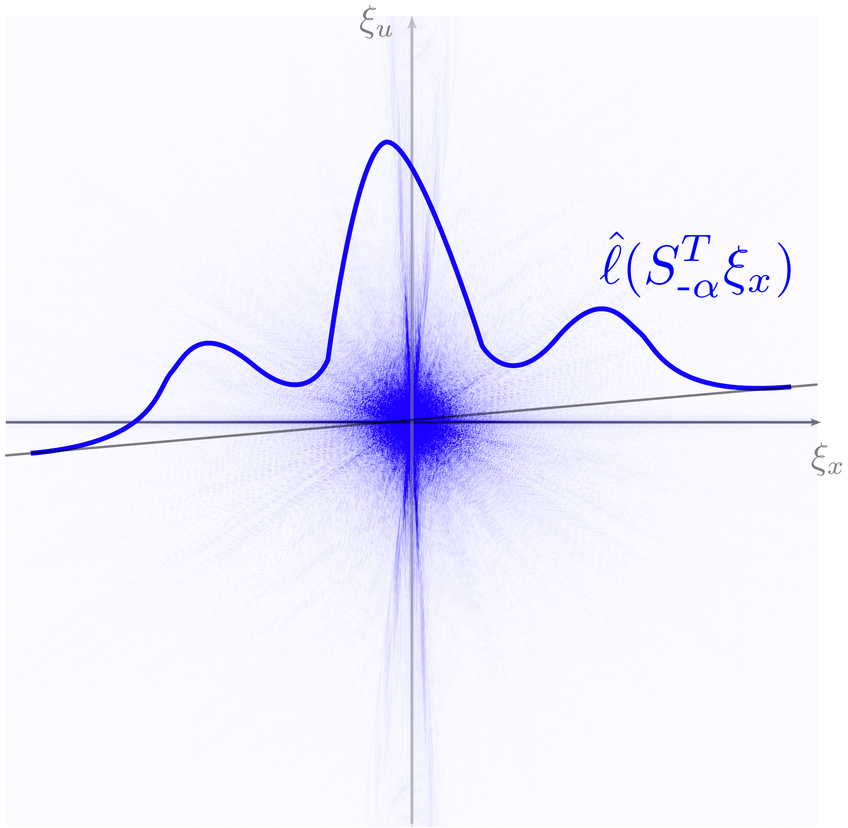
Polar coordinates for images in Fourier domain

- C. Fefferman. A note on spherical summation multipliers. Israel J. of Math., 15(1), 1973
- W. T. Freeman and E. H. Adelson. The design and use of steerable filters. PAMI, 13(9), 1991
- P. Perona. Deformable kernels for early vision. In Proceedings. 1991 IEEE CVPR, 1991.
- E. J. Candes and D. L. Donoho. Curvelets: A Surprisingly Effective Nonadaptive Representation of Objects with Edges. in Curves and Surfaces, 1999.
- M. N. Do and M. Vetterli. Contourlets. In J. Stoeckler and G. V. Welland, editors, Beyond Wavelets, Studies in Computational Mathematics. 2003.
- D. Labate, W.-Q. Lim, G. Kutyniok, and G. Weiss. Sparse Multidimensional Representation using Shear- lets. In M. Papadakis, A. F. Laine, and M. A. Unser, editors, Wavelets XI, pages 254–262. International Society for Optics and Photonics, aug 2005.
- ...

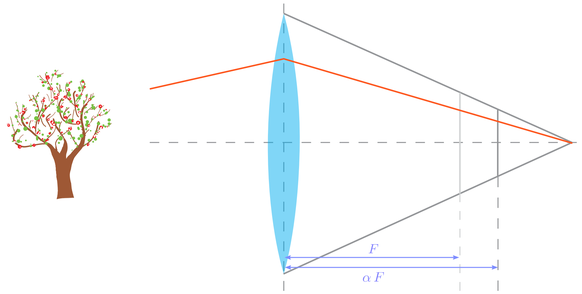
Light field refocusing



$$
\begin{aligned}
I(x) &= \frac{1}{\alpha^2} \int_{\mathbb{R}_u} \ell_\alpha(x,u)\,\mathrm{d}u \\
&= \frac{1}{\alpha^2} \int_{\mathbb{R}_u} \mathcal{F}_{2\mathrm{d}}^{-1}\big(\hat{\ell}_\alpha(\xi_x,\xi_u)\big)\,\mathrm{d}u \\
&= \frac{1}{\alpha^2} \mathcal{F}_{1\mathrm{d}}^{-1}\big(\hat{\ell}_\alpha(\xi_x,0)\big) \\
&= \frac{1}{\alpha^2} \mathcal{F}_{1\mathrm{d}}^{-1}\big(\hat{\ell}(S_{-\alpha}^T\xi_x)\big)
\end{aligned}
$$



Light field refocusing



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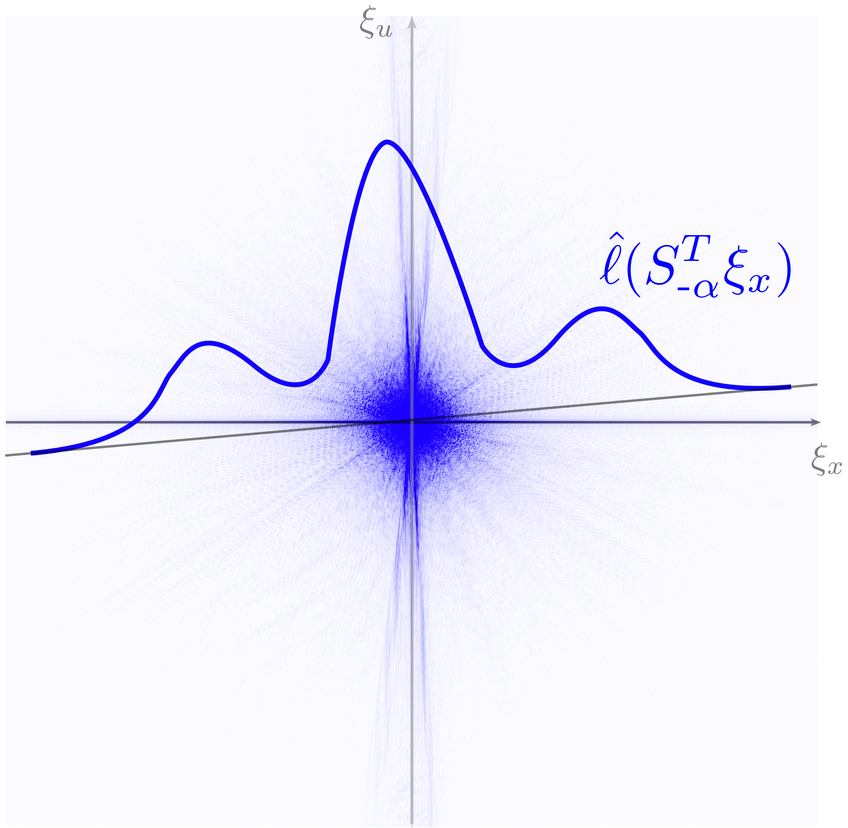
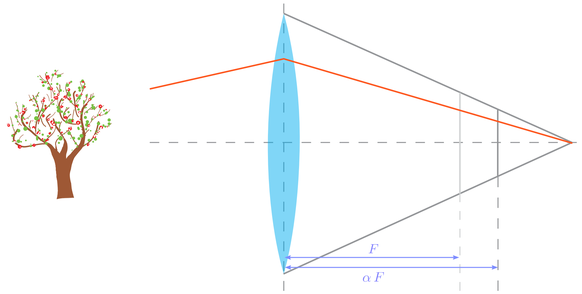


Light field refocusing



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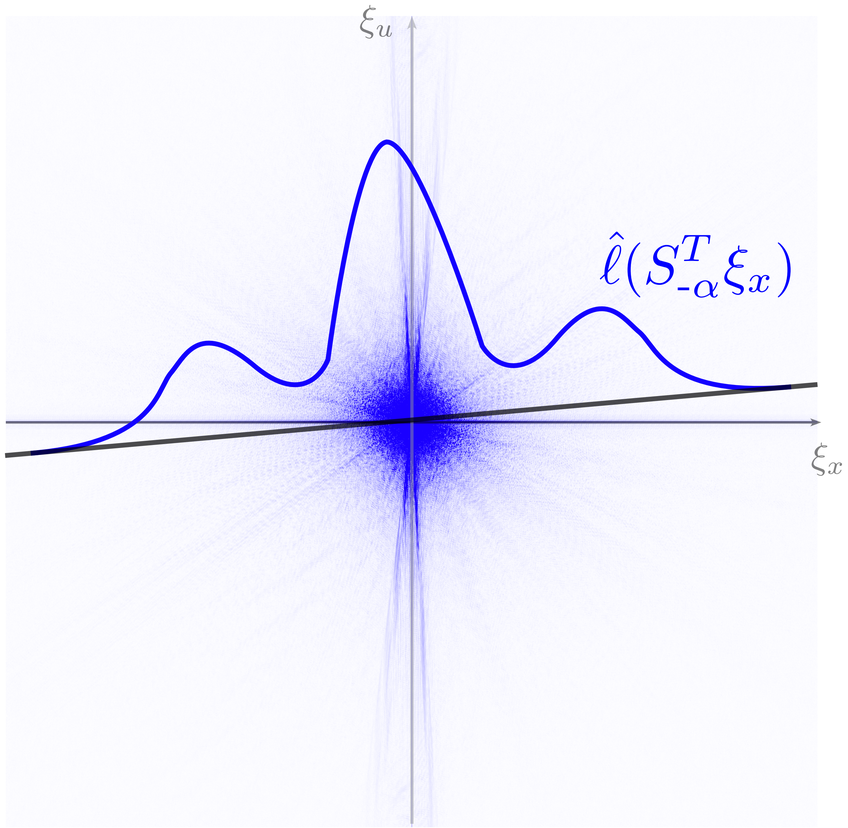
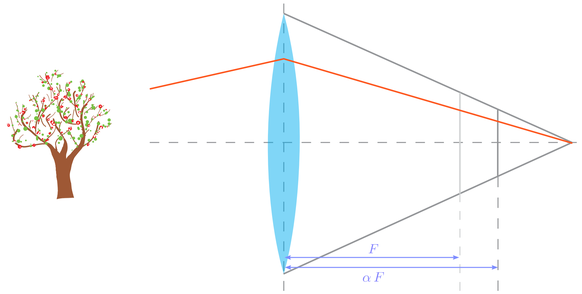


Light field refocusing



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\end{aligned}
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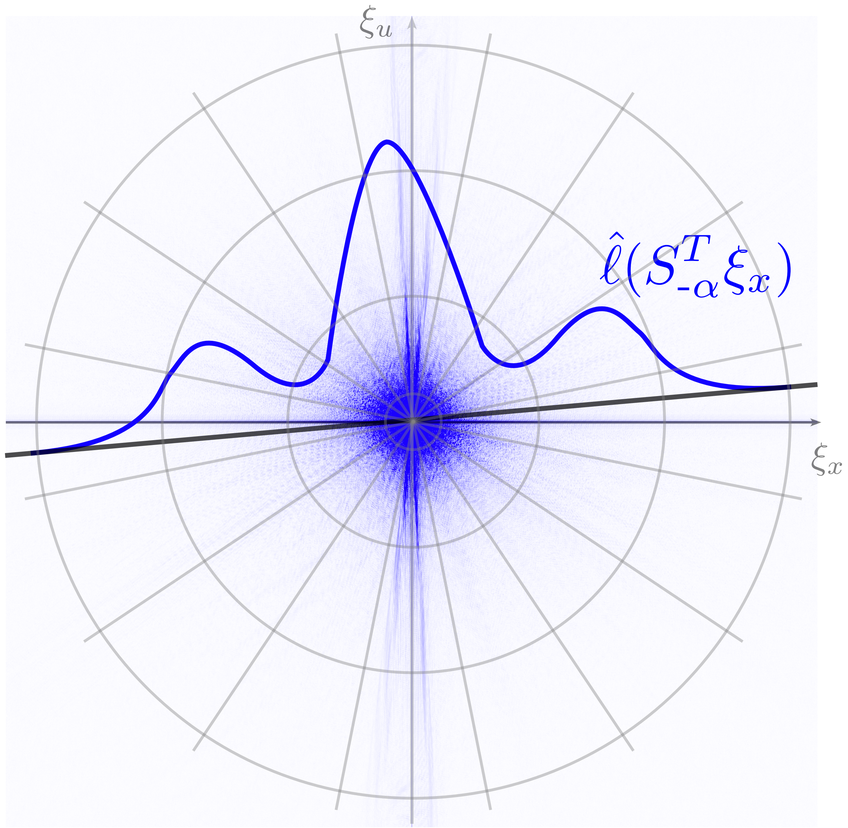
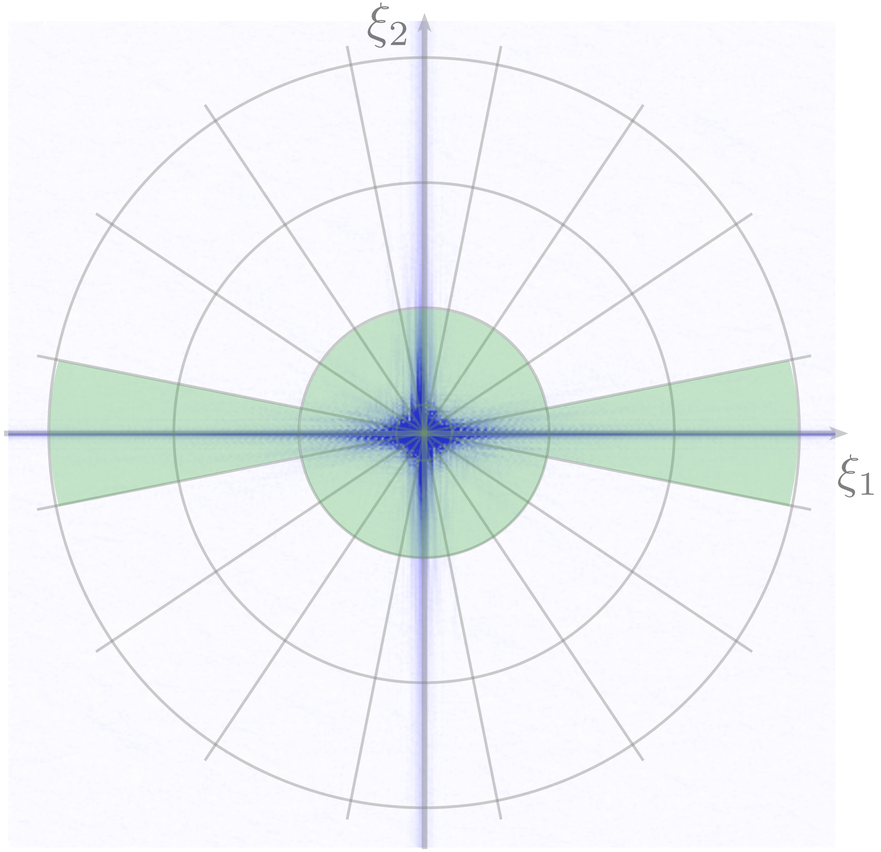
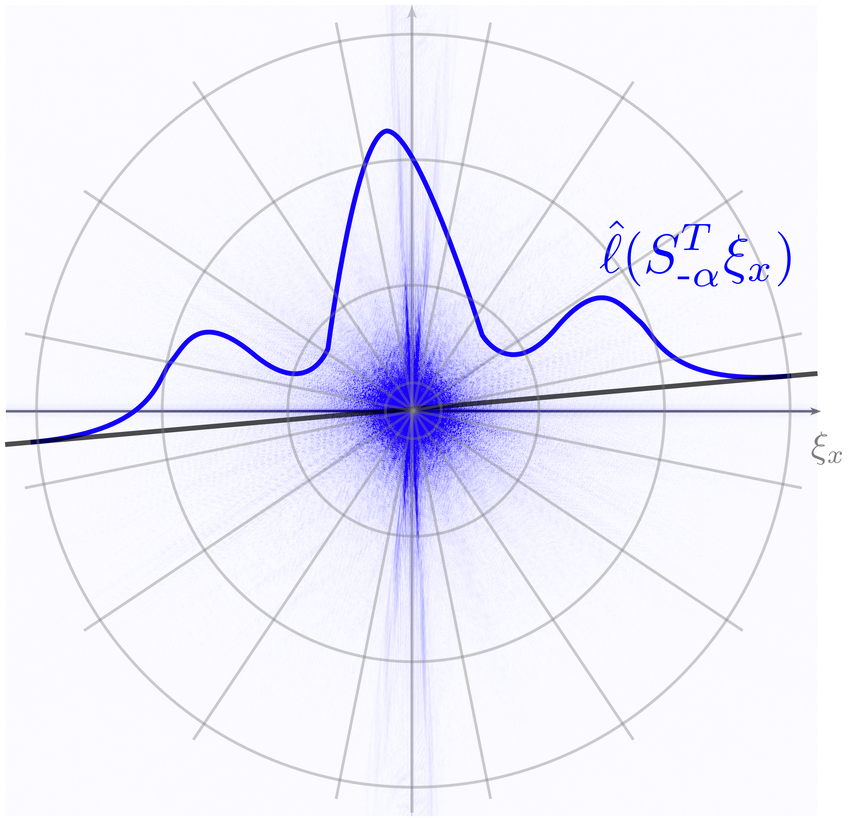


Restriction to 1d subspace in polar coordinates

Polar coordinates to the rescue

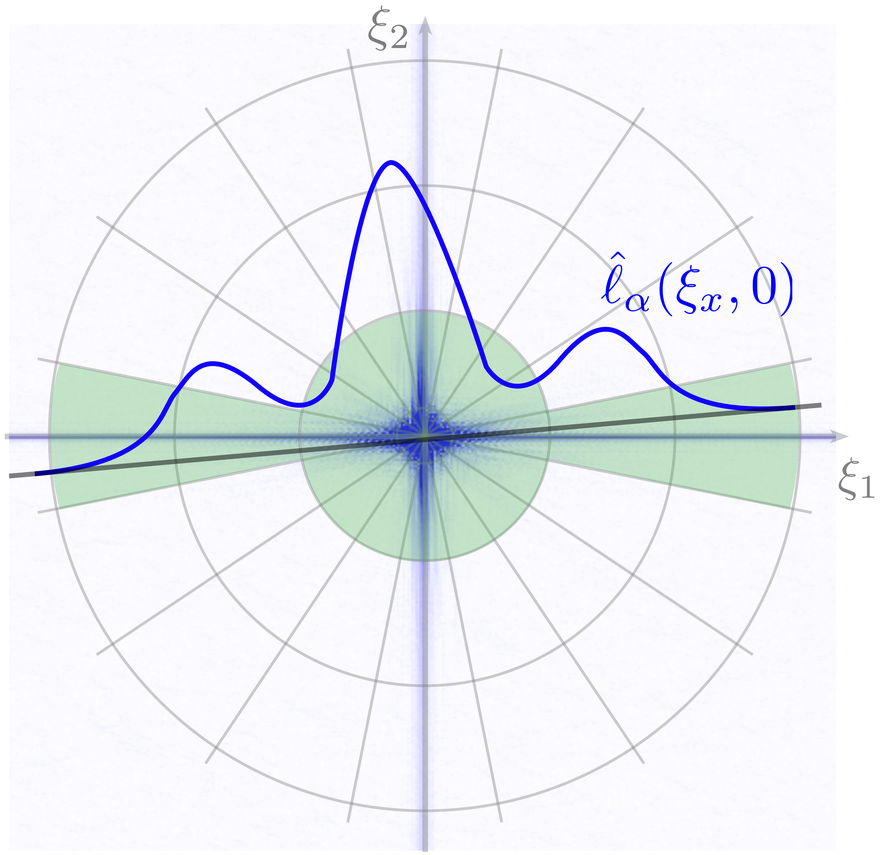


compression



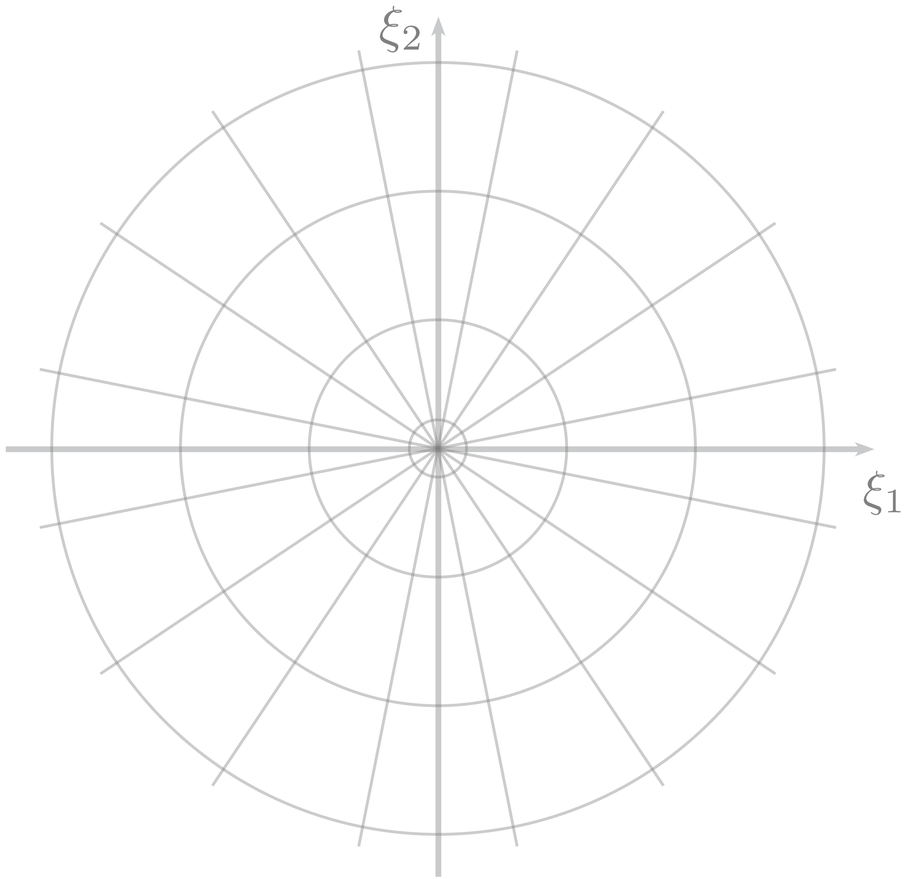
refocusing

Polar coordinates to the rescue

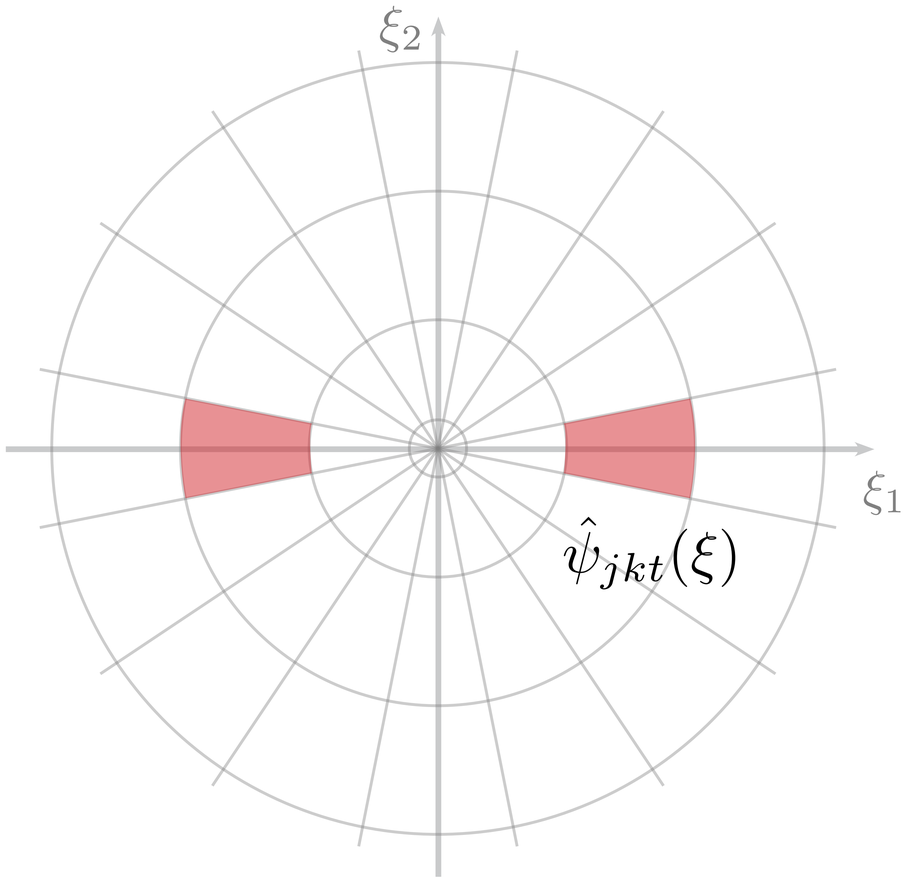


C. Lessig. A local Fourier slice equation. Optics Express, 26(23):29769–29783, nov 2018.

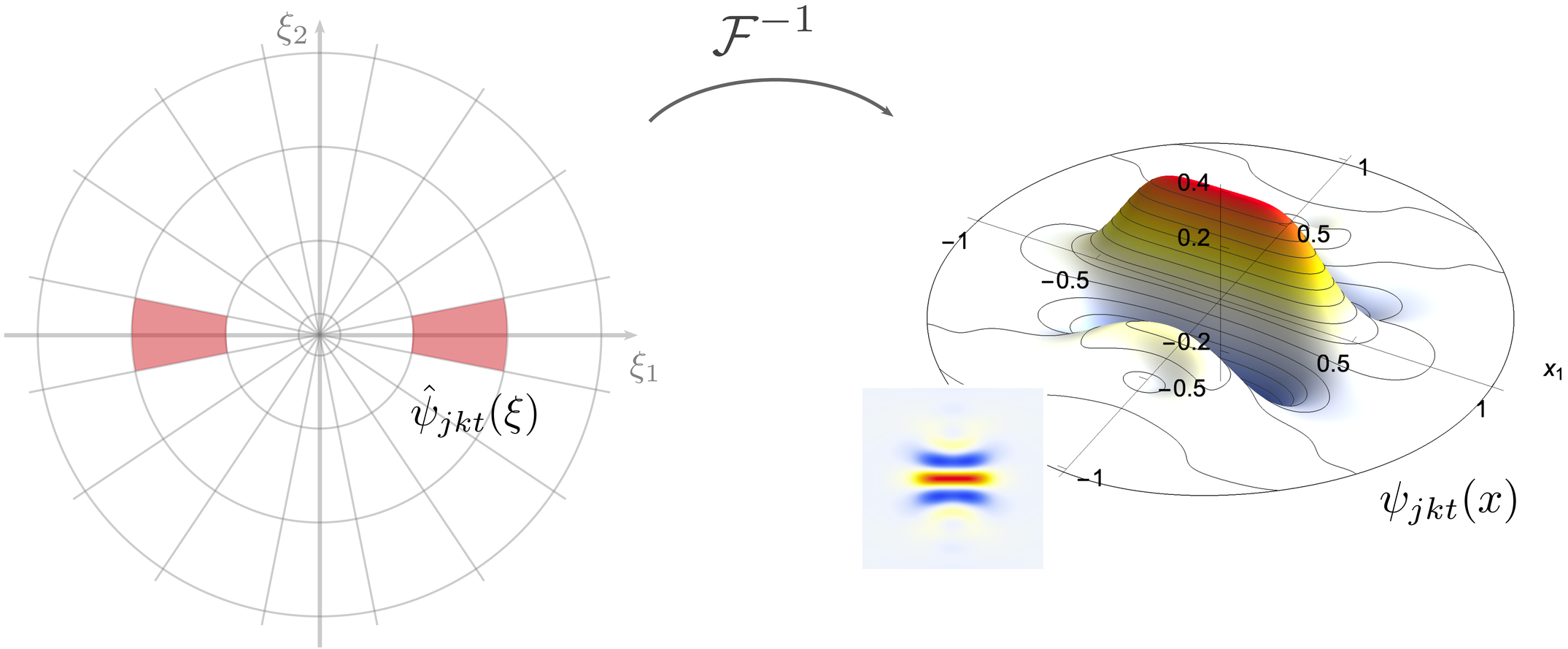
Polar wavelets



Polar wavelets

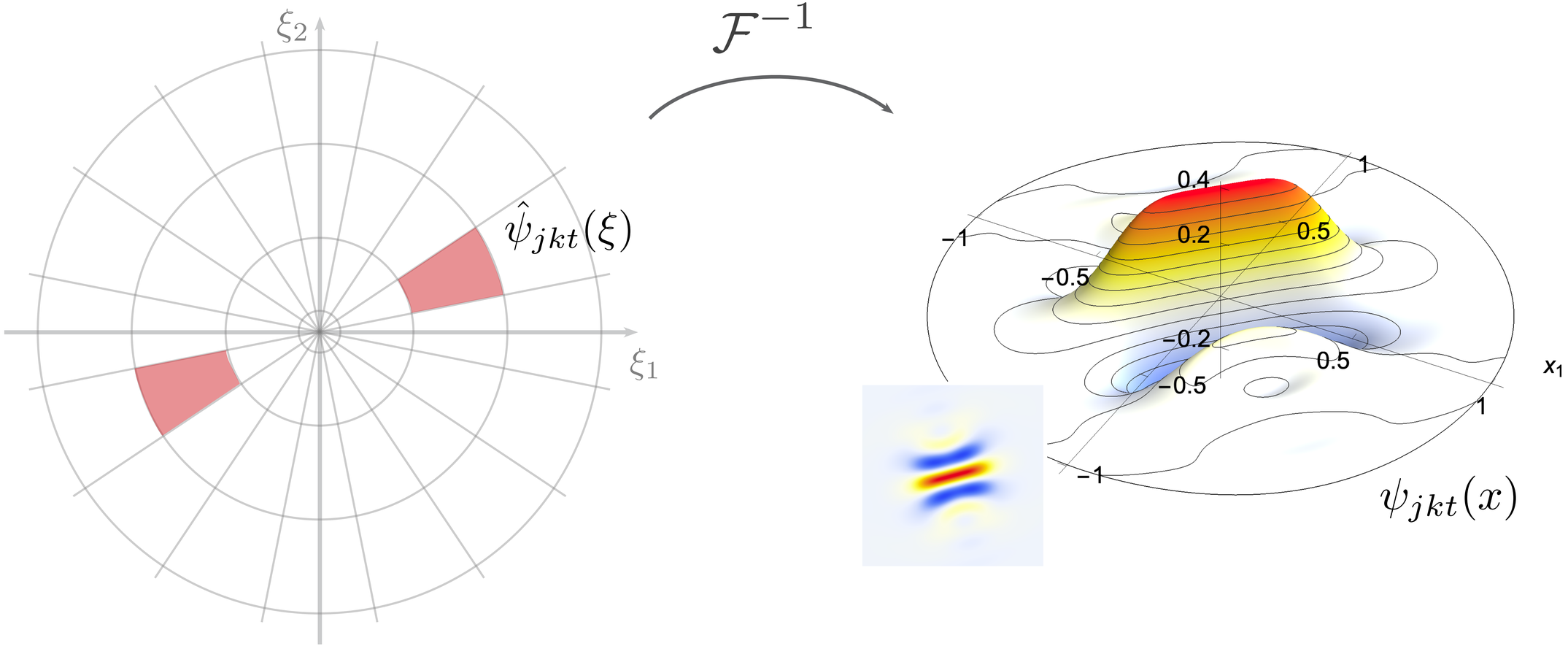


Polar wavelets



M. Unser and N. Chenouard. A Unifying Parametric Framework for 2D Steerable Wavelet Transforms. SIAM Journal on Imaging Sciences, 6(1):102–135, jan 2013.
C. Lessig. Polar Wavelets in Space. Technical report, 2018.

Polar wavelets



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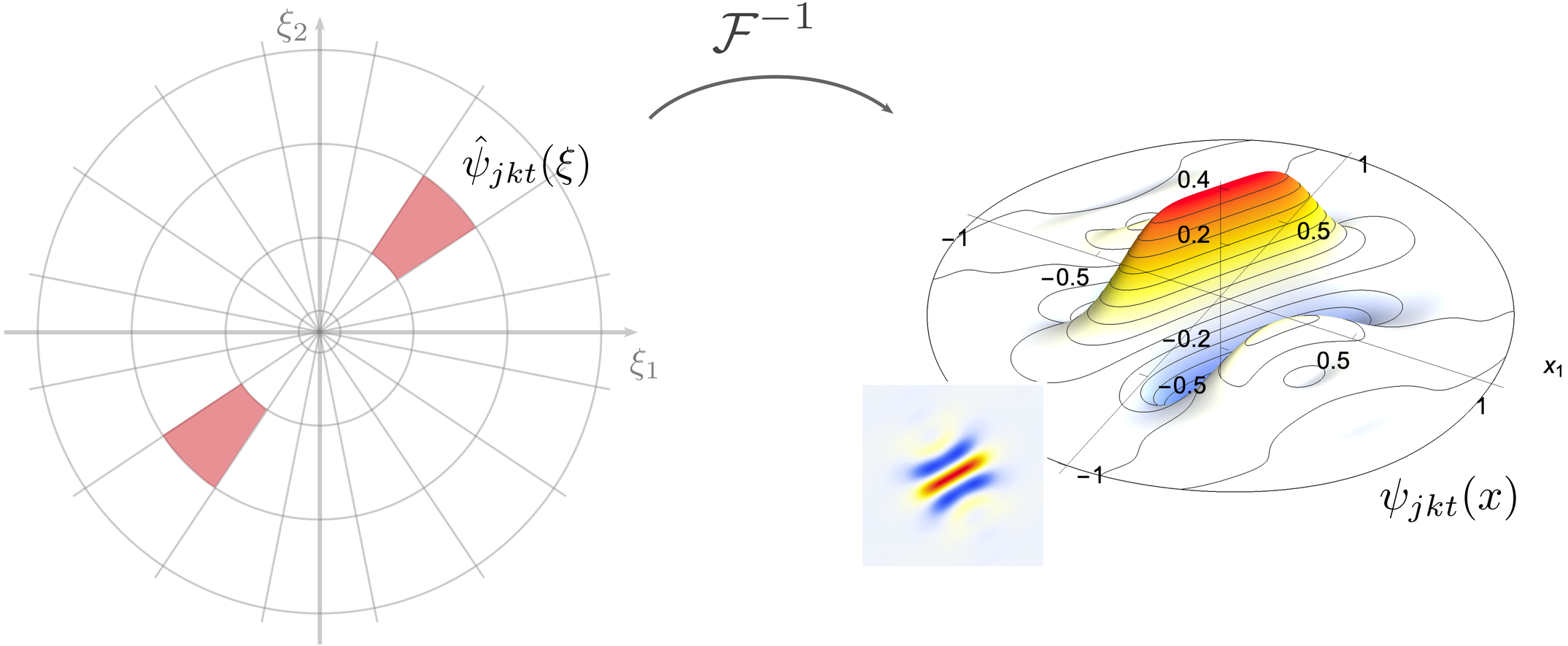

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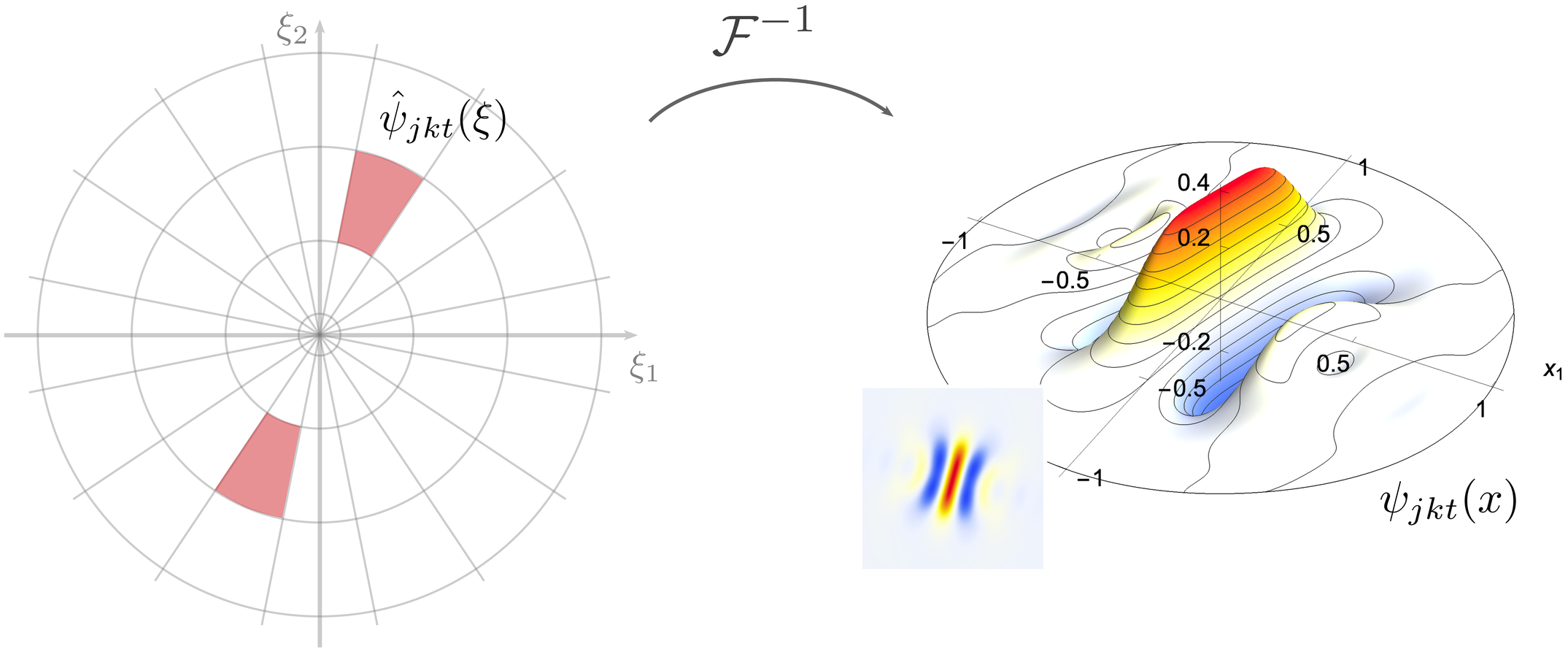
Polar wavelets



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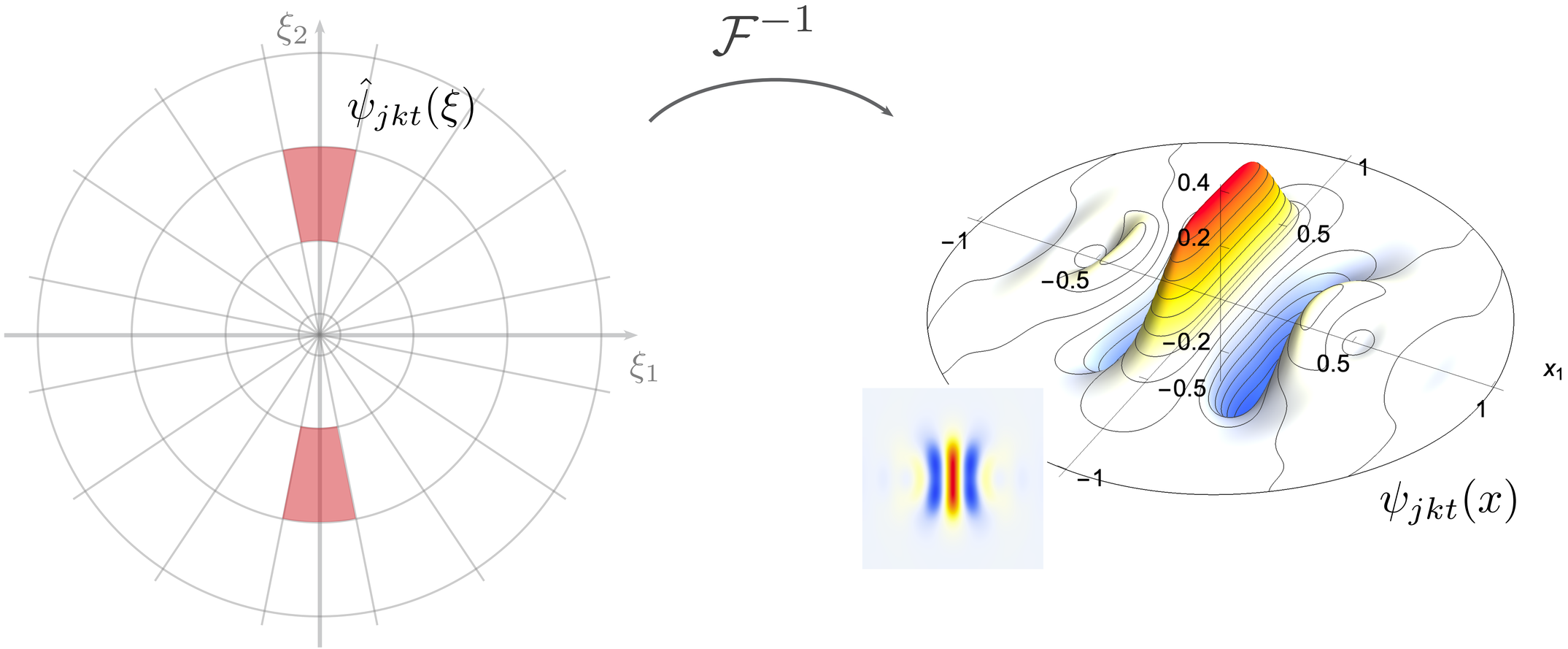

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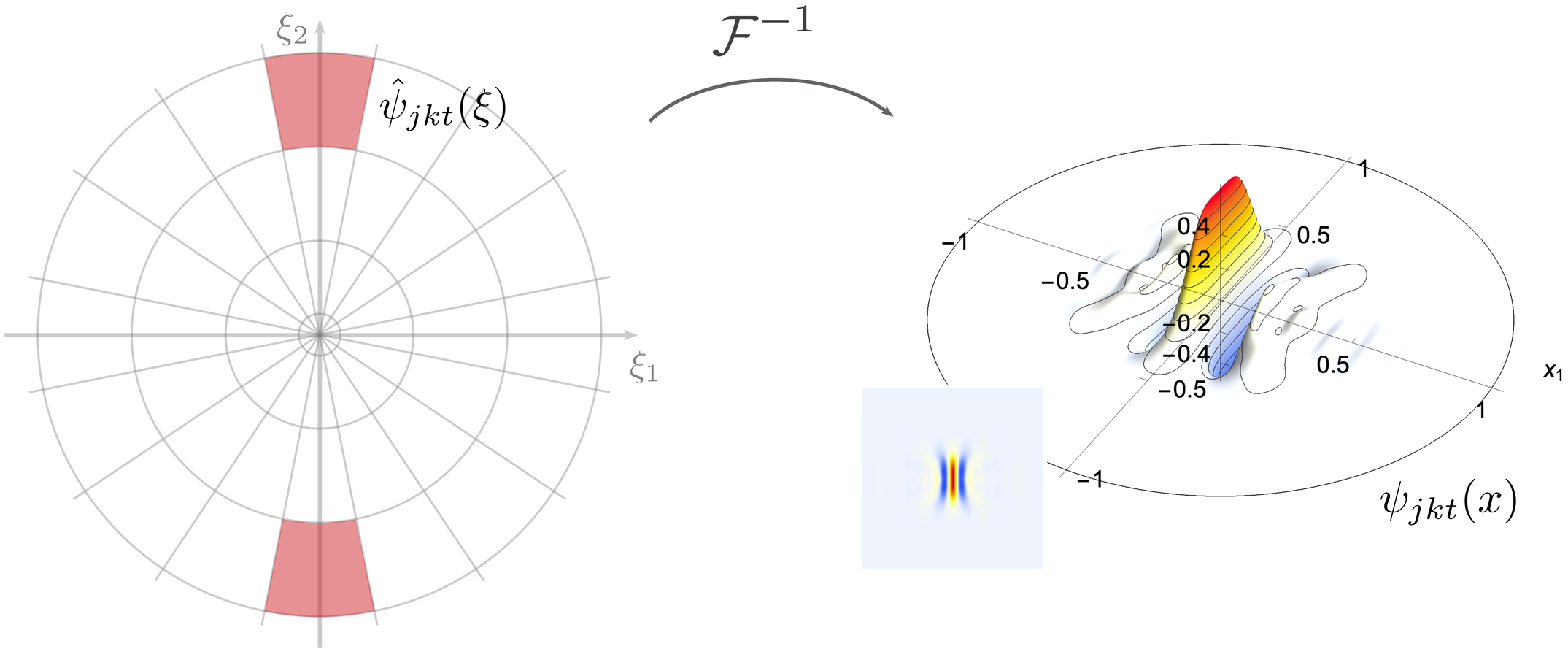

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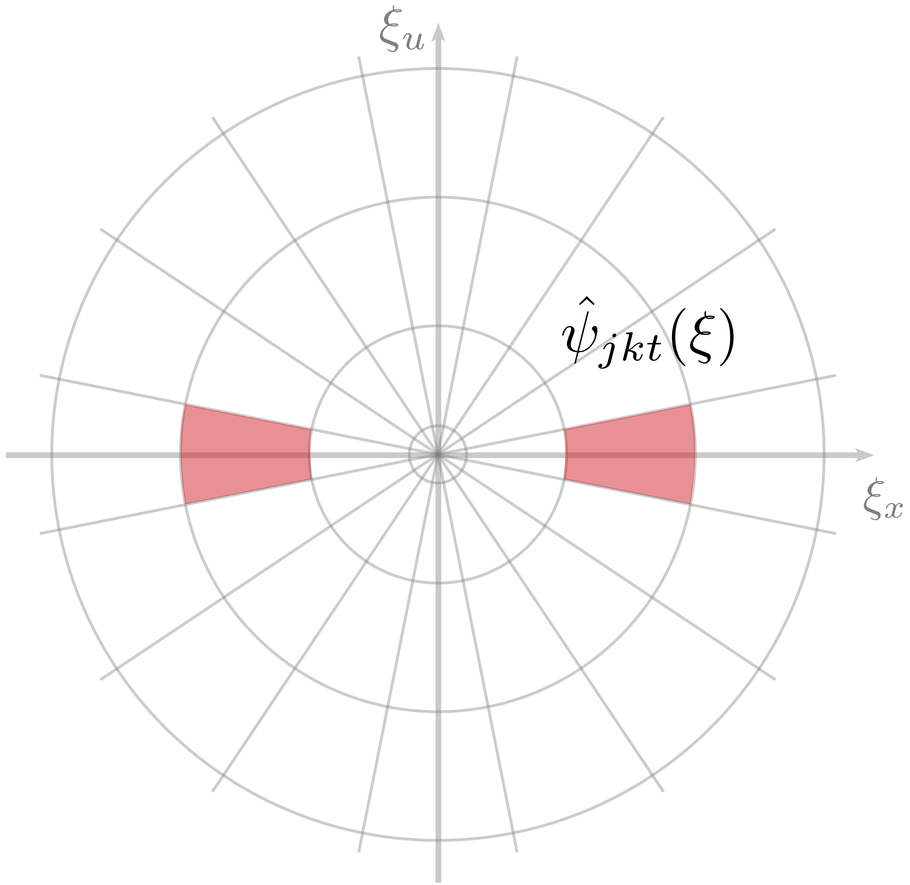

Compressed representation:

$$\ell(x,u) = \sum_{j=0}^{J} \sum_{k\in\mathbb{Z}^2} \sum_{t=0}^{T_j} \ell_{jkt}\, \psi_{jkt}(x,u)$$

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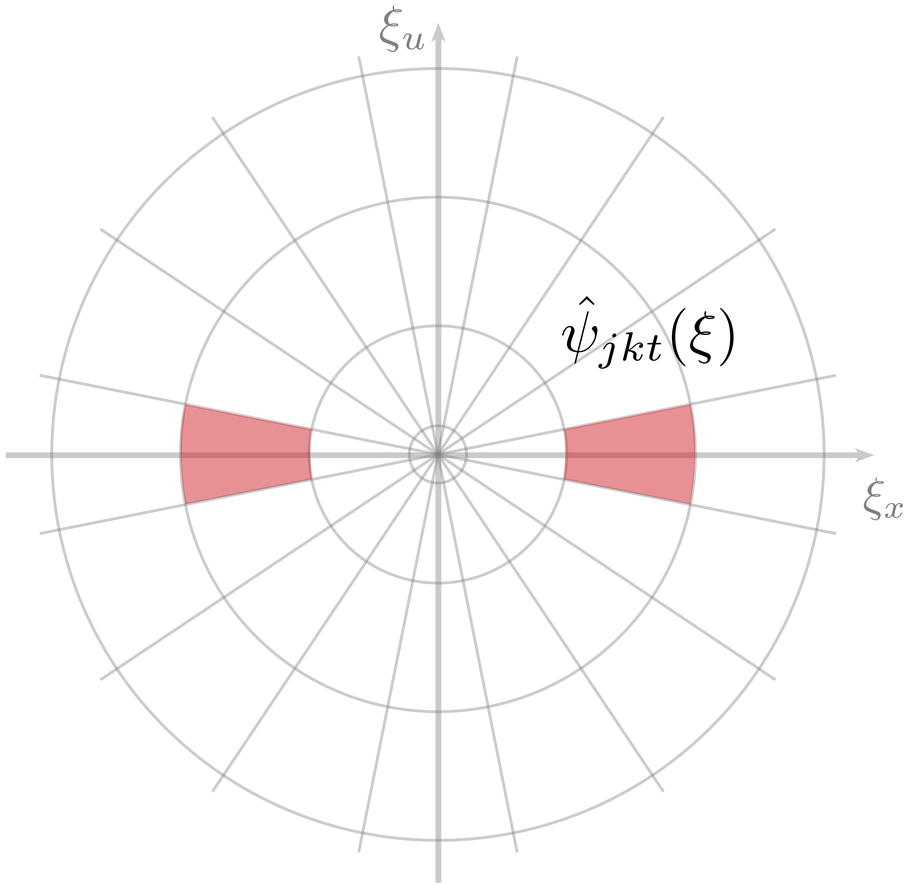

Compressed representation:

$$\ell(x,u) = \sum_{j=0}^{J} \sum_{k\in\mathbb{Z}^2} \sum_{t=0}^{T_j} \ell_{jkt}\, \psi_{jkt}(x,u)$$

scale

position

orientation

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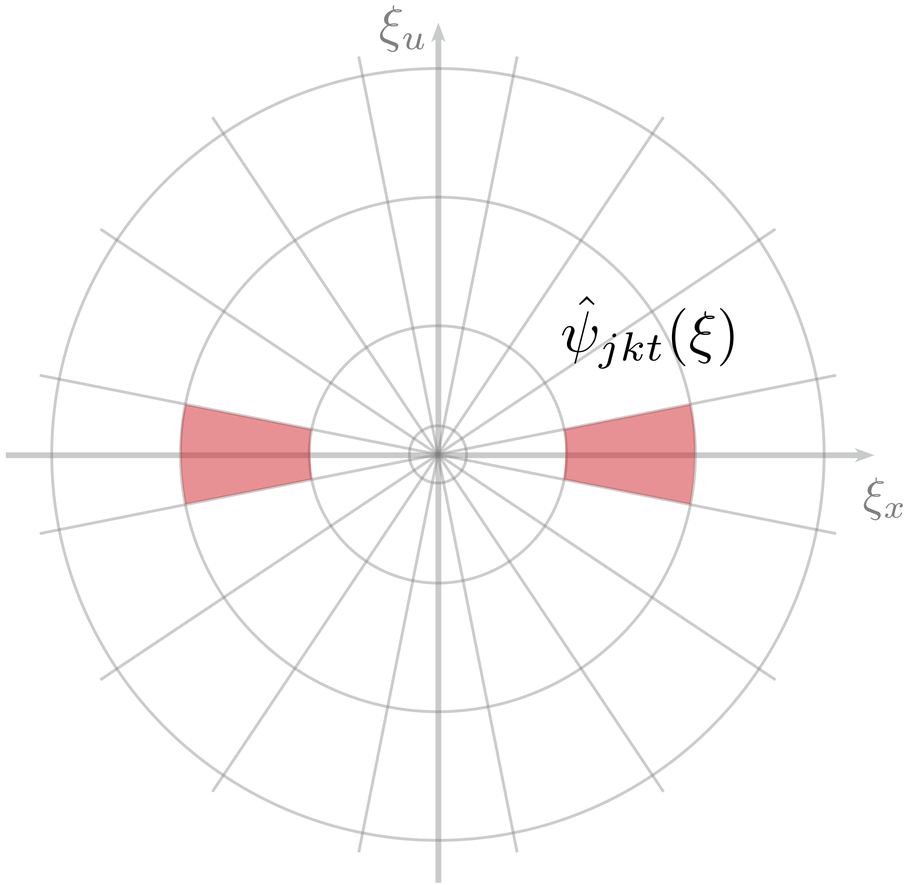
C. Lessig. Polar Wavelets in Space. Technical report, 2018.

Polar wavelets



Compressed representation:

$$\ell(x,u) = \sum_{j=0}^{J} \sum_{k\in\mathbb{Z}^2} \sum_{t=0}^{T_j} \ell_{jkt}\, \psi_{jkt}(x,u)$$

$$= \sum_{s\in\mathcal{S}_\epsilon} \ell_s\, \psi_s(x,u)$$

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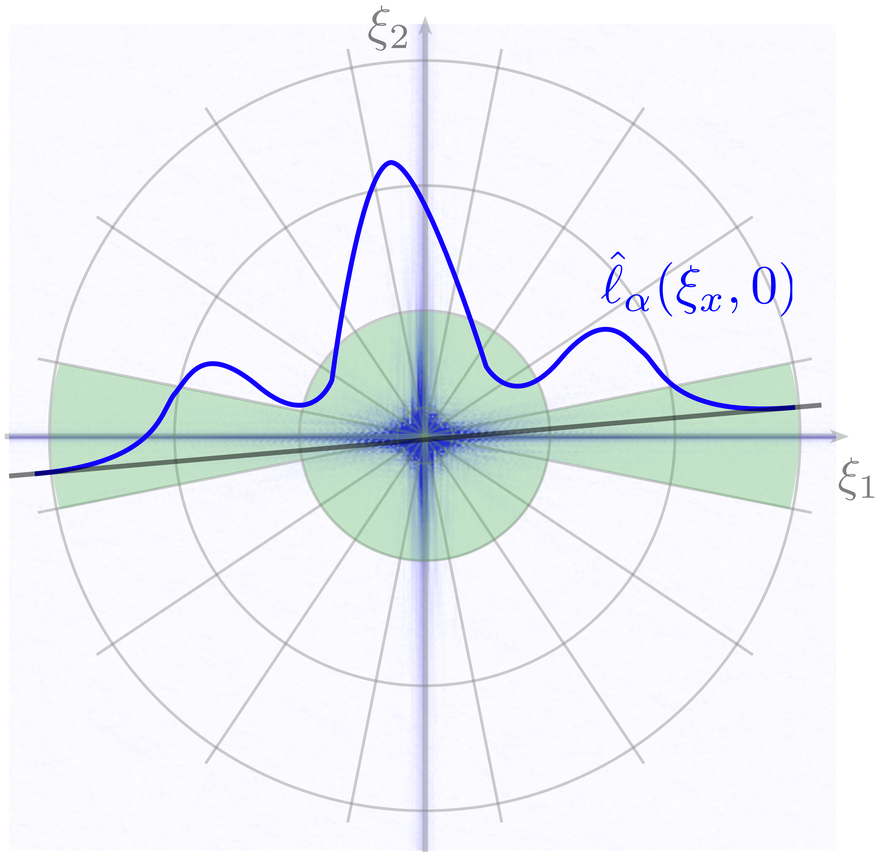
Polar coordinates to the rescue



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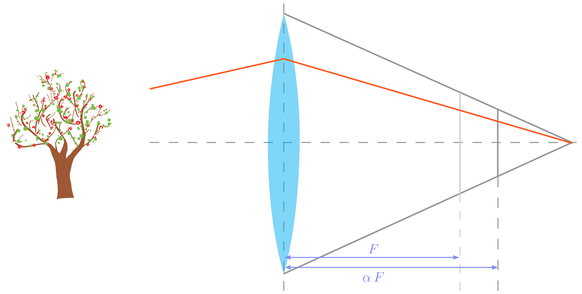
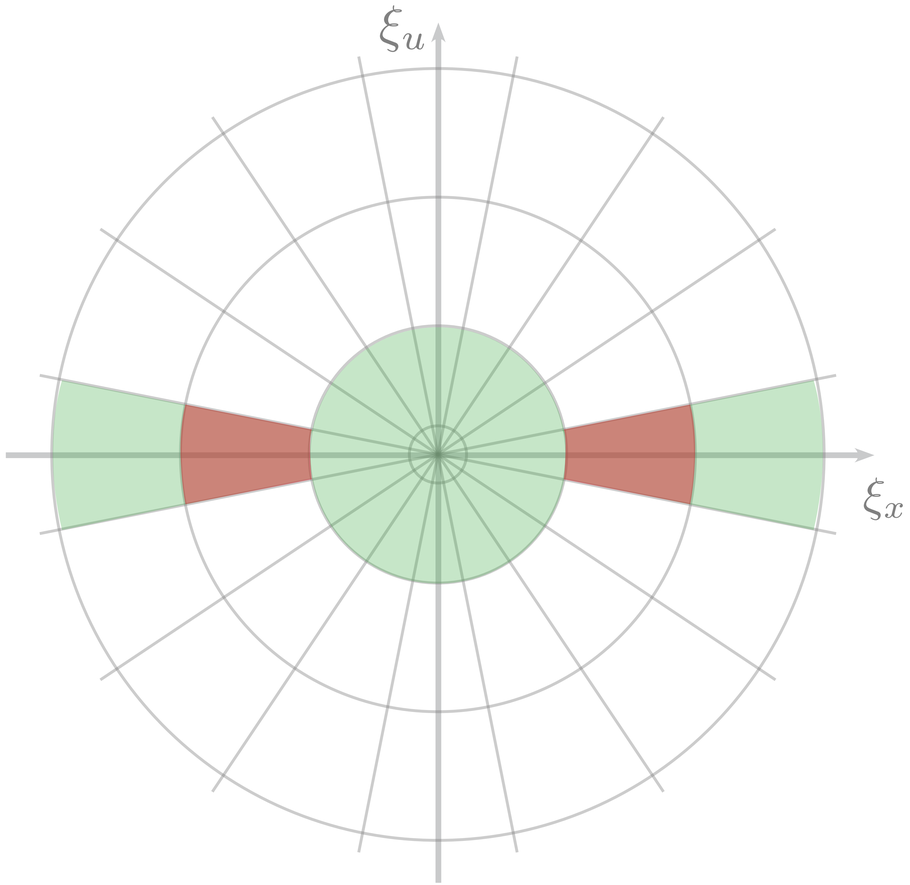
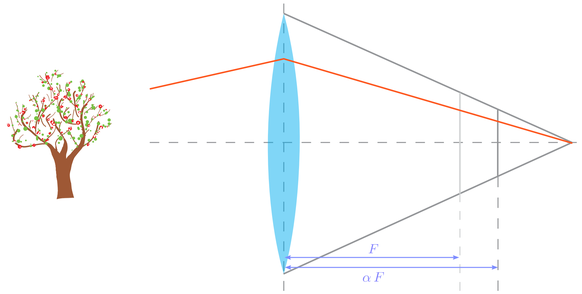
Light field refocusing

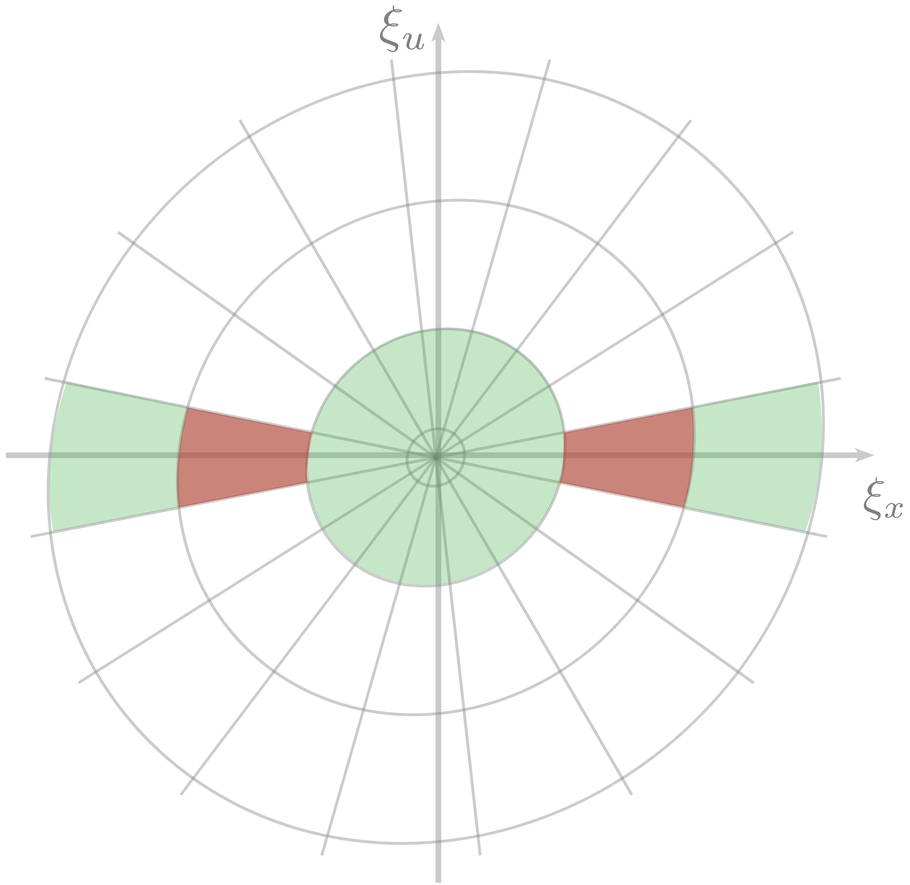




$$I(x) = \frac{1}{\alpha^2} \int_{\mathbb{R}_u} \ell_\alpha(x, u)\, \mathrm{d}u$$

Light field refocusing





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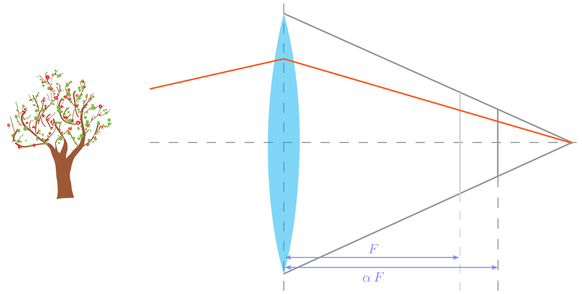
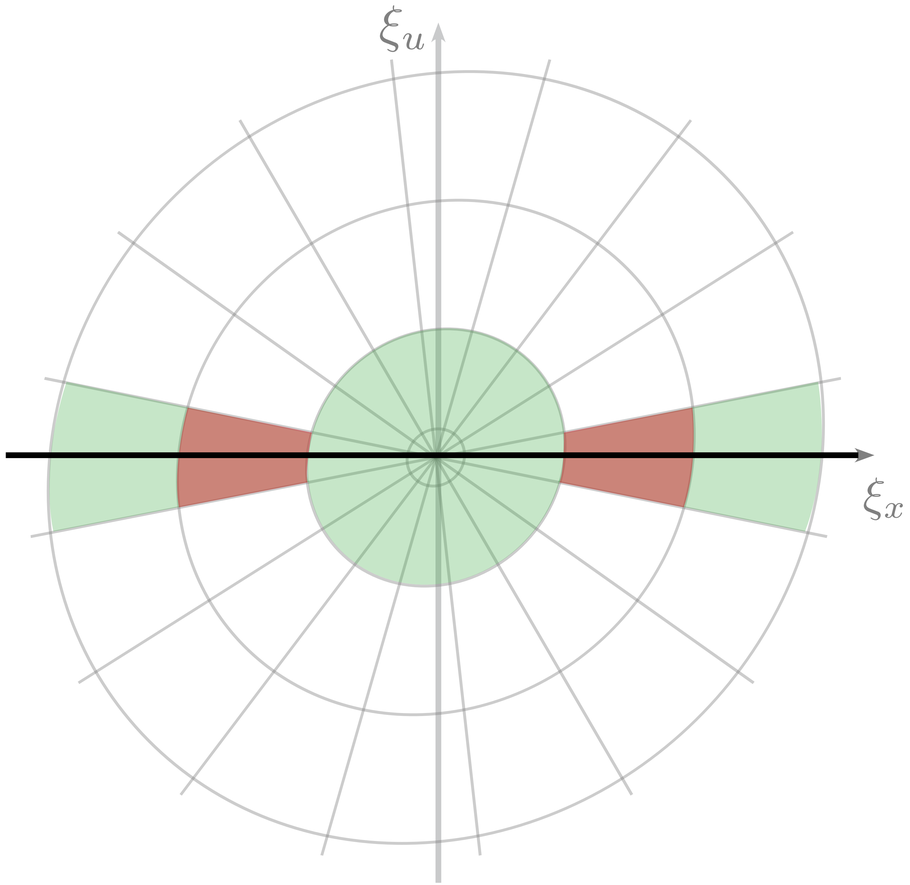
$$= \frac{1}{\alpha^2} \mathcal{F}_{1\mathrm{d}}^{-1}\left(\hat{\ell}(S_{-\alpha}^T \xi_x)\right)$$

Light field refocusing





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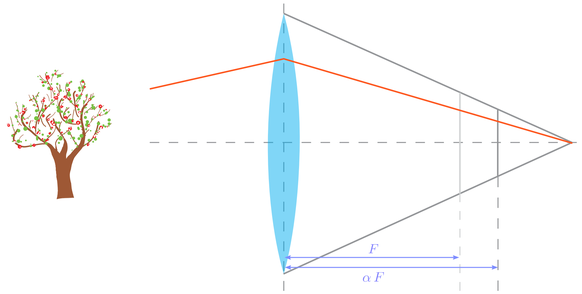

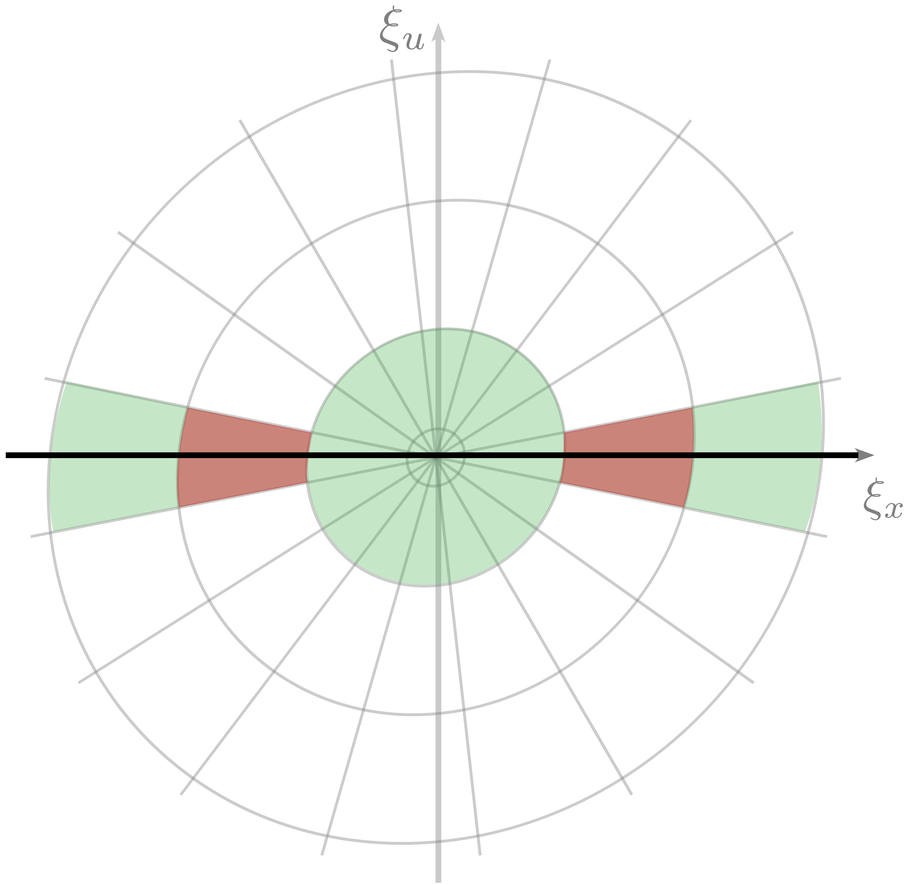

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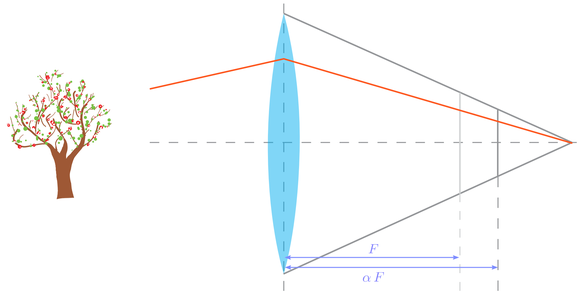
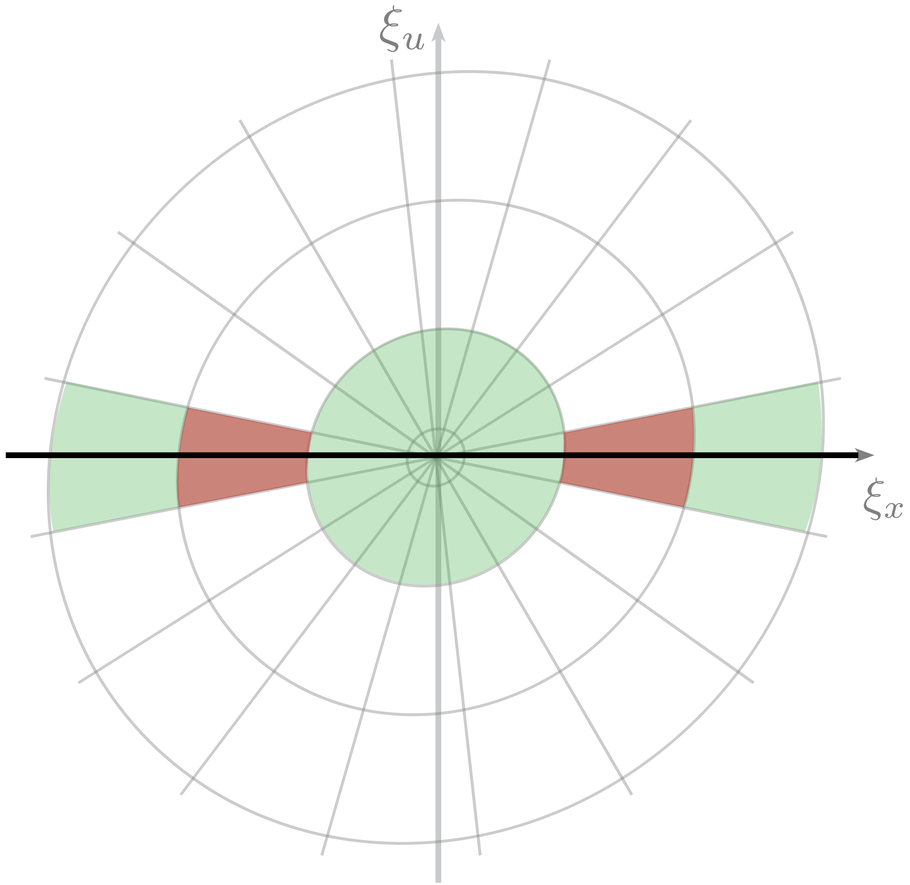
Light field refocusing





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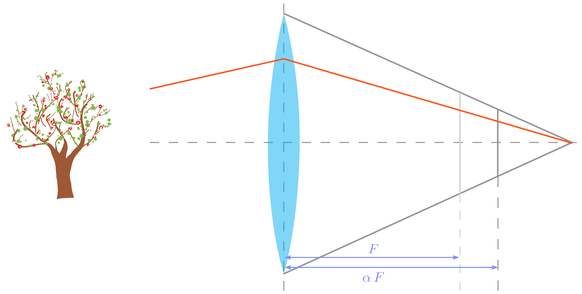
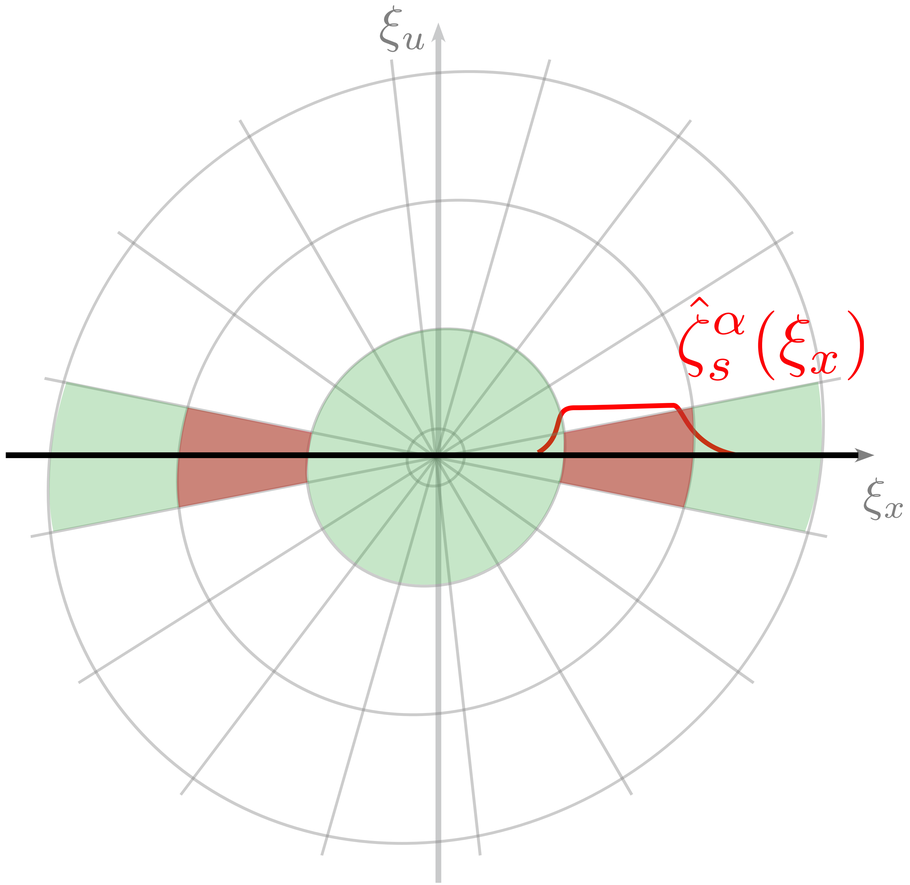
Light field refocusing





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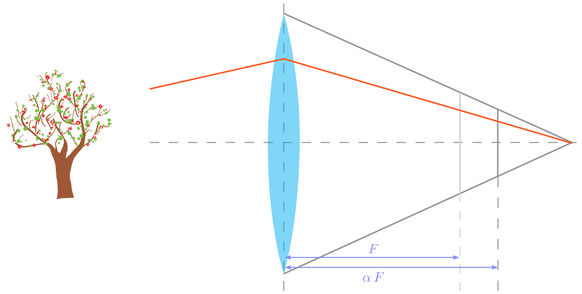
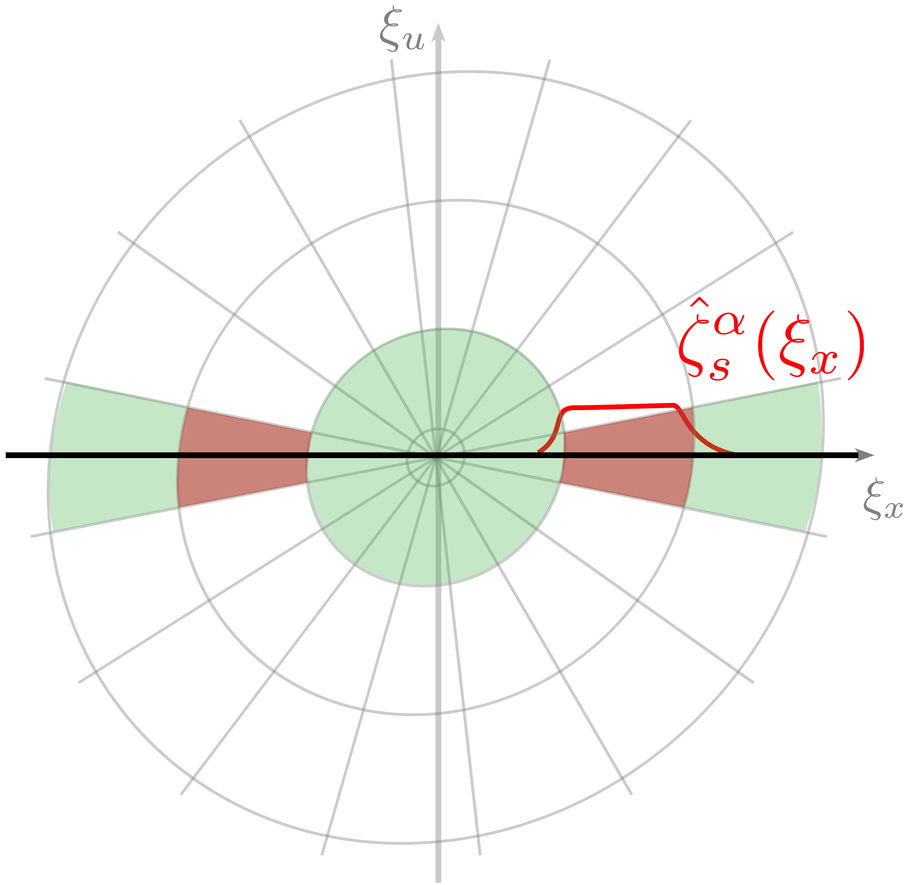
Light field refocusing





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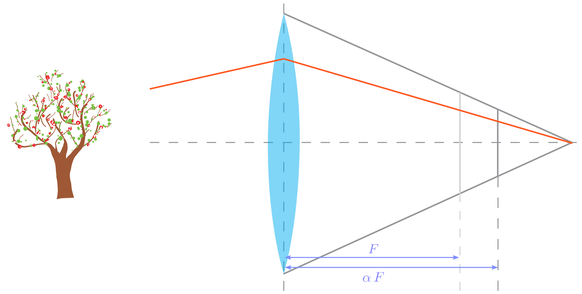
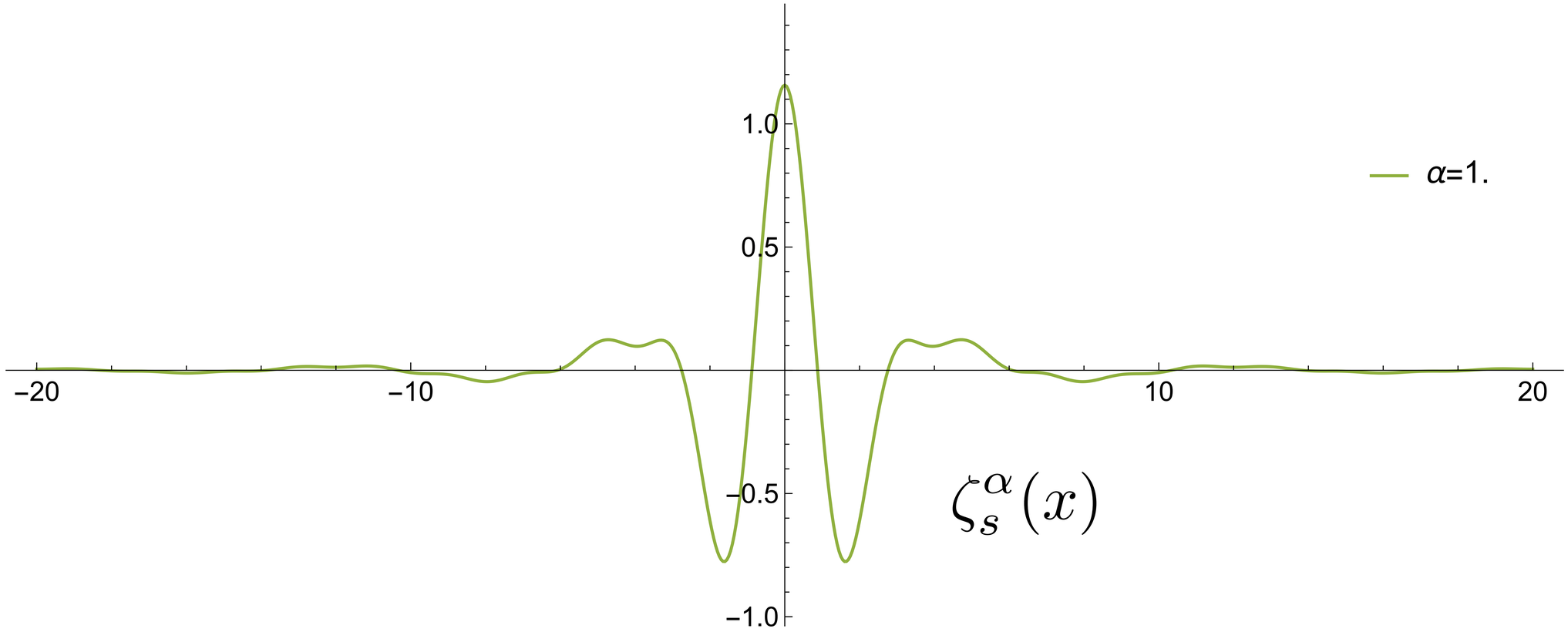
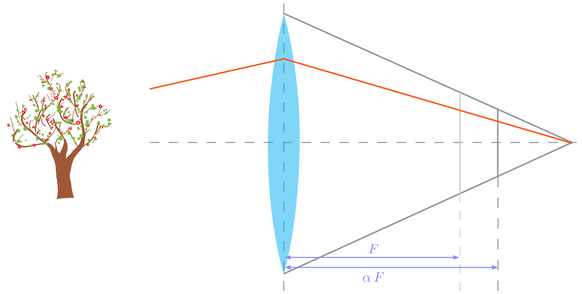
analytic inverse
Fourier transform

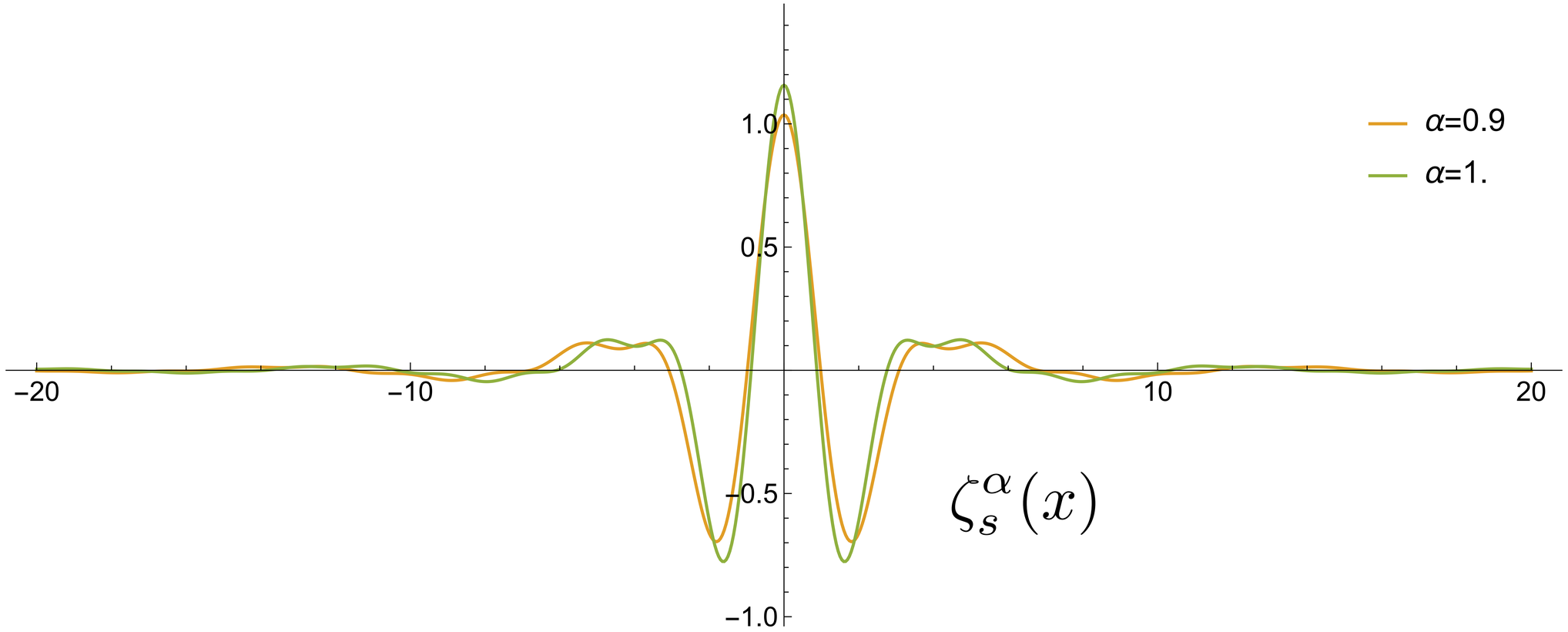
Light field refocusing



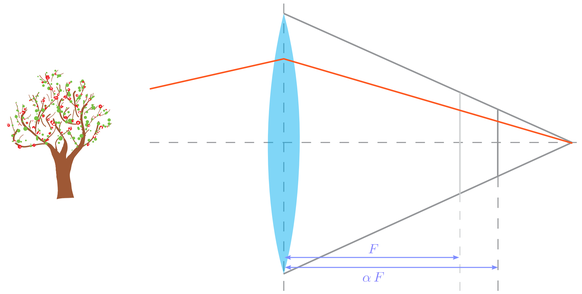


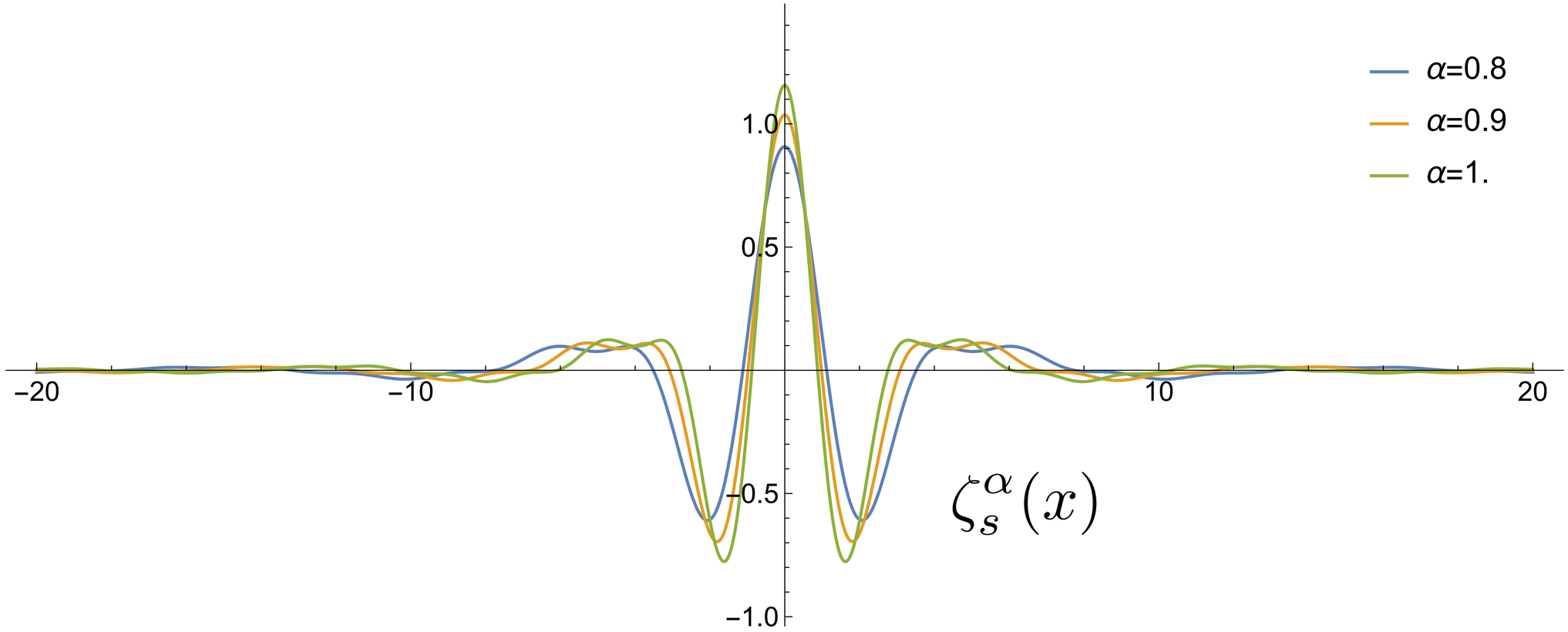
Light field refocusing



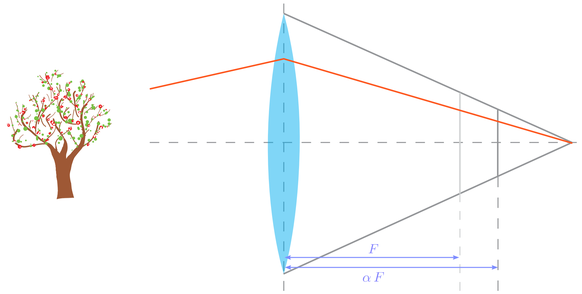


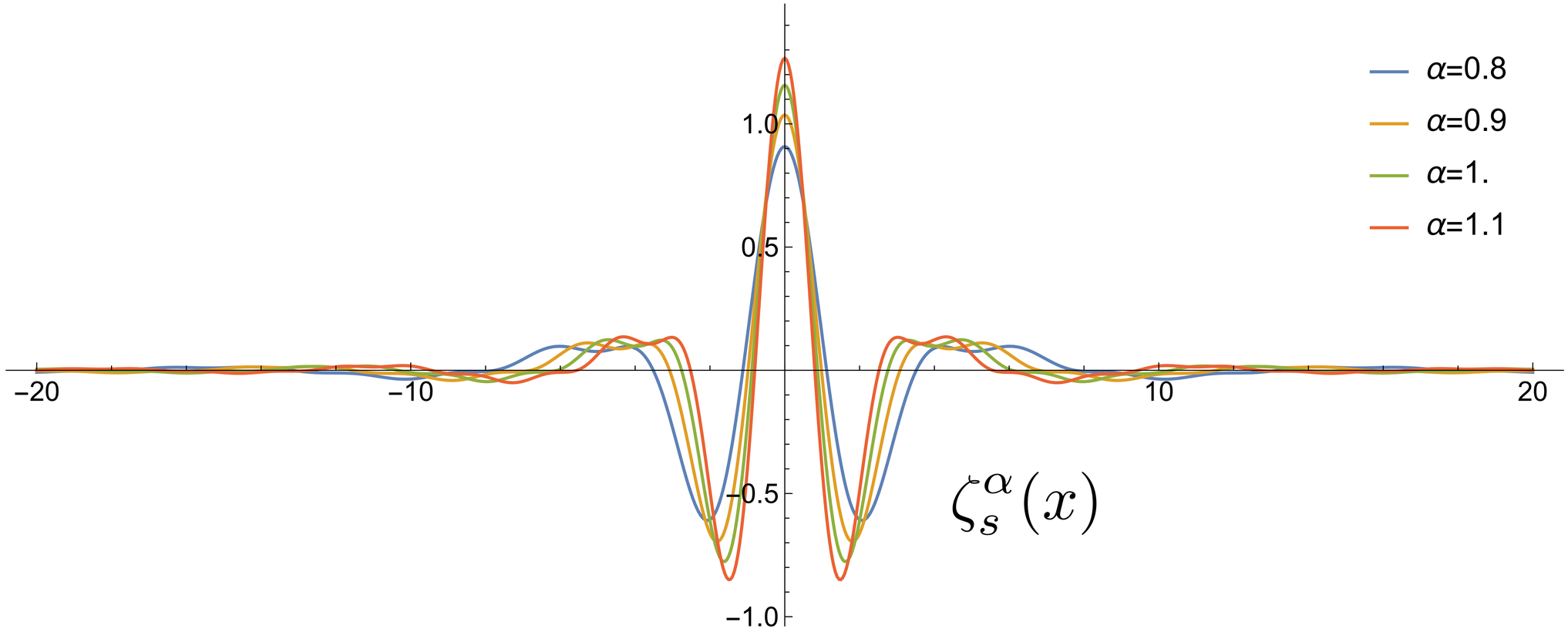
Light field refocusing



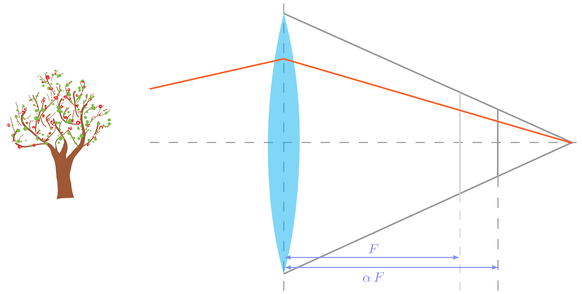


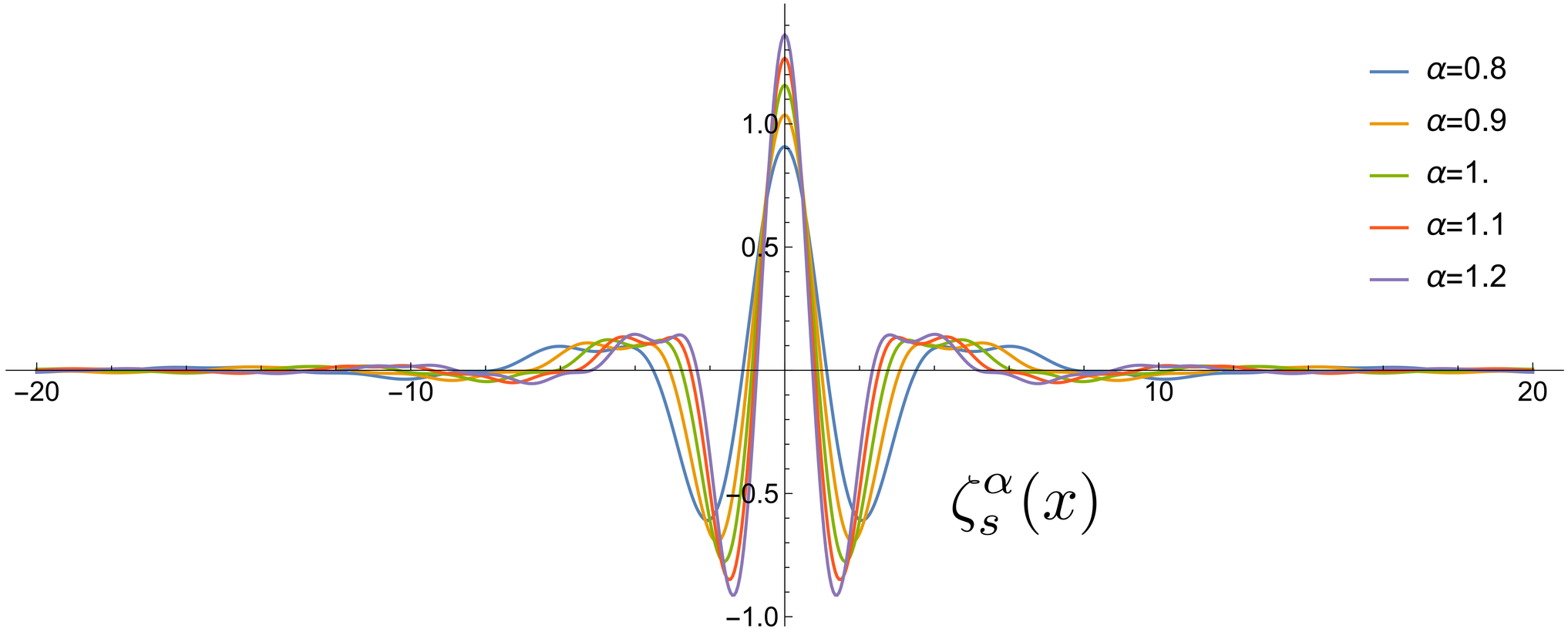
Light field refocusing





Light field refocusing





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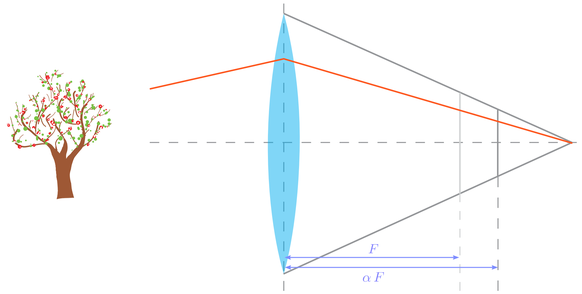

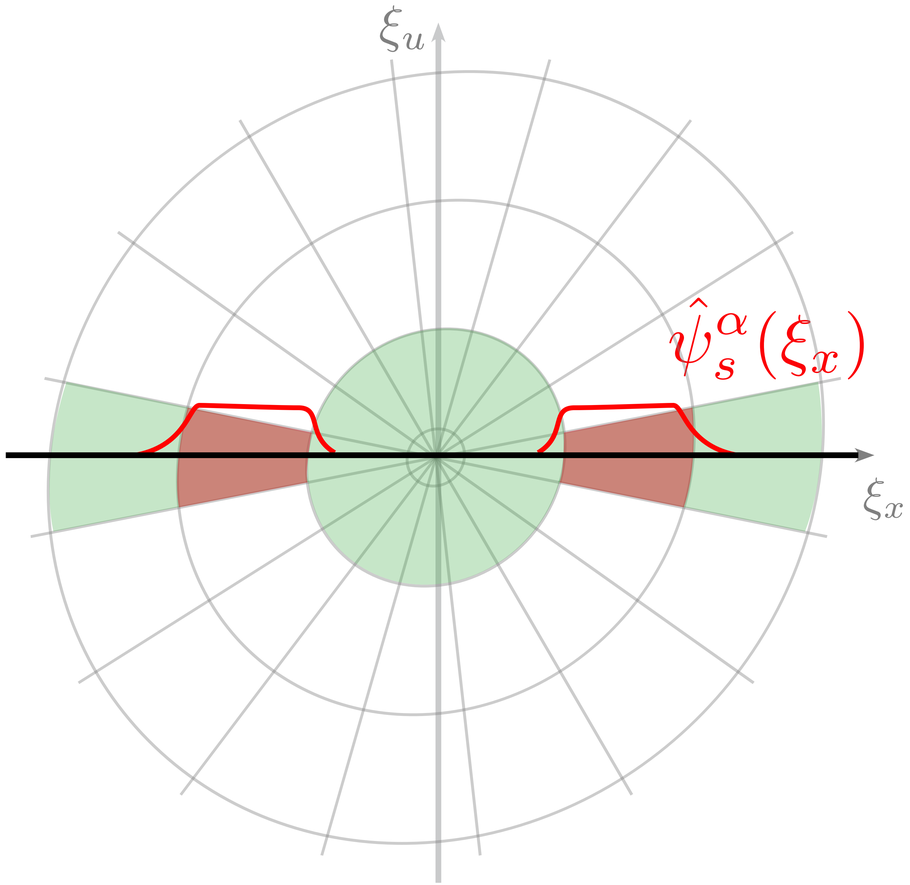

$$
\begin{aligned}
I(x) &= \frac{1}{\alpha^2} \int_{\mathbb{R}_u} \ell_\alpha(x,u)\,\mathrm{d}u \\
&= \frac{1}{\alpha^2} \mathcal{F}_{1\mathrm{d}}^{-1}\big(\hat{\ell}(S_{-\alpha}^T \xi_x)\big) \\
&= \frac{1}{\alpha^2} \sum_{s \in \mathcal{S}_\epsilon} \ell_s\, \mathcal{F}_{1\mathrm{d}}^{-1}\big(\hat{\psi}_s(S_{-\alpha}^T \xi_x)\big) \\
&= \frac{1}{\alpha^2} \sum_{s \in \mathcal{S}_\epsilon} \ell_s\, \zeta_s^\alpha(x)
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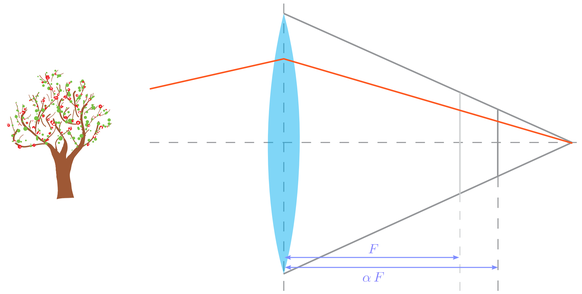
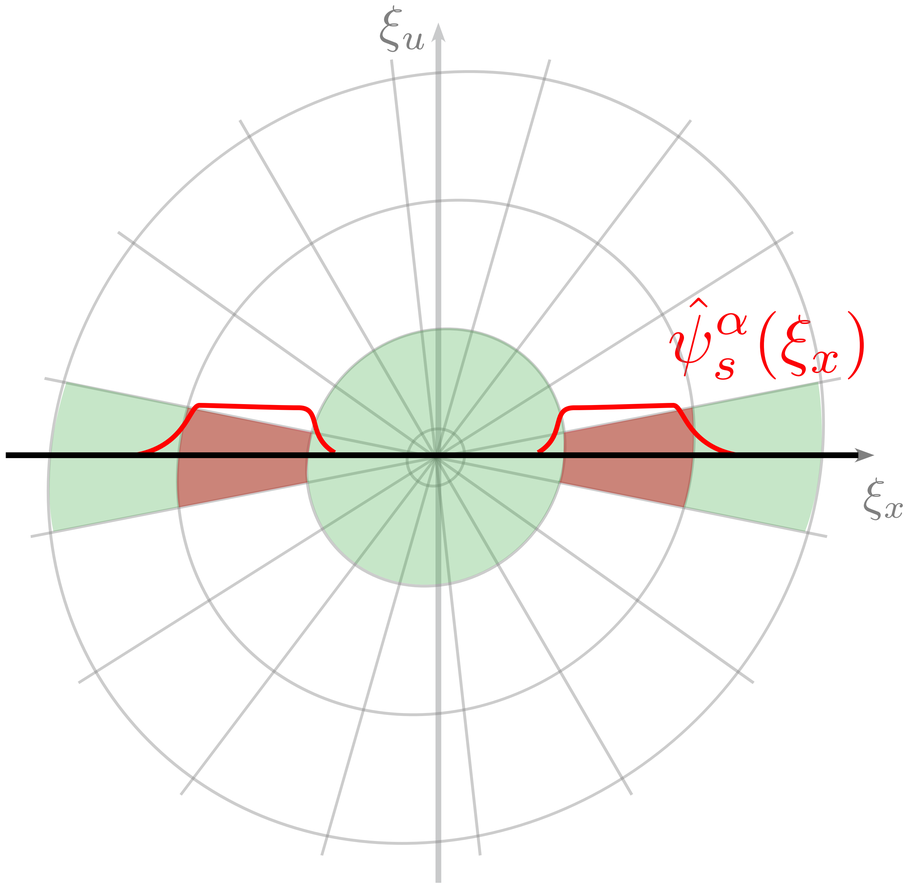
Light field refocusing





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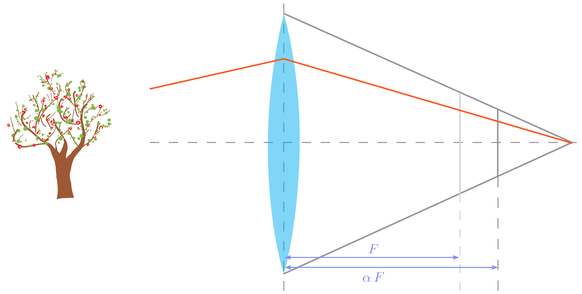
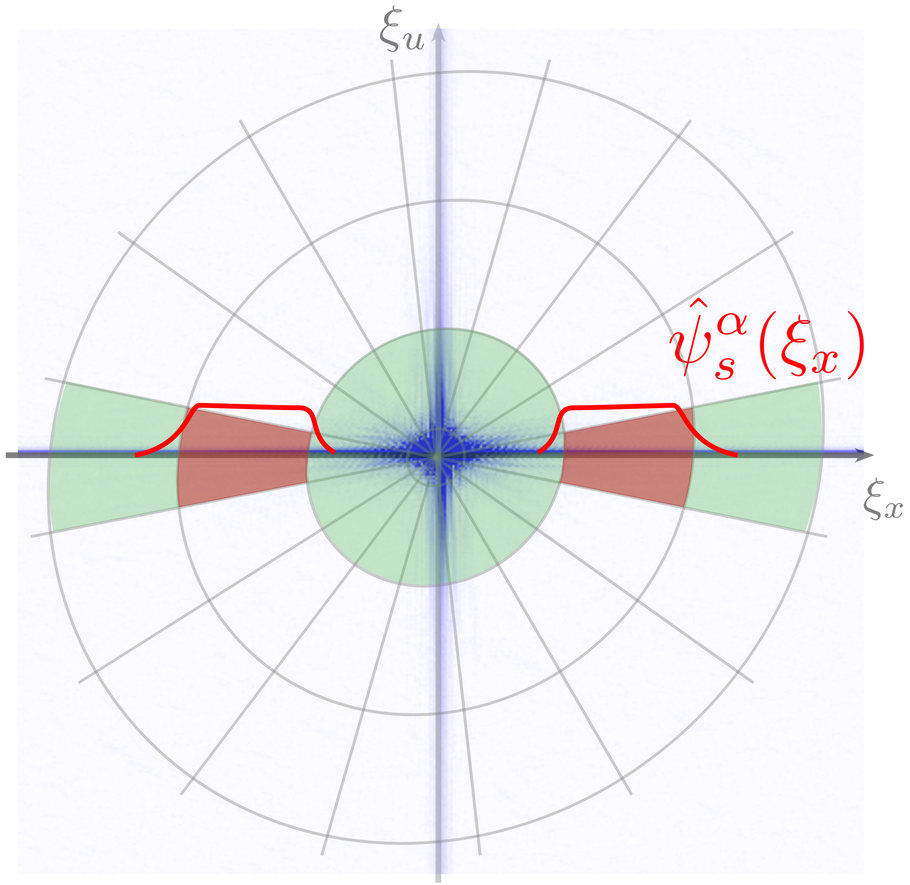
Light field refocusing





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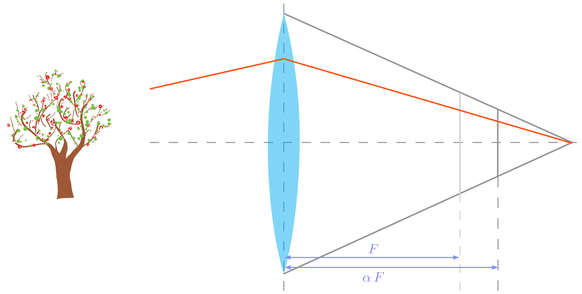
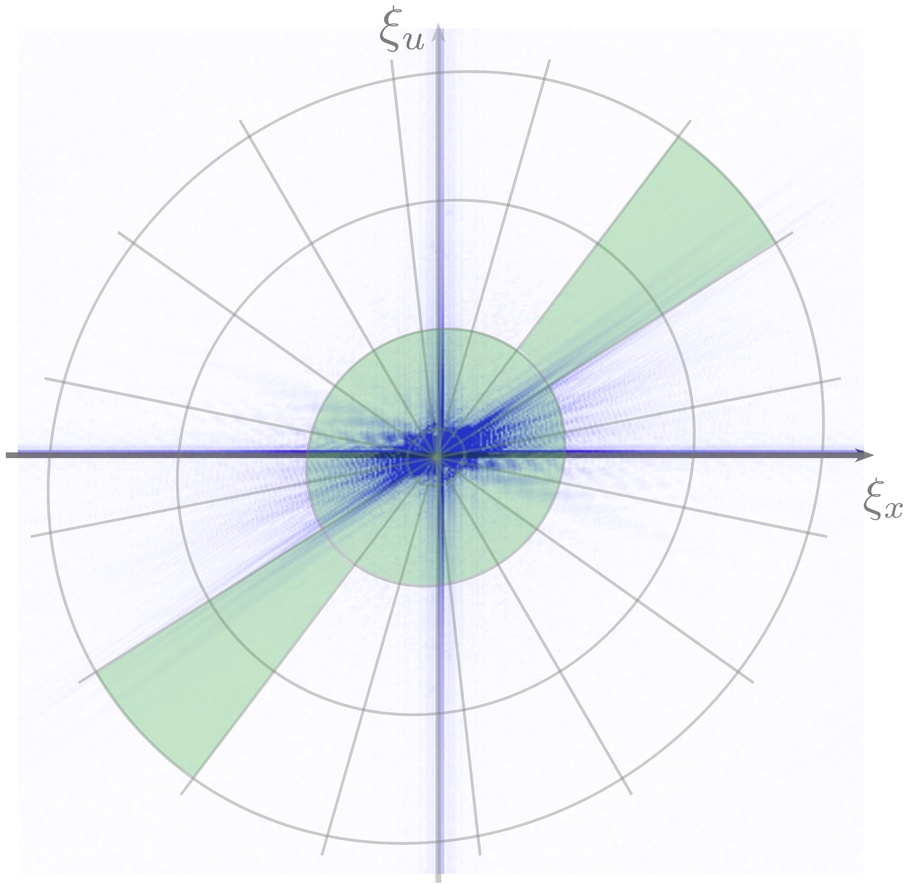
Light field refocusing





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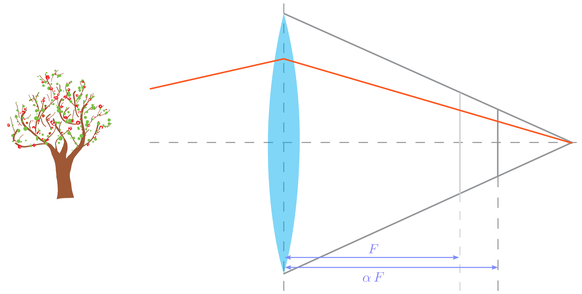
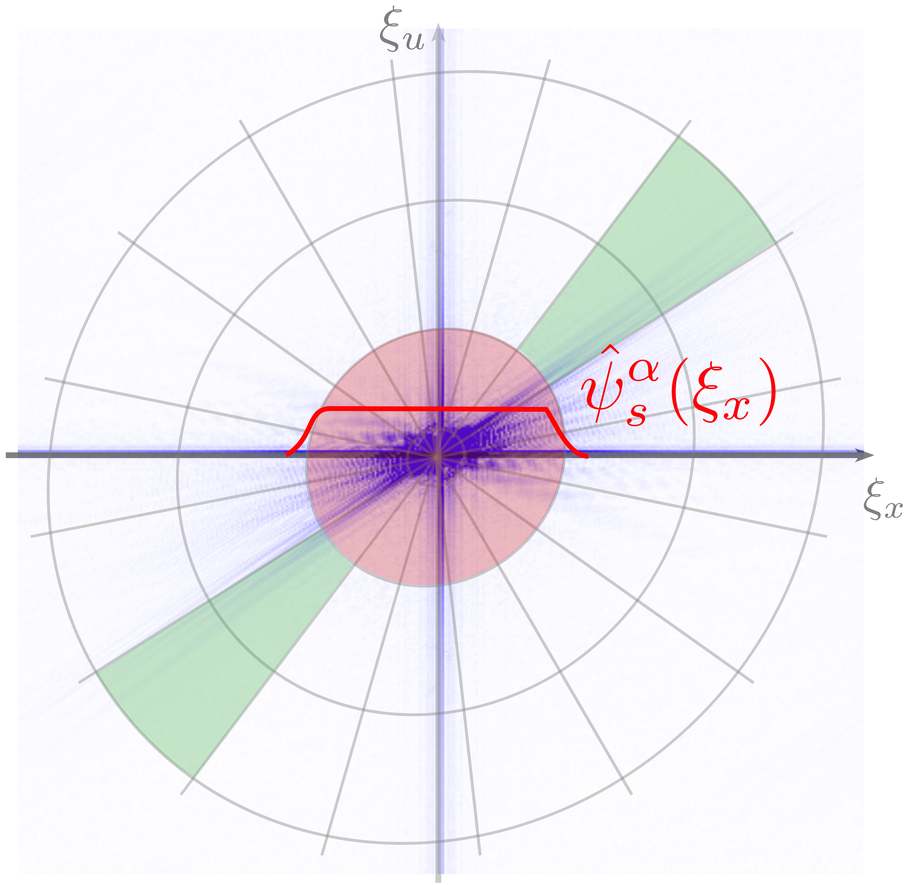
Light field refocusing





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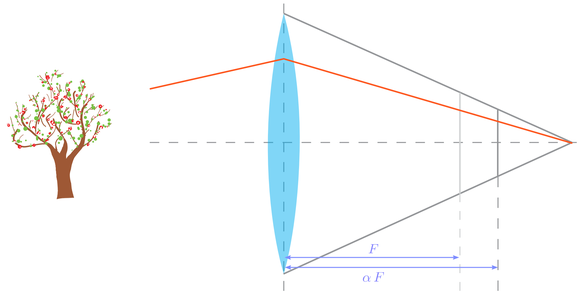
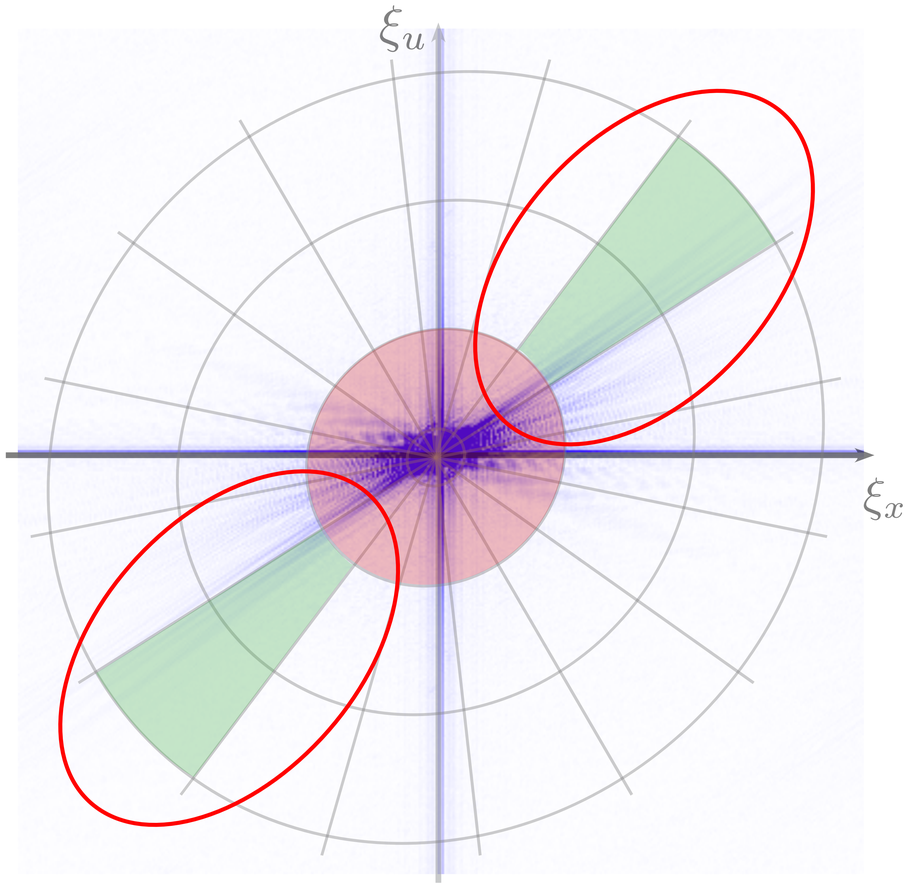
Light field refocusing





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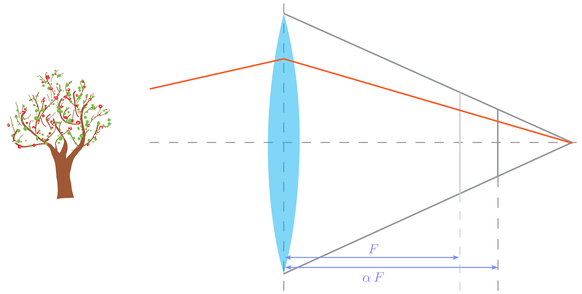
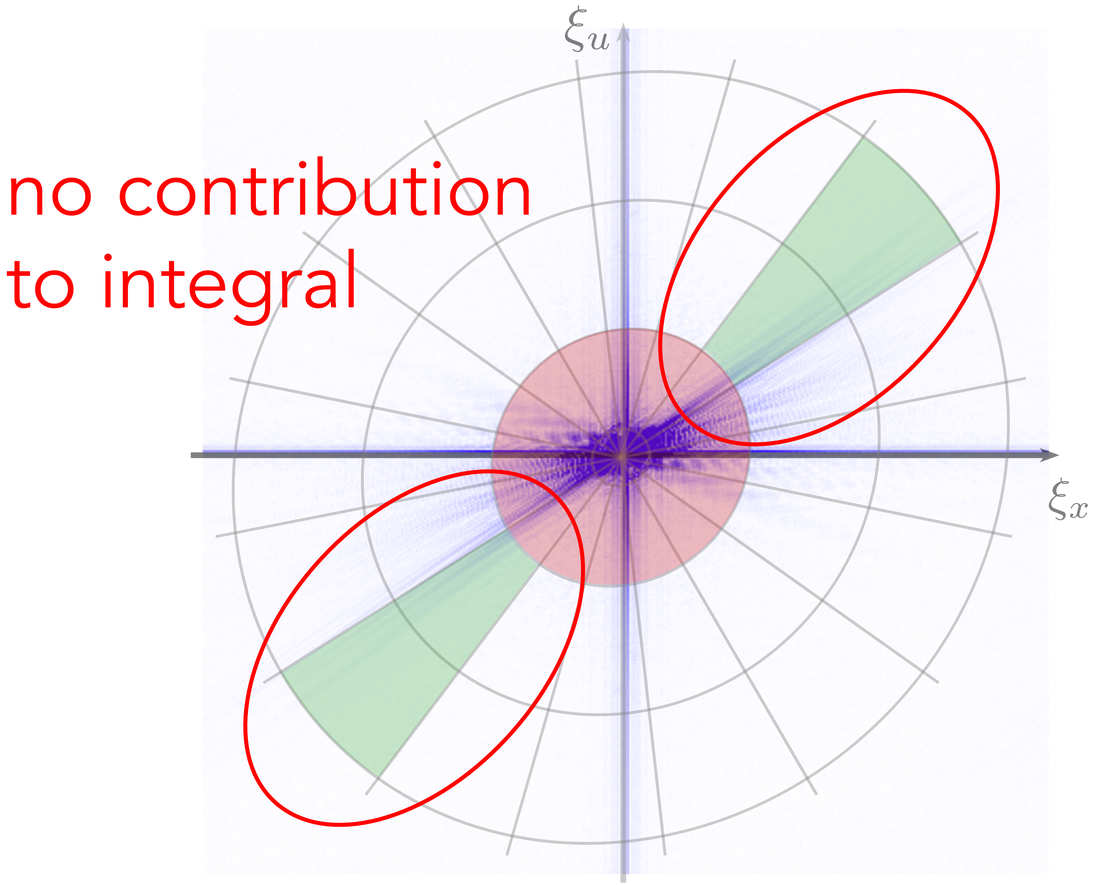
Light field refocusing





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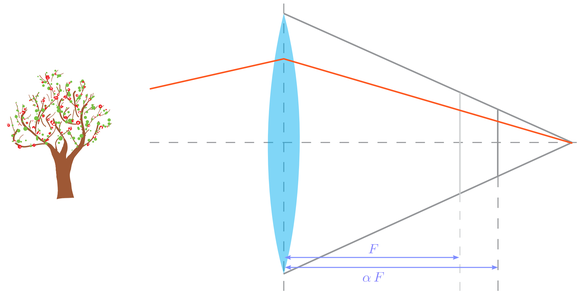
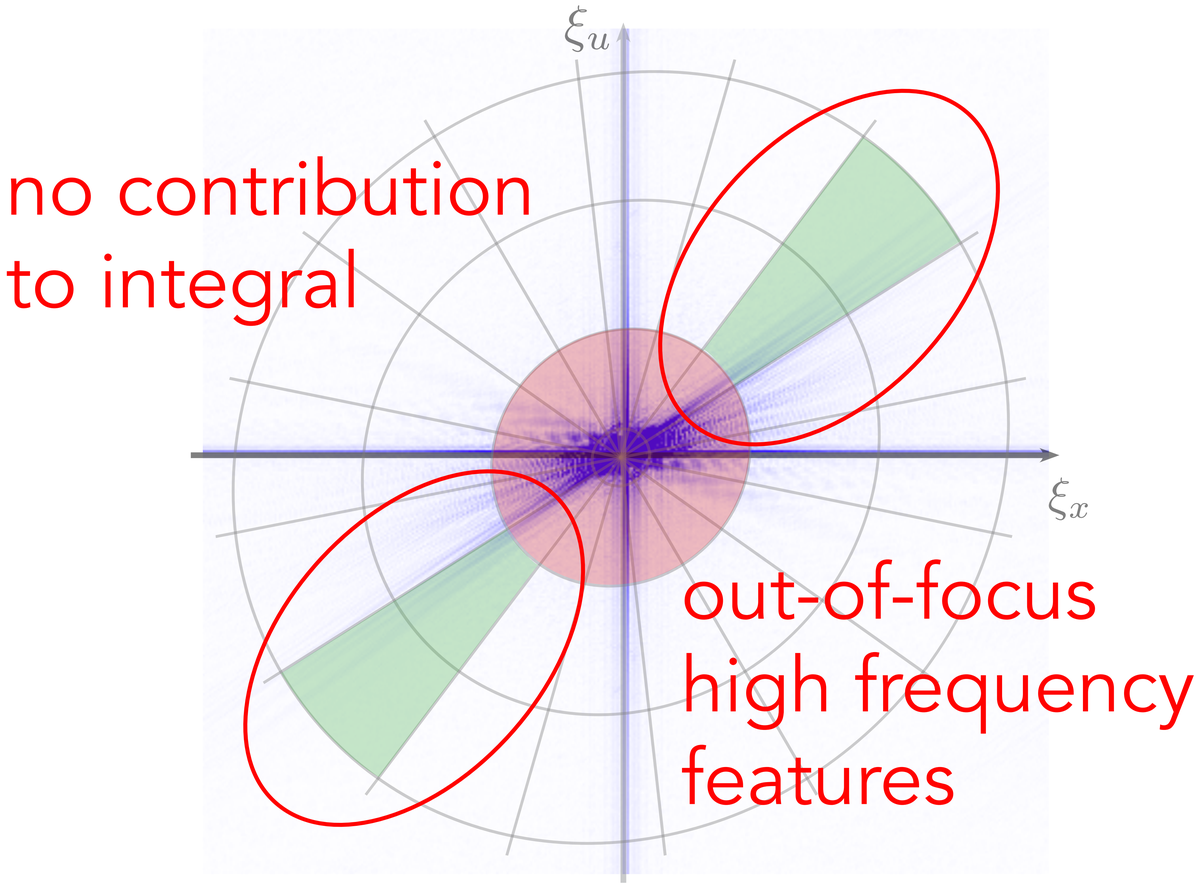
Light field refocusing





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Local Fourier slice photography

Local Fourier slice photography

1. Project light field into (x,u) / (y,v) separable polar wavelet representation.

Local Fourier slice photography

1. Project light field into (x,u) / (y,v) separable polar wavelet representation.

2. Compress light field.

Local Fourier slice photography

1. Project light field into (x,u) / (y,v) separable polar wavelet representation.

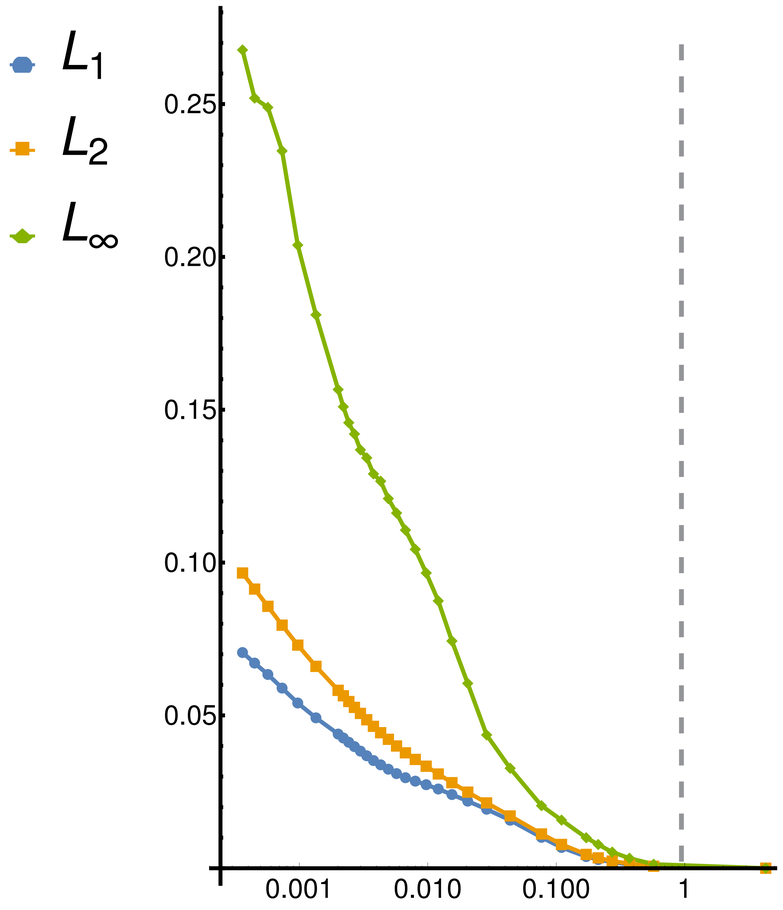
2. Compress light field.

3. Reconstruct image for focusing parameter α:

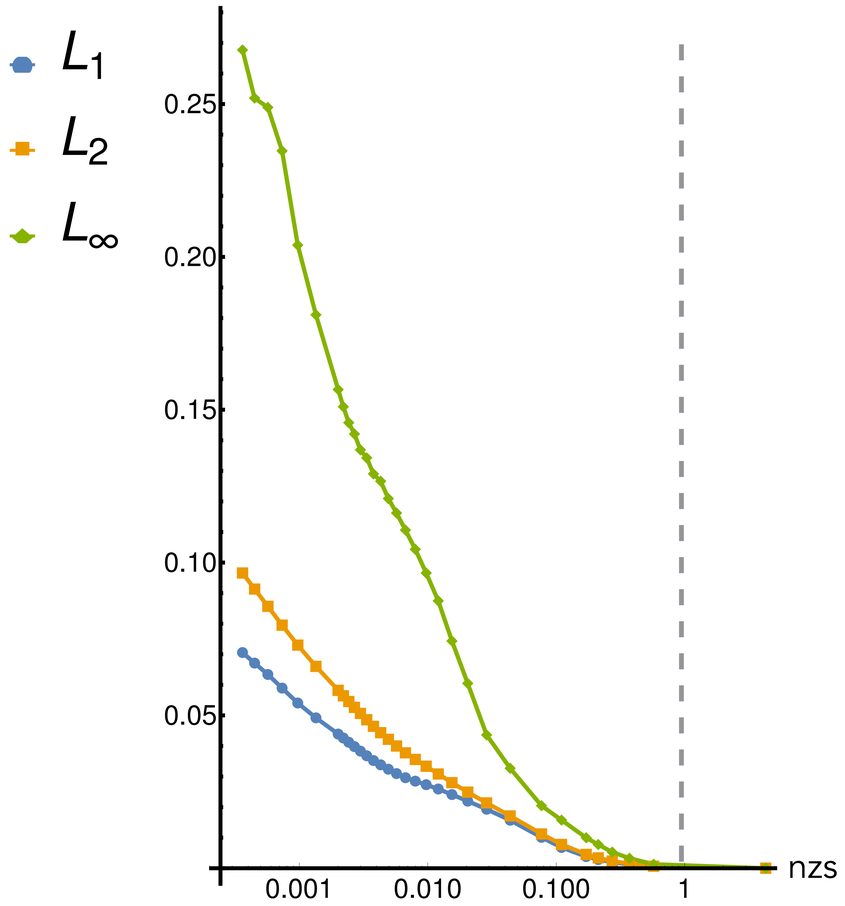
$$I(x,y) = \frac{1}{\alpha^2} \sum_{s \in \mathcal{S}_\epsilon^\alpha} \sum_{r \in \mathcal{R}_\epsilon^\alpha} \ell_{s,r} \, \zeta_s^\alpha(x) \, \zeta_r^\alpha(y)$$

Visual fidelity versus compression rate

Visual fidelity versus compression rate



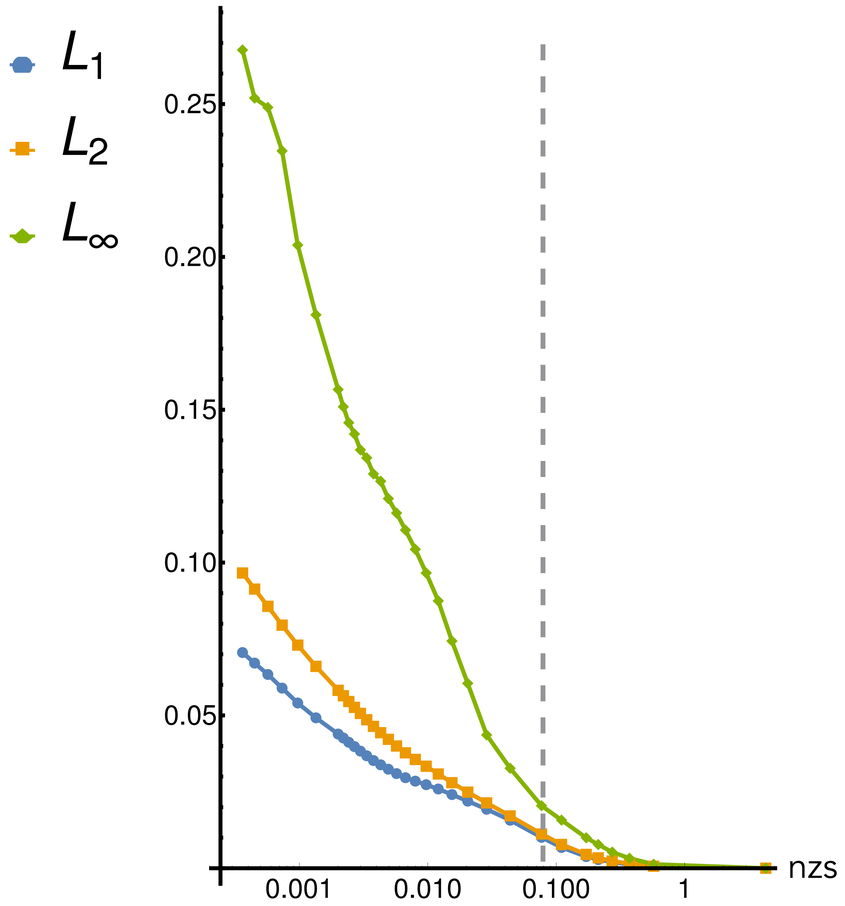
Visual fidelity versus compression rate



cr = 1.

error

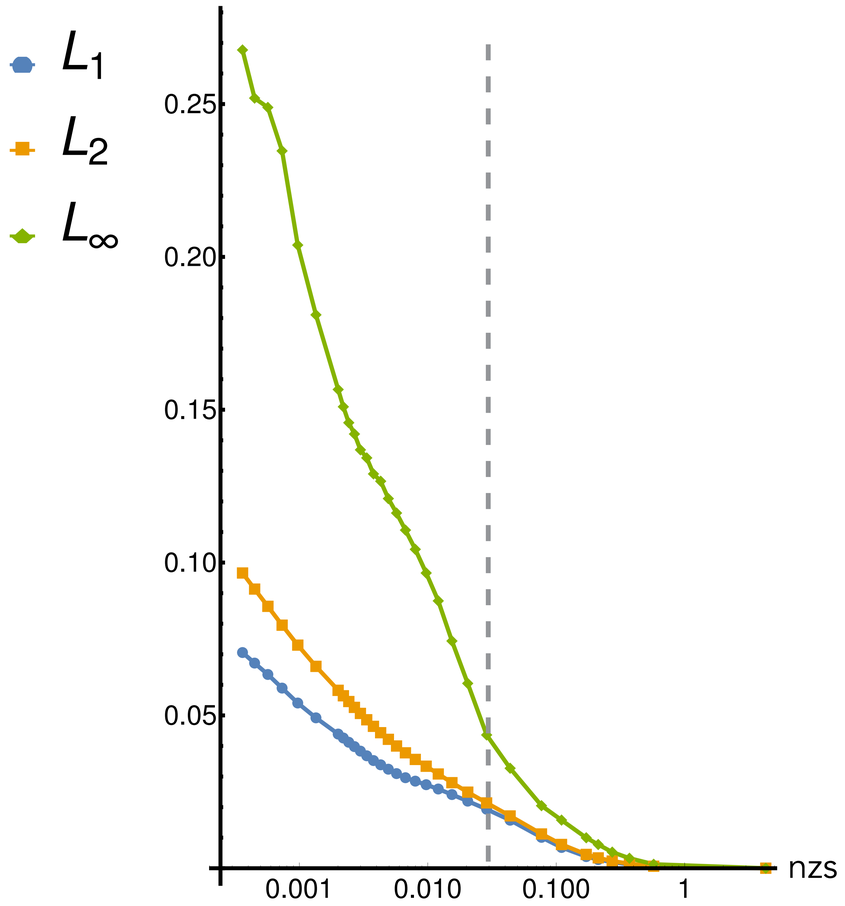
Visual fidelity versus compression rate



cr = 11.52

error

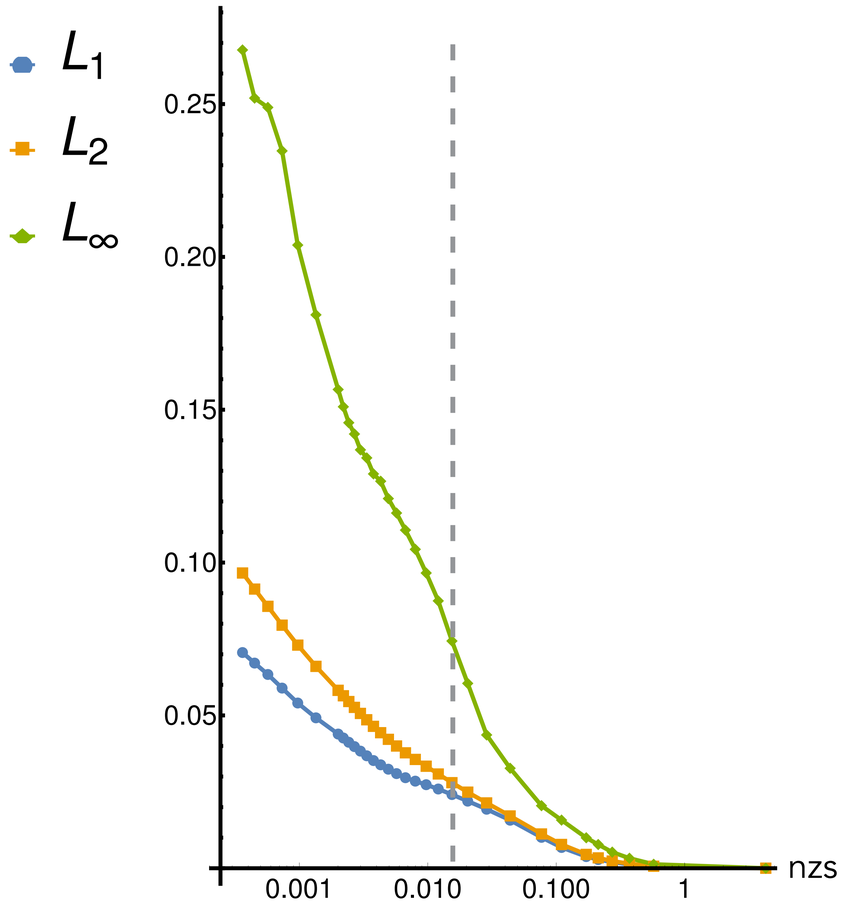
Visual fidelity versus compression rate



cr = 20.17

error

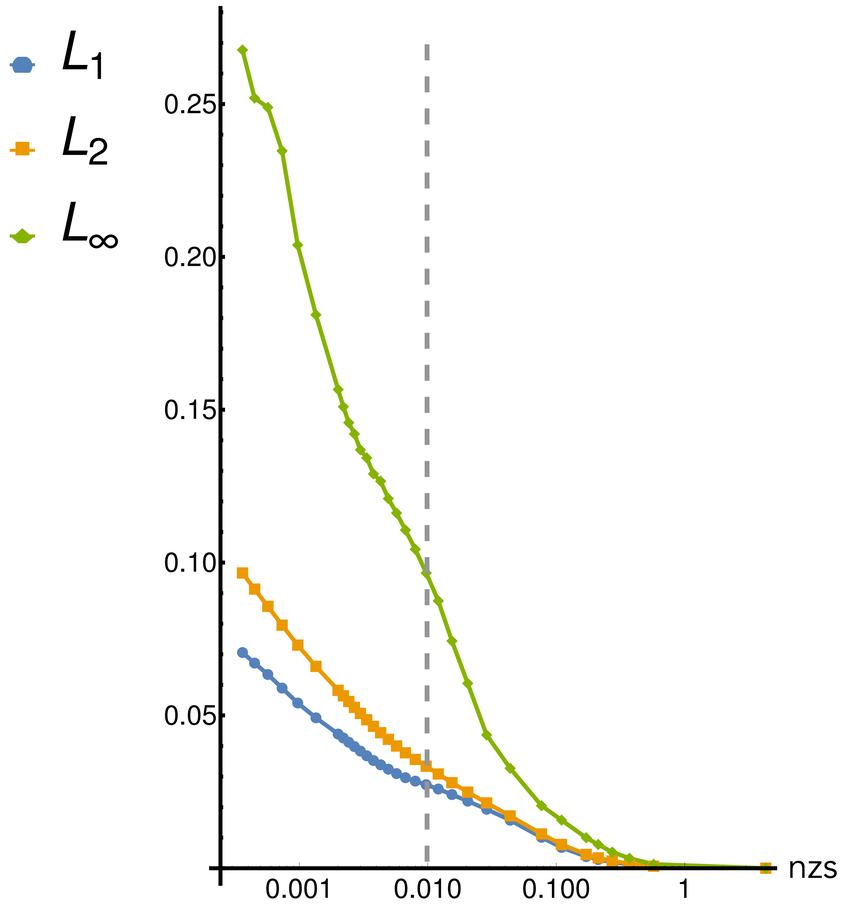
Visual fidelity versus compression rate



cr = 38.92

error

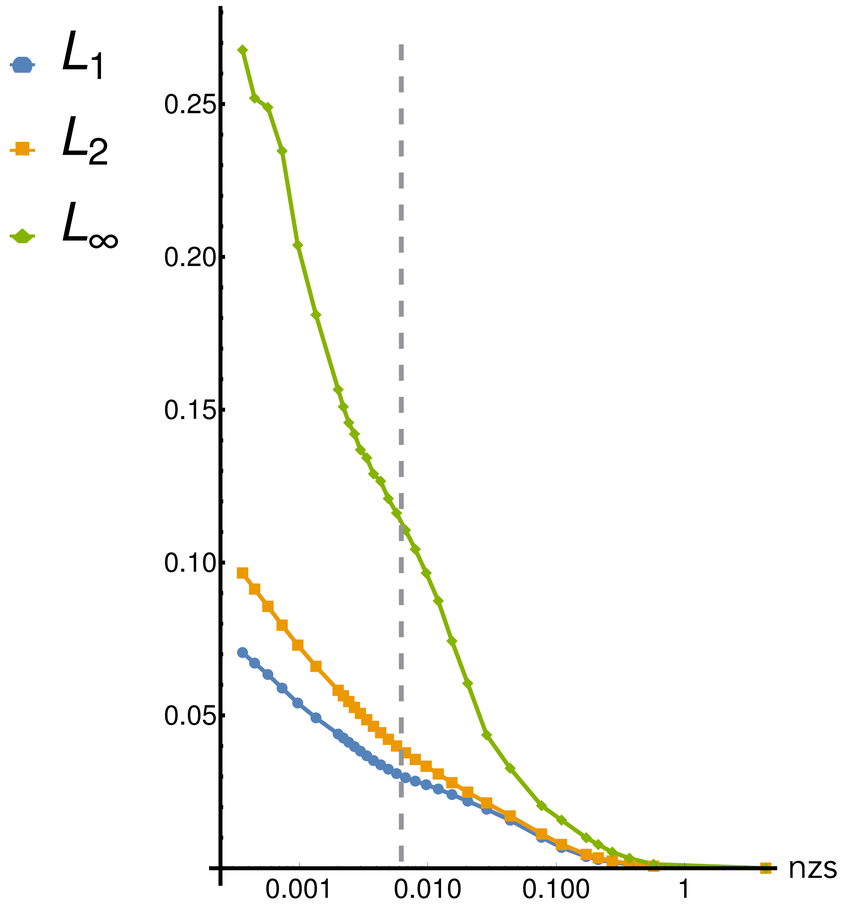
Visual fidelity versus compression rate



cr = 97.81

error

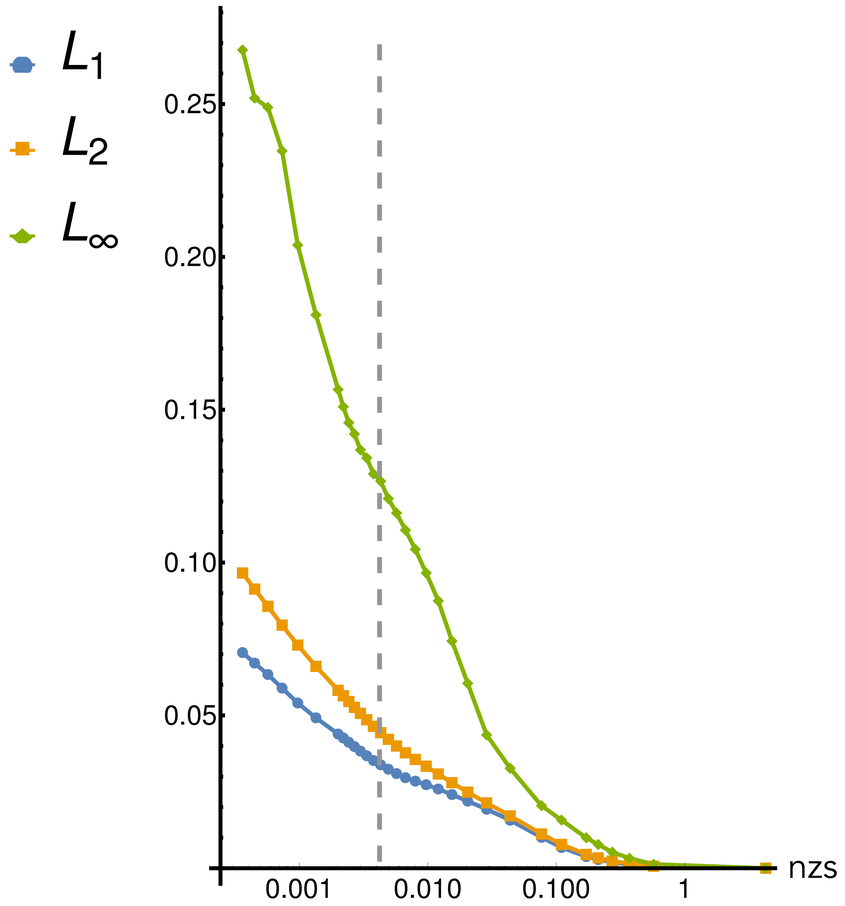
Visual fidelity versus compression rate



cr = 209.25

error

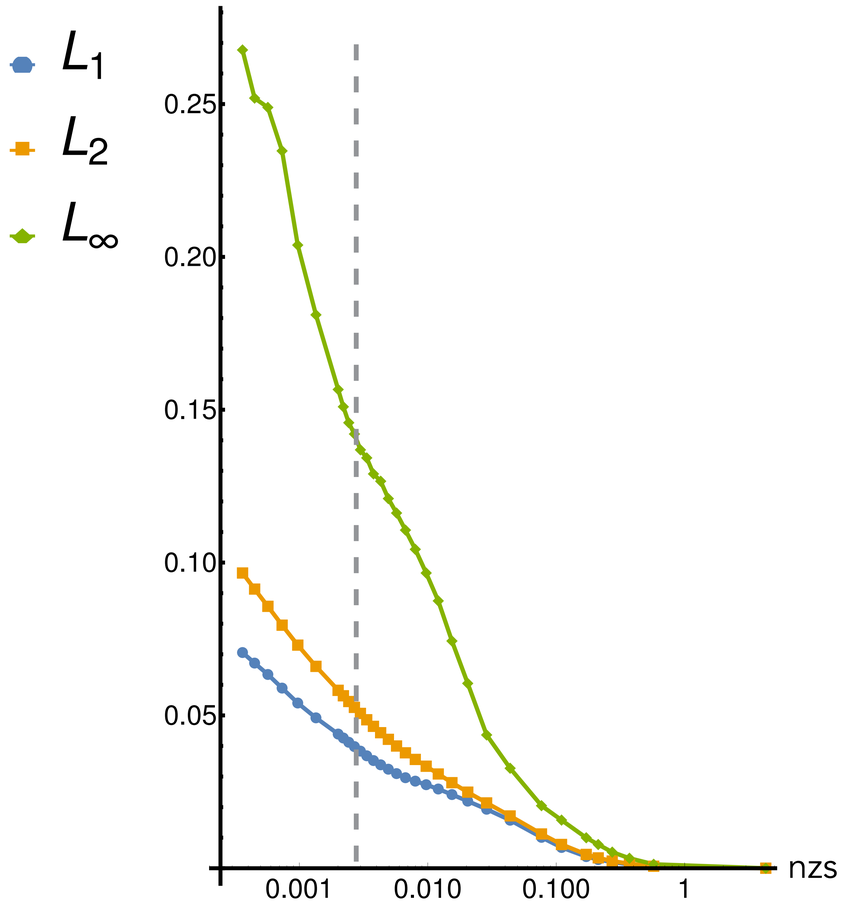
Visual fidelity versus compression rate



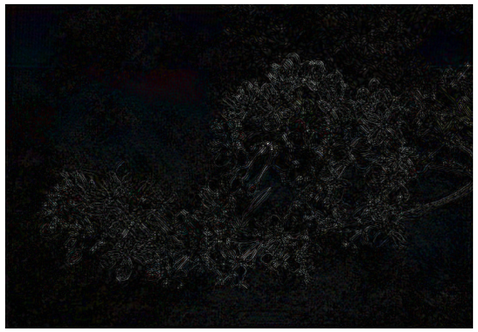
cr = 354.75

error

Visual fidelity versus compression rate

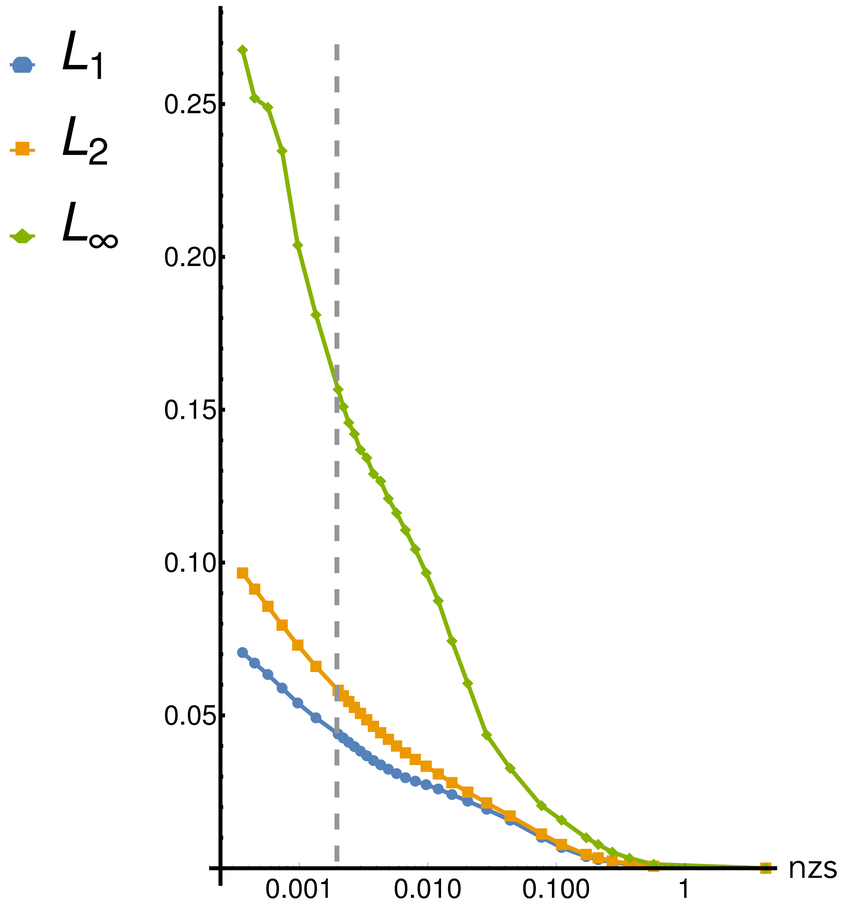


cr = 535.62

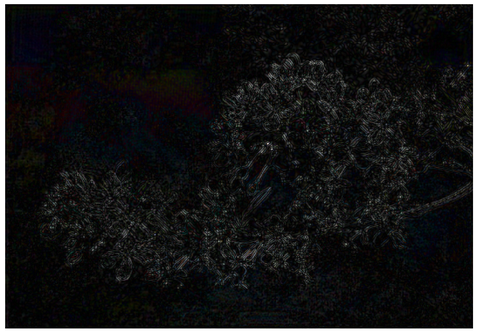


error

Visual fidelity versus compression rate

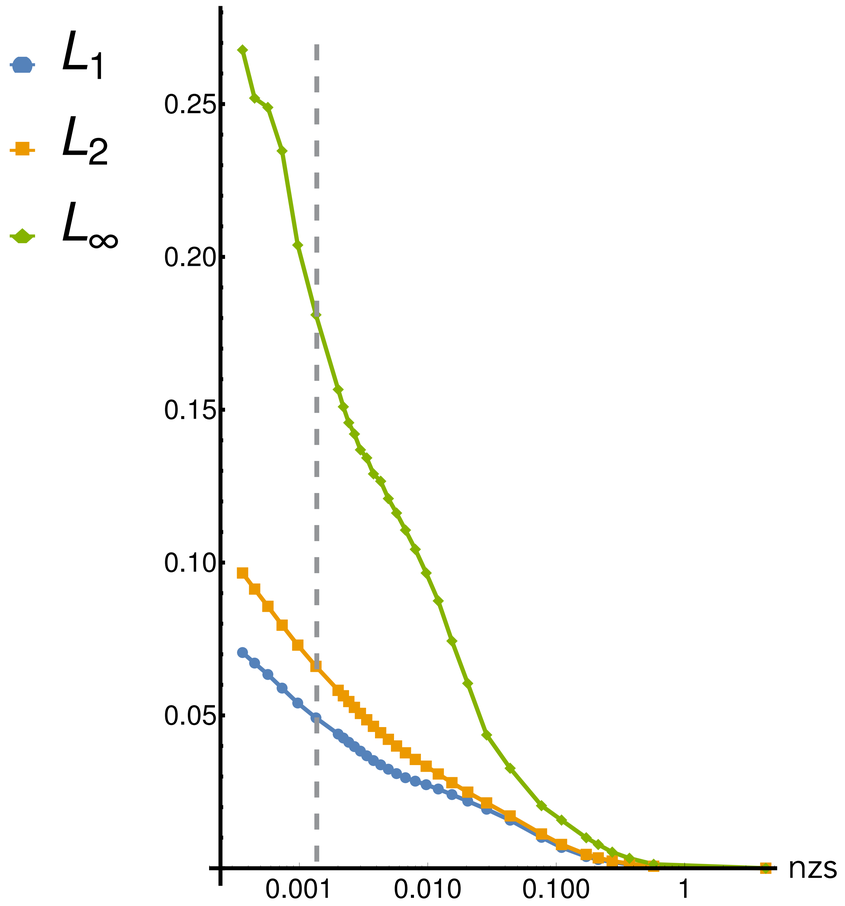


cr = 749.51

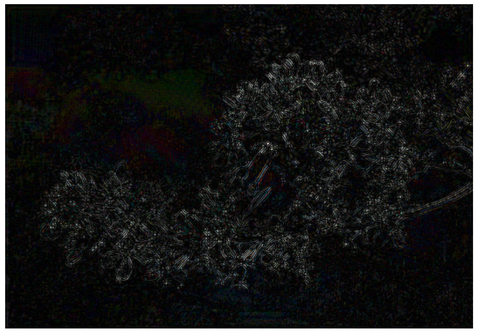


error

Visual fidelity versus compression rate

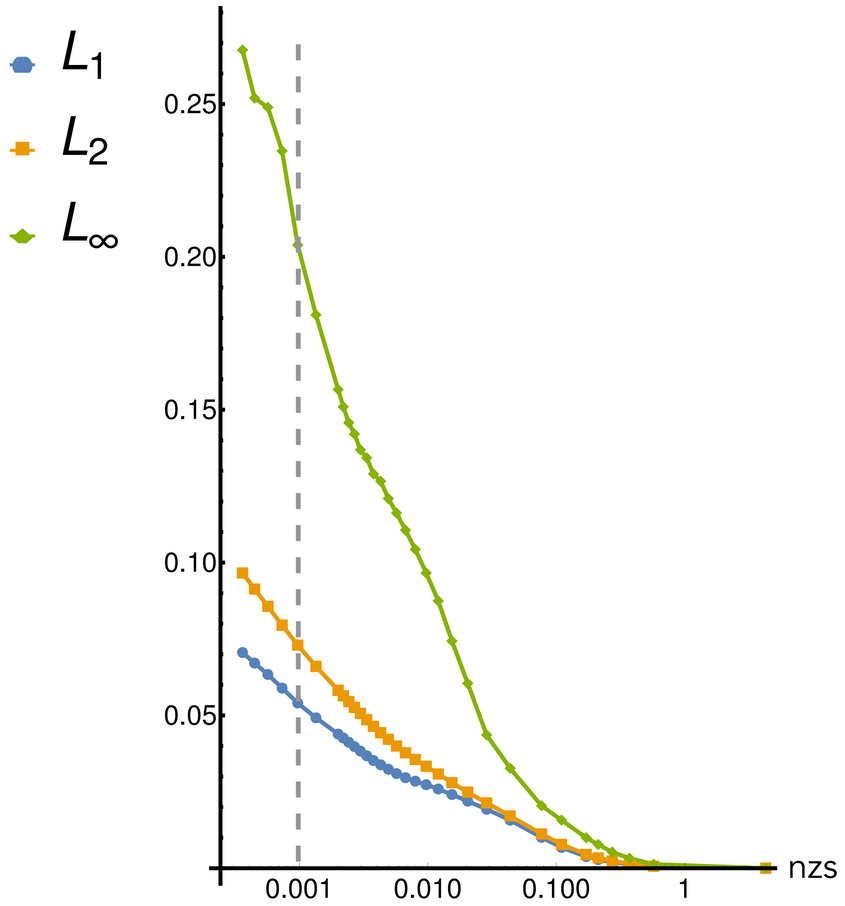


cr = 995.99

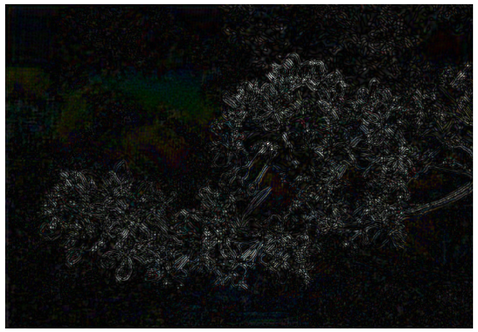


error

Visual fidelity versus compression rate

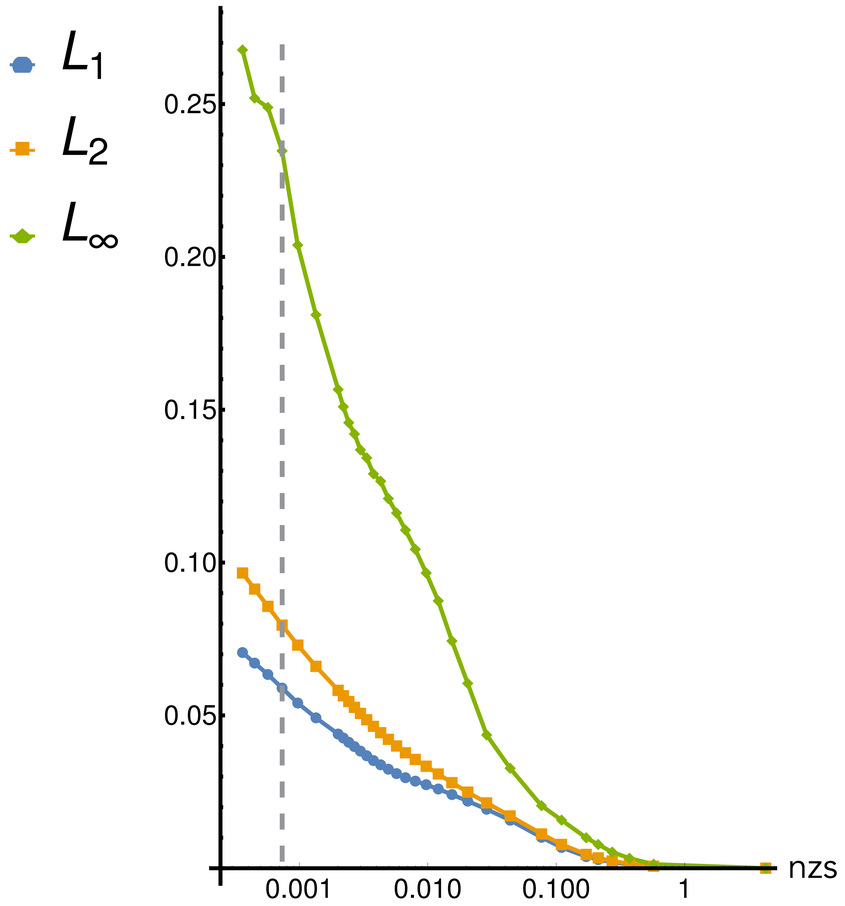


cr = 1278

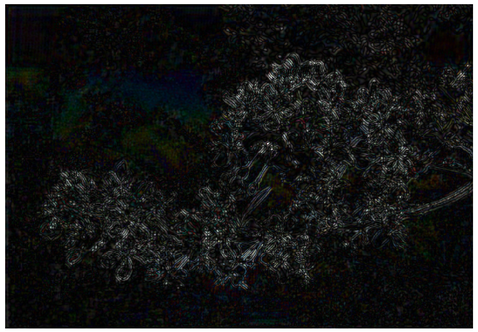


error

Visual fidelity versus compression rate

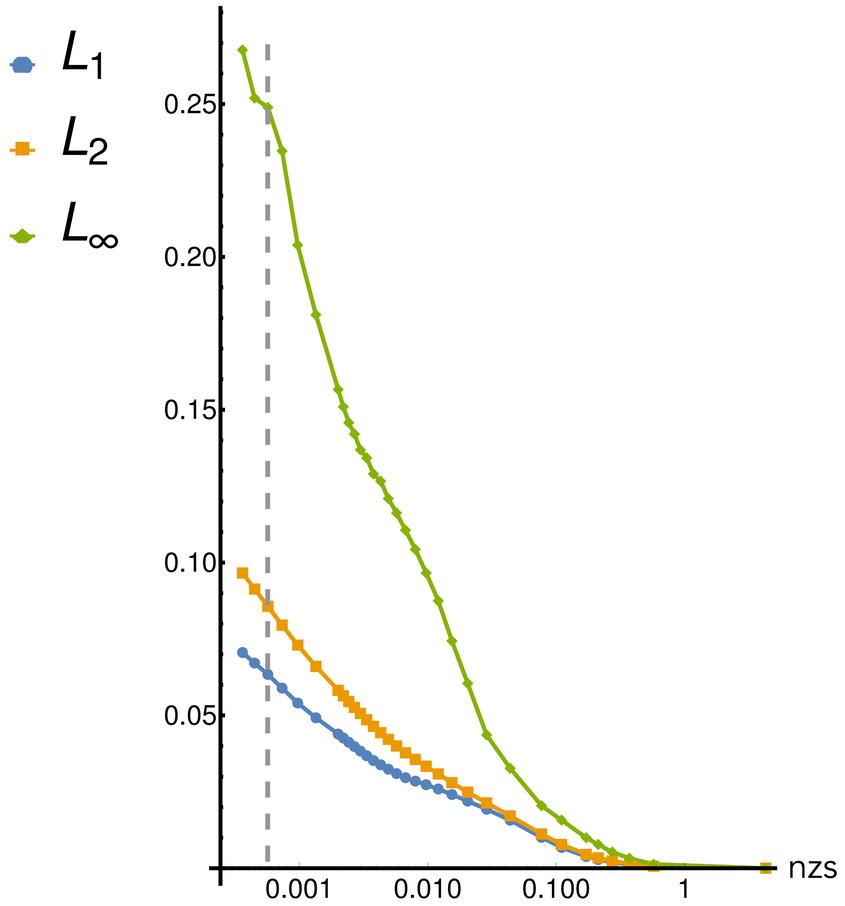


cr = 1432

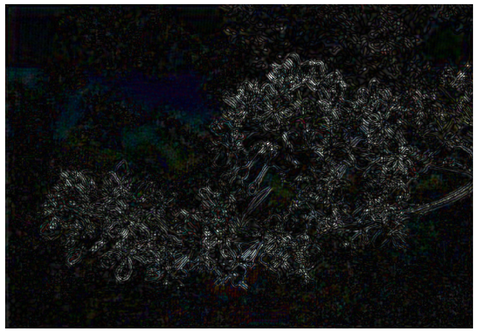


error

Visual fidelity versus compression rate

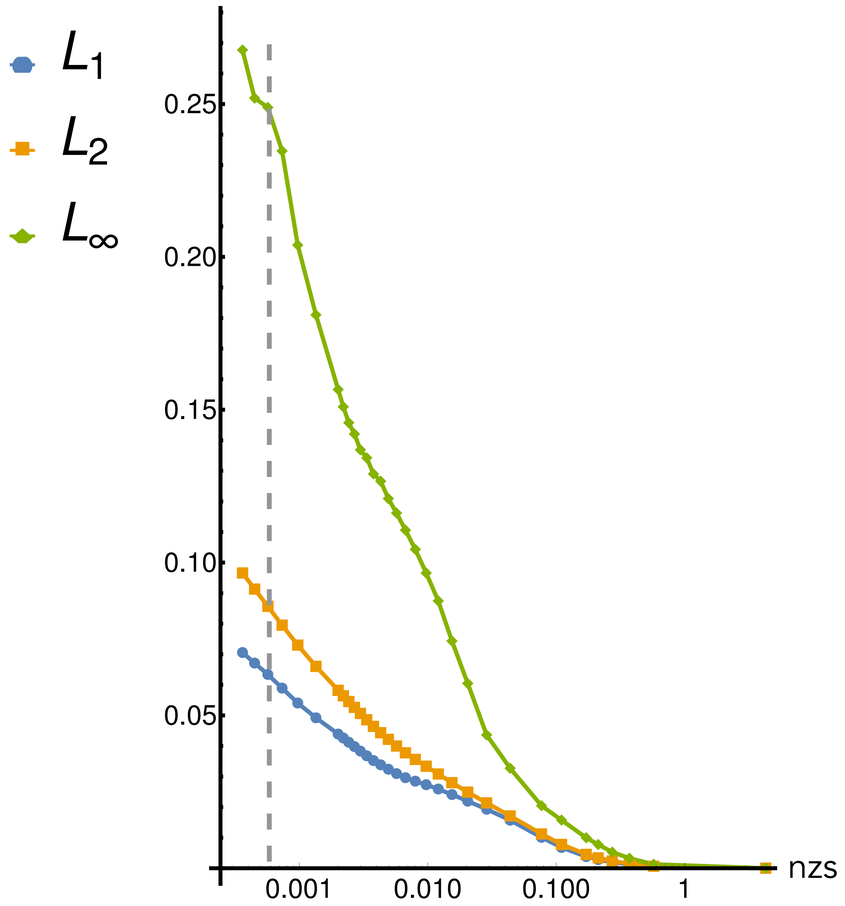


cr = 1765

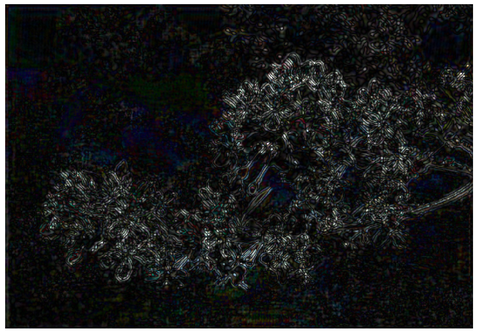


error

Visual fidelity versus compression rate

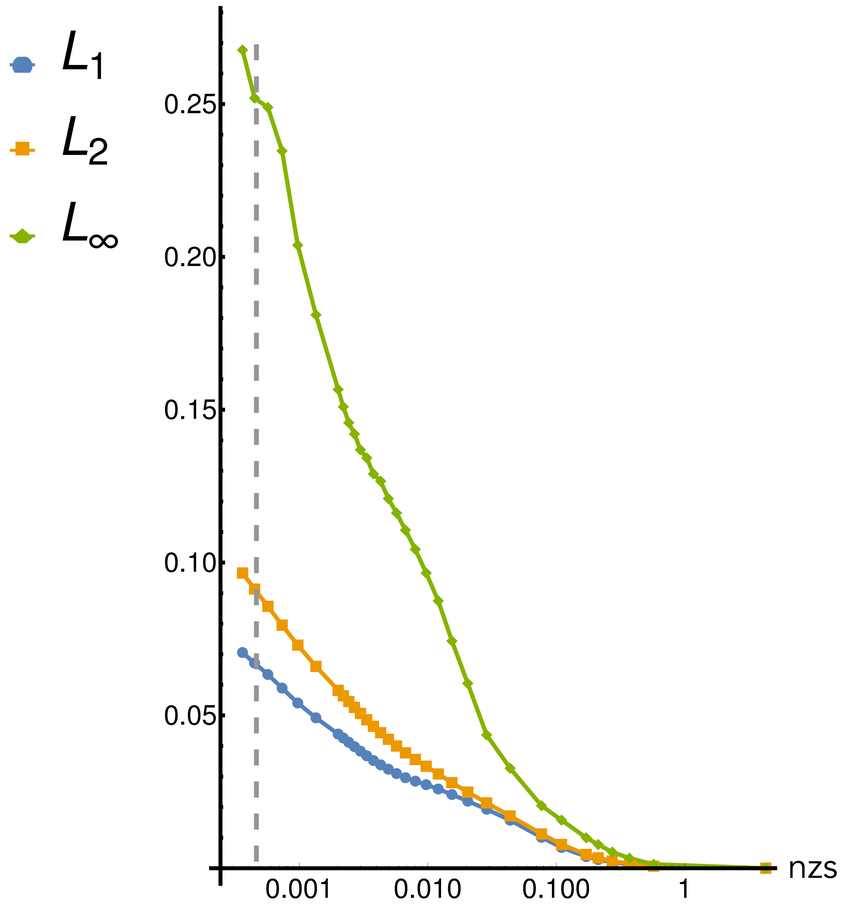


cr = 2134

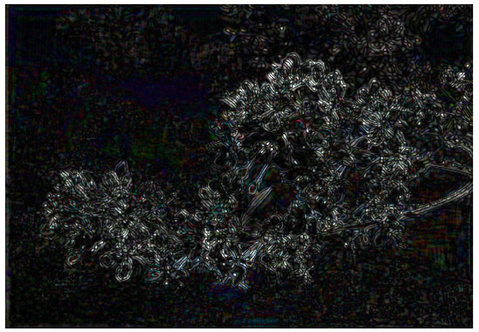


error

Visual fidelity versus compression rate

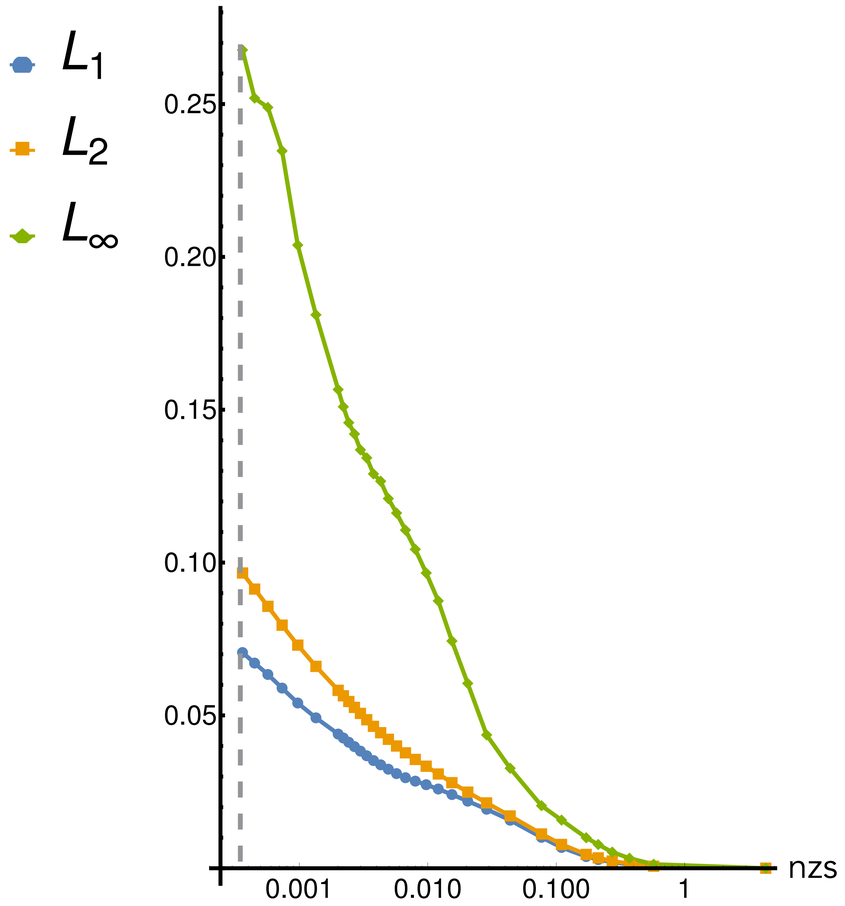


cr = 4406

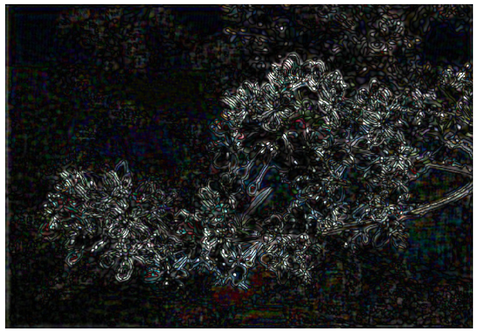


error

Visual fidelity versus compression rate

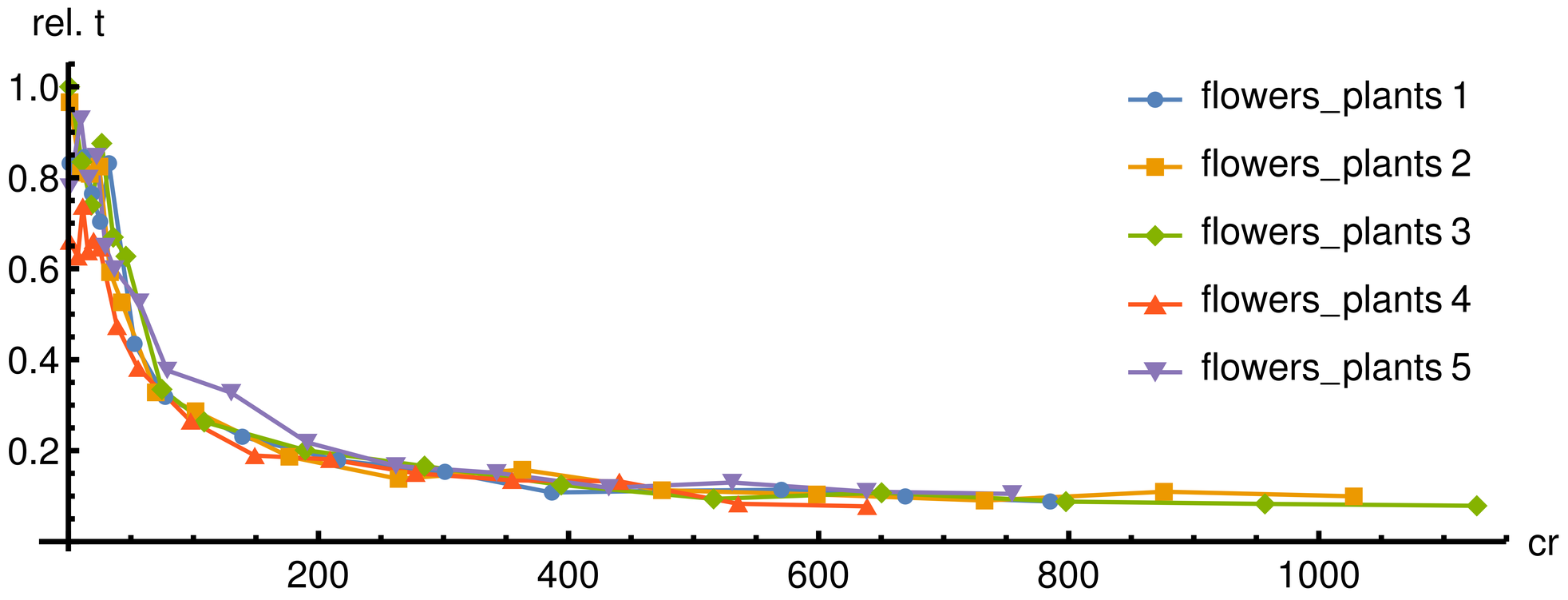


cr = 9547



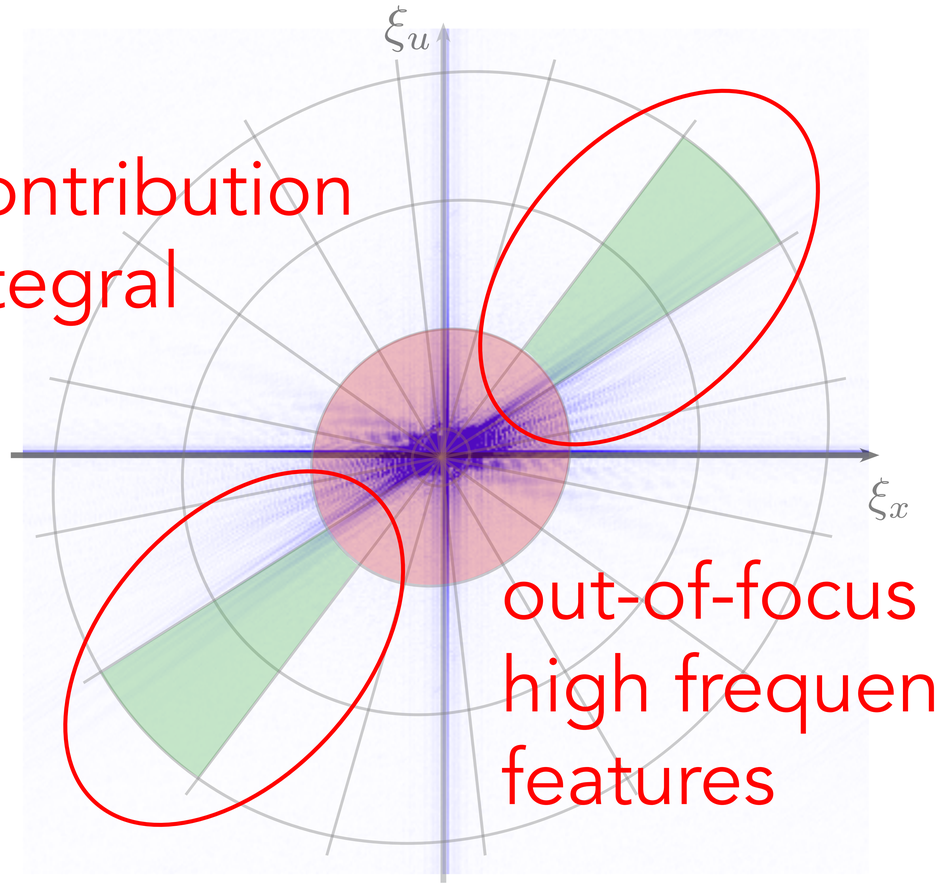
error

Computation time versus compression rate



Output sensitivity of computations

Output sensitivity of computations



no contribution to integral

out-of-focus high frequency features

$$I(x) = \frac{1}{\alpha^2} \int_{\mathbb{R}_u} \ell_\alpha(x,u)\,\mathrm{d}u$$

$$= \frac{1}{\alpha^2} \mathcal{F}_{1\mathrm{d}}^{-1}\big(\hat{\ell}(S_{-\alpha}^T \xi_x)\big)$$

$$= \frac{1}{\alpha^2} \sum_{s \in \mathcal{S}_\epsilon} \ell_s\, \mathcal{F}_{1\mathrm{d}}^{-1}\big(\hat{\psi}_s(S_{-\alpha}^T \xi_x)\big)$$

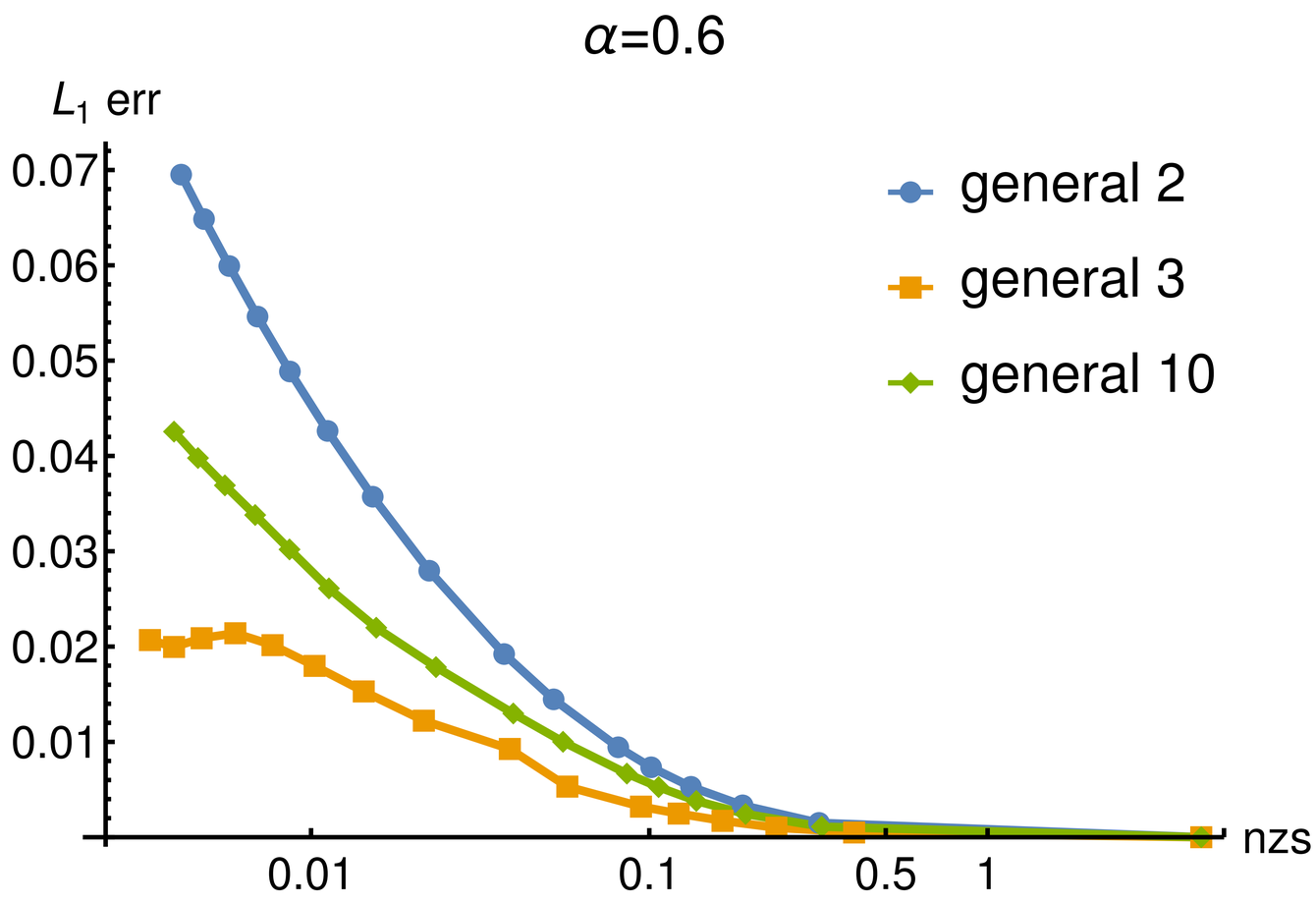
$$\boxed{I(x) = \frac{1}{\alpha^2} \sum_{s \in \mathcal{S}_\epsilon} \ell_s\, \zeta_s^\alpha(x)}$$

Output sensitivity of computations

Output sensitivity of computations

Output sensitivity of computations

Output sensitivity of computations



Output sensitivity of computations

$\alpha = 0.6$

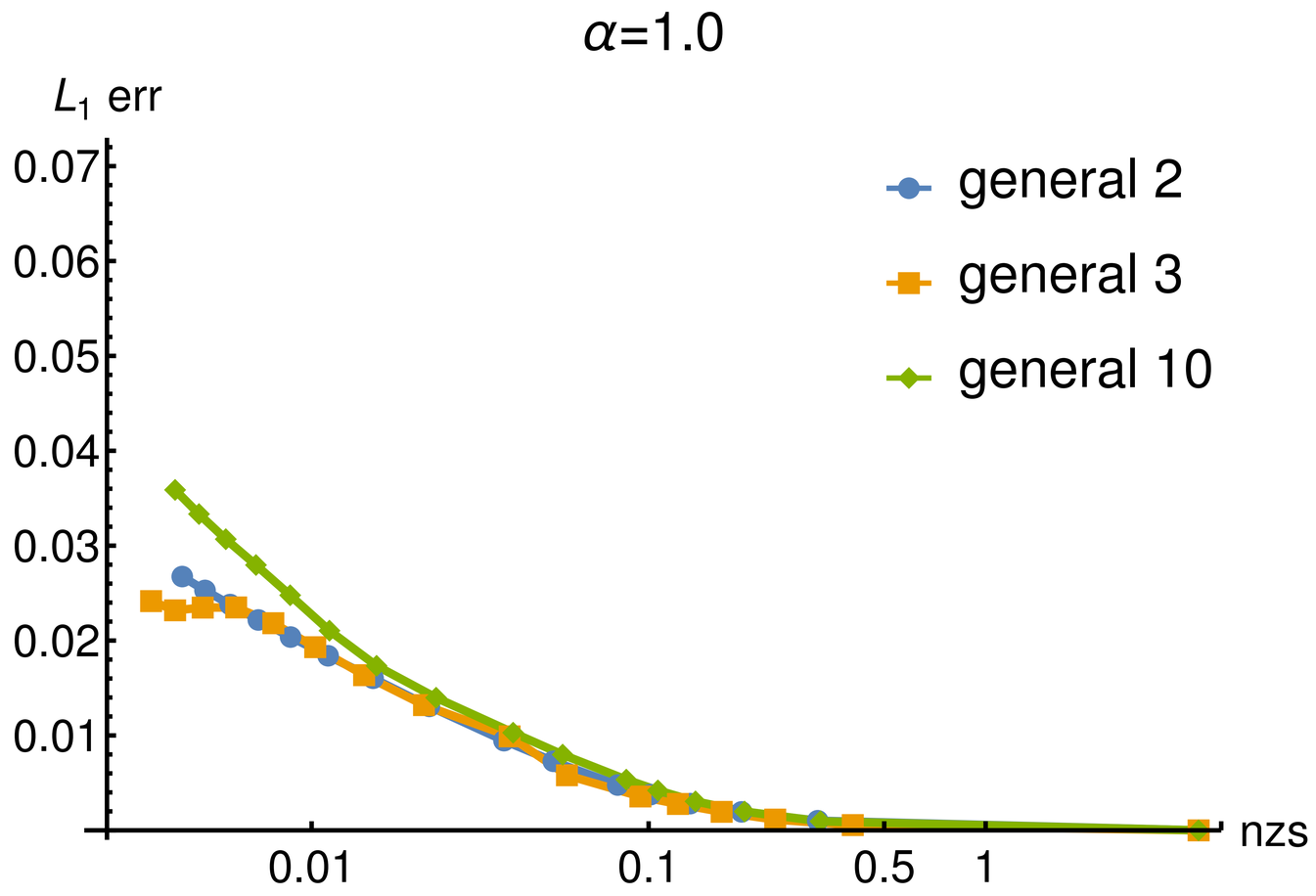
Output sensitivity of computations

$\alpha = 0.6$

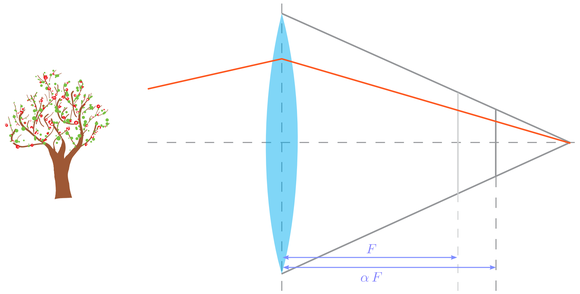
Output sensitivity of computations

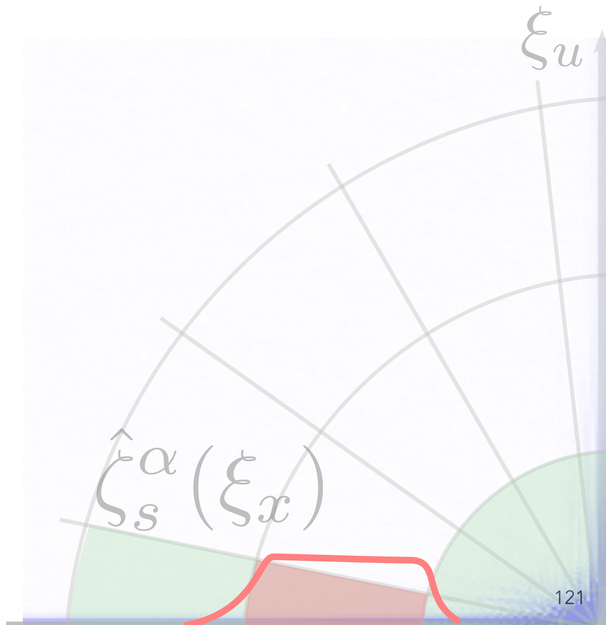
$\alpha = 1.0$

Output sensitivity of computations

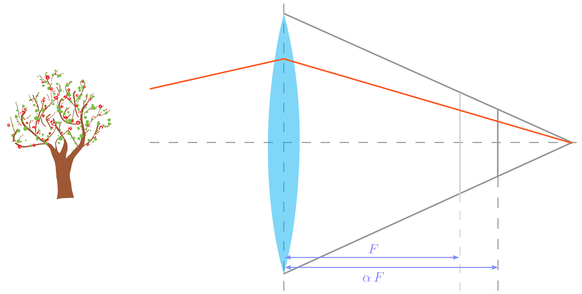


Limitations and Outlook

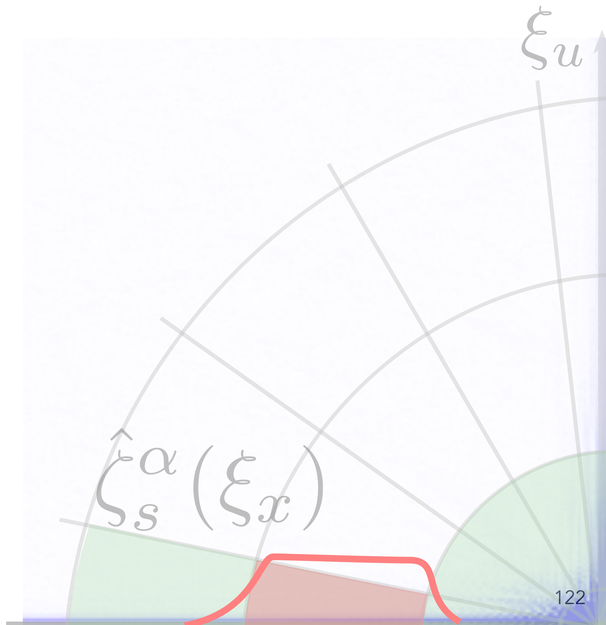




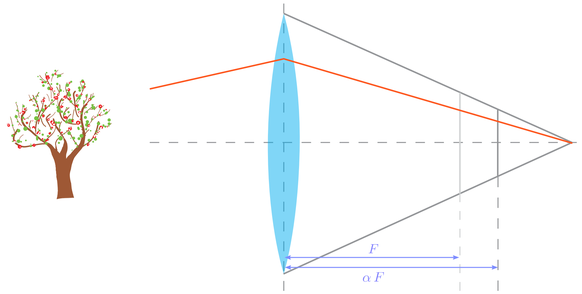
Limitations and Outlook



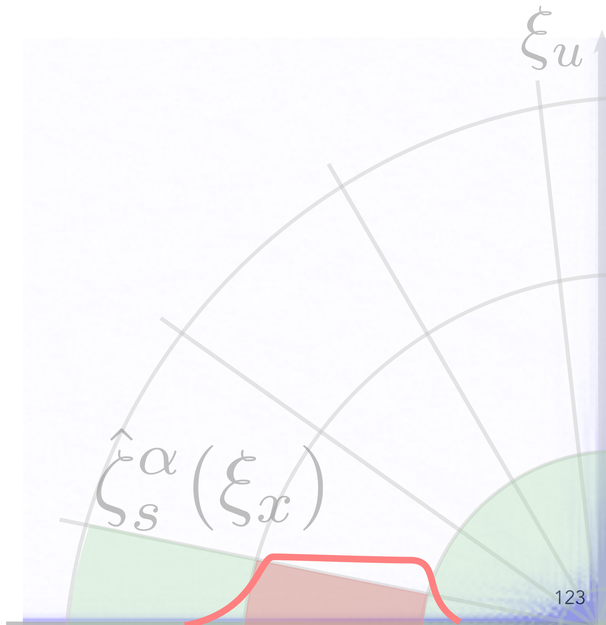
- Compactly supported polar wavelets
  - Lack of it hampers efficiency at the moment



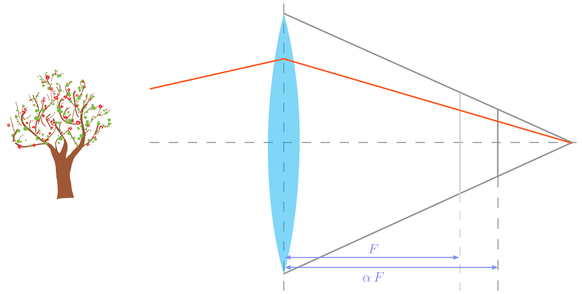
Limitations and Outlook



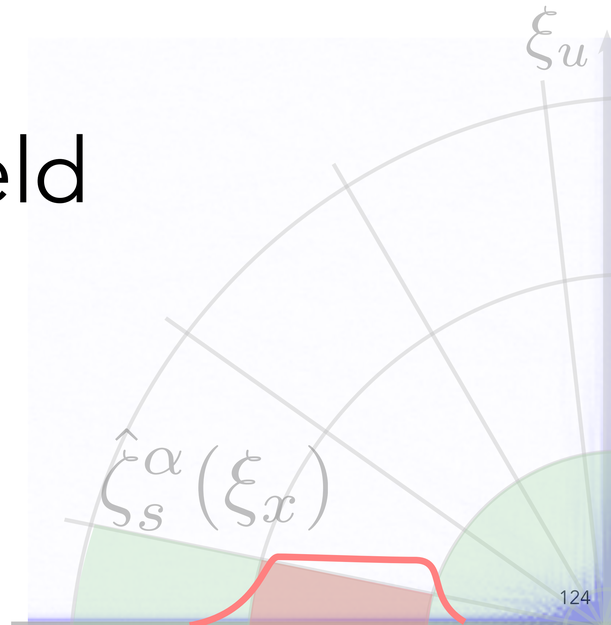
- Compactly supported polar wavelets
  - Lack of it hampers efficiency at the moment
- Other light field processing in polar wavelet representation (novel view synthesis, ...)



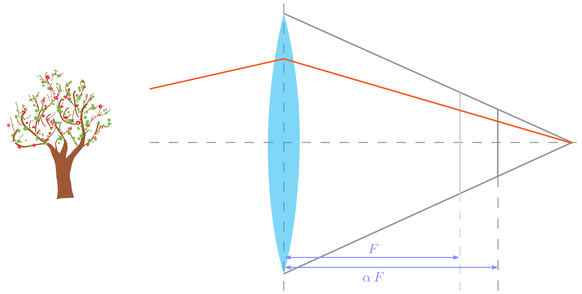
Limitations and Outlook



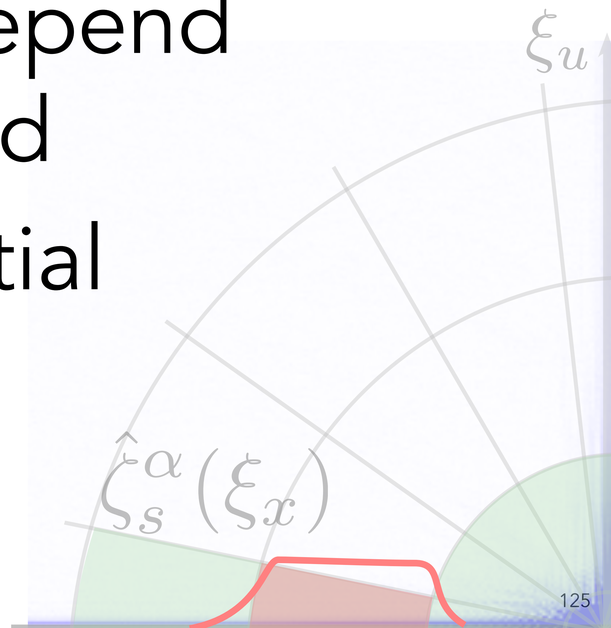
- Compactly supported polar wavelets
  - Lack of it hampers efficiency at the moment
- Other light field processing in polar wavelet representation (novel view synthesis, ...)
- Non-separable representation of light field (4D polar wavelets)

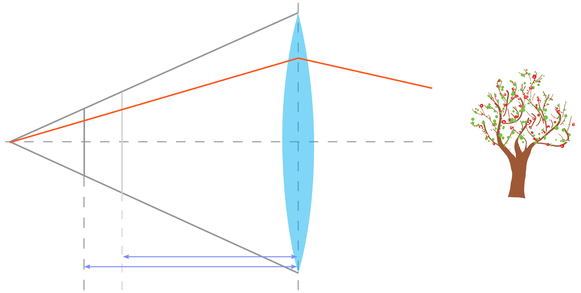


Conclusion



- Refocusing from wavelet representation
  - Restriction and compression are both naturally expressed in polar coordinates
- Output sensitive: computational costs depend on complexity of image to be synthesized
- Experimental results demonstrate potential of approach





Pre-print, slides, and source code:

http://graphics.cs.ovgu.de/projects/slice_alpha/

First ideas for the project were developed during a post-doc in Marc Alexa's computer graphics group at TU Berlin. Many thanks also to Eugene Fiume for continuing support.

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