Geophysical Research Abstracts
Vol. 21, EGU2019-3547-5, 2019
EGU General Assembly 2019

A Local Spectral Exterior Calculus for the Primitive Equations

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An accurate and efficient simulation of the equations governing global climate dynamics is highly challenging. This is due to their intrinsic complexity, such as their nonlinearity and intricate geometric structure, and the spherical domain over which they are defined. Existing global climate simulation models use finite elements or spectral discretizations, or a mixture of both, to face the challenges. As an alternative, we propose a local spectral wavelet discretization that respects the intrinsic structure of the governing equations, is able to adapt to the local regularity of a solution, and is intrinsically defined on the sphere. The favorable properties of our formulation arise from a local spectral exterior calculus, that is a wavelet-based "discretization" of Cartan's exterior calculus, for the sphere S^2. It consists of dedicated wavelets $\psi_{jk}^{q,\nu} \in \Omega_{\nu}^{q}(S^2)$ for 0-forms, exact and co-exact 1-forms, and 2-forms (corresponding to scalar functions, divergence-free and curl-free vector fields and densities) that satisfy essential properties of the exterior calculus, for example that the exterior derivative of a co-exact q-form wavelet is an exact $(q+1)$-form, i.e. $\mathrm{d}\psi_{jk}^{q,\delta} = \psi_{jk}^{q+1,\mathrm{d}}$, and that for an exact wavelet $\mathrm{d}\psi_{jk}^{q,\mathrm{d}} = 0$. With these properties, the local spectral exterior calculus facilitates a faithful discretization of the dynamic equations. At the same time, the space-frequency localization of the wavelets enables us to adapt to the local regularity of a solution by using more wavelet levels in regions with high frequency details, such as around a storm front or a coast line. Our approach thereby preserves many of the advantages of classical spectral formulations, for instance finite closure of the nonlinear advection terms. Inspired by the fast transform method of Orszag and Eliasen, also in our wavelet-based discretization we evaluate these in the spatial domain, with the fast wavelet transform providing the means to convert between spatial and spectral representations.

Our current implementation provides a proof-of-concept for our approach for the primitive equations and we are working on a thorough validation to understand the trade-offs involved compared to existing methods. In future work, we will consider more complex dynamical models and we will couple our formulation to a learning-based technique for data-assisted adjustment of the analytical simulations. Using our local spectral exterior calculus for the sphere, we will also investigate structure preserving numerical time integration schemes that conserve, for example, energy and circulation.