EGSR 2017

Helsinki, Finland, June 19–21

Controlling and Sampling Visibility on the Image Plane

Christian Lessig

Otto-von-Guericke-Universität Magdeburg

Idea

EGSR 2017

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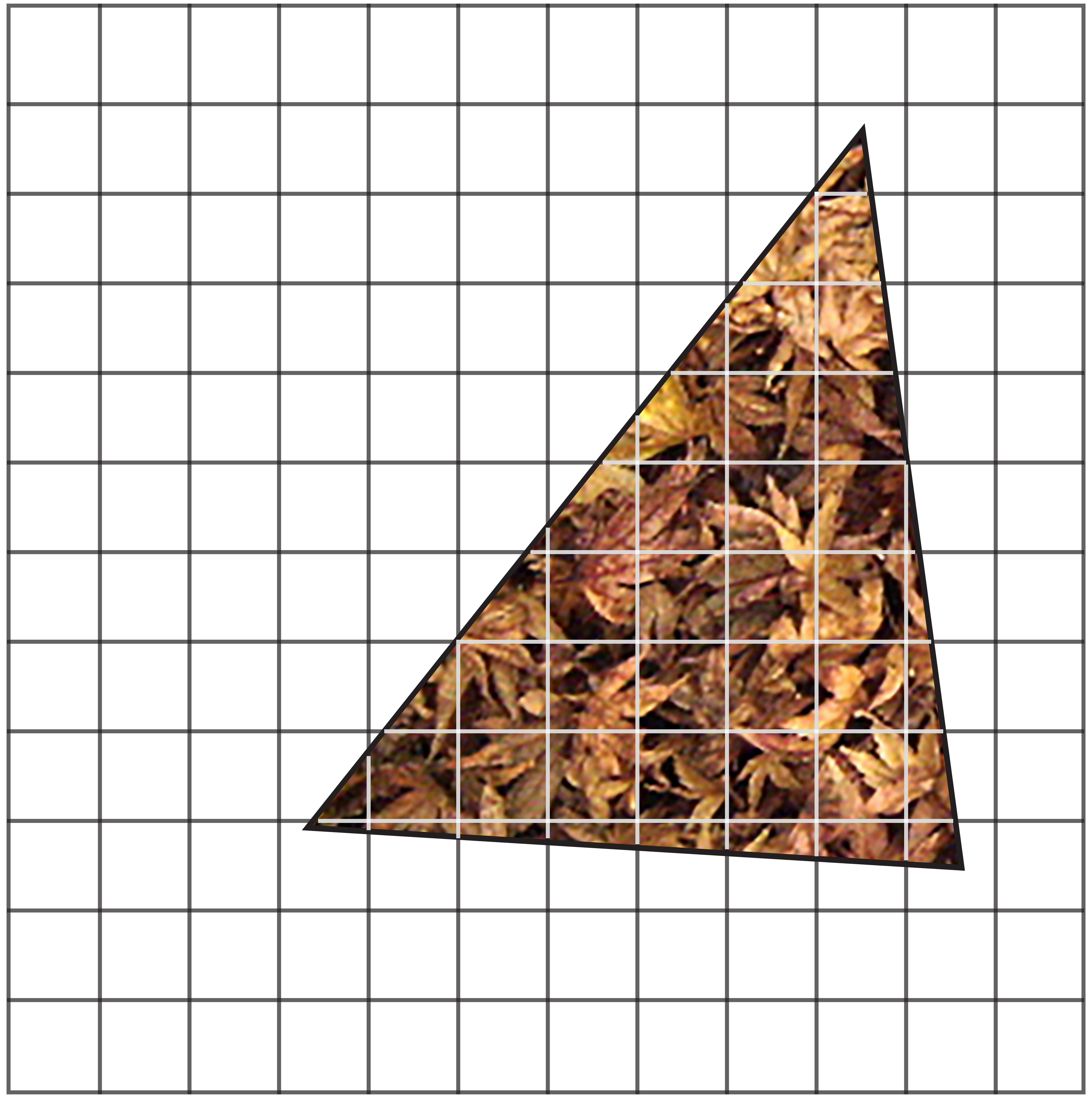

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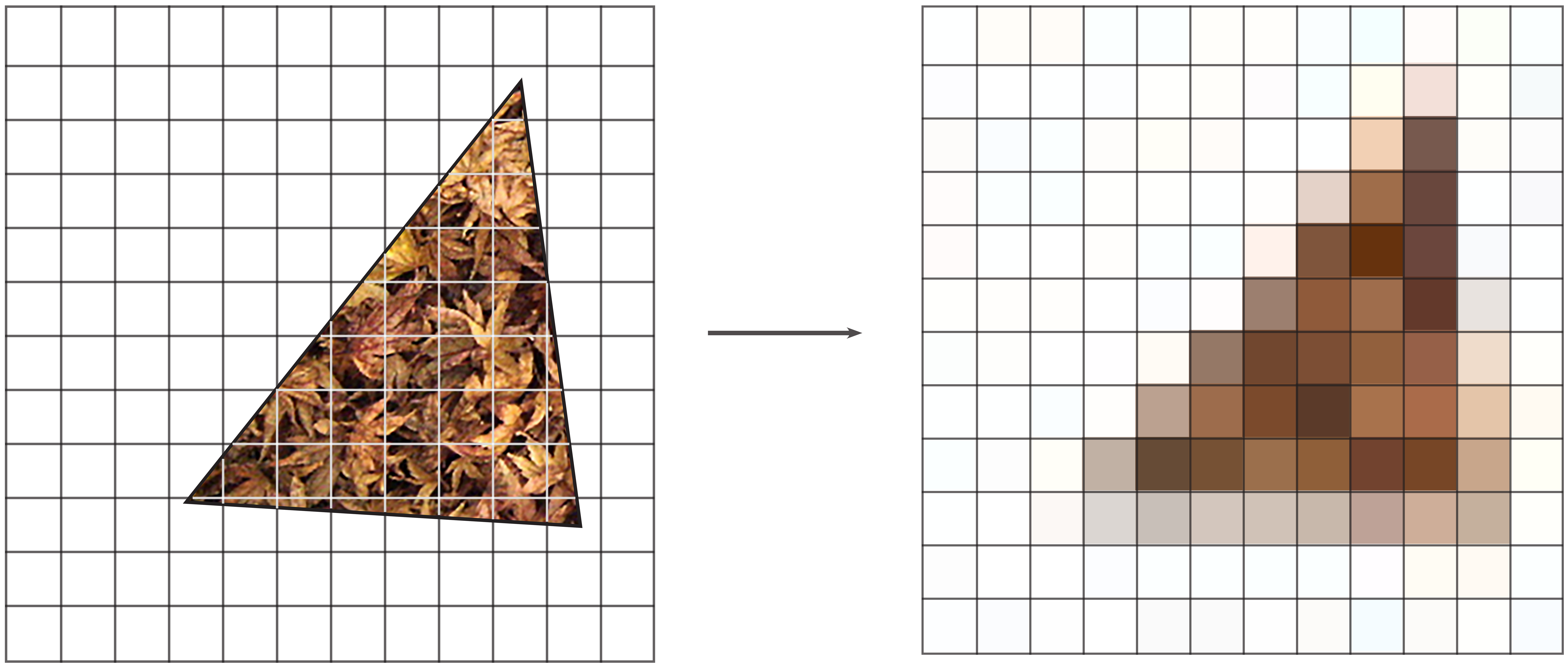

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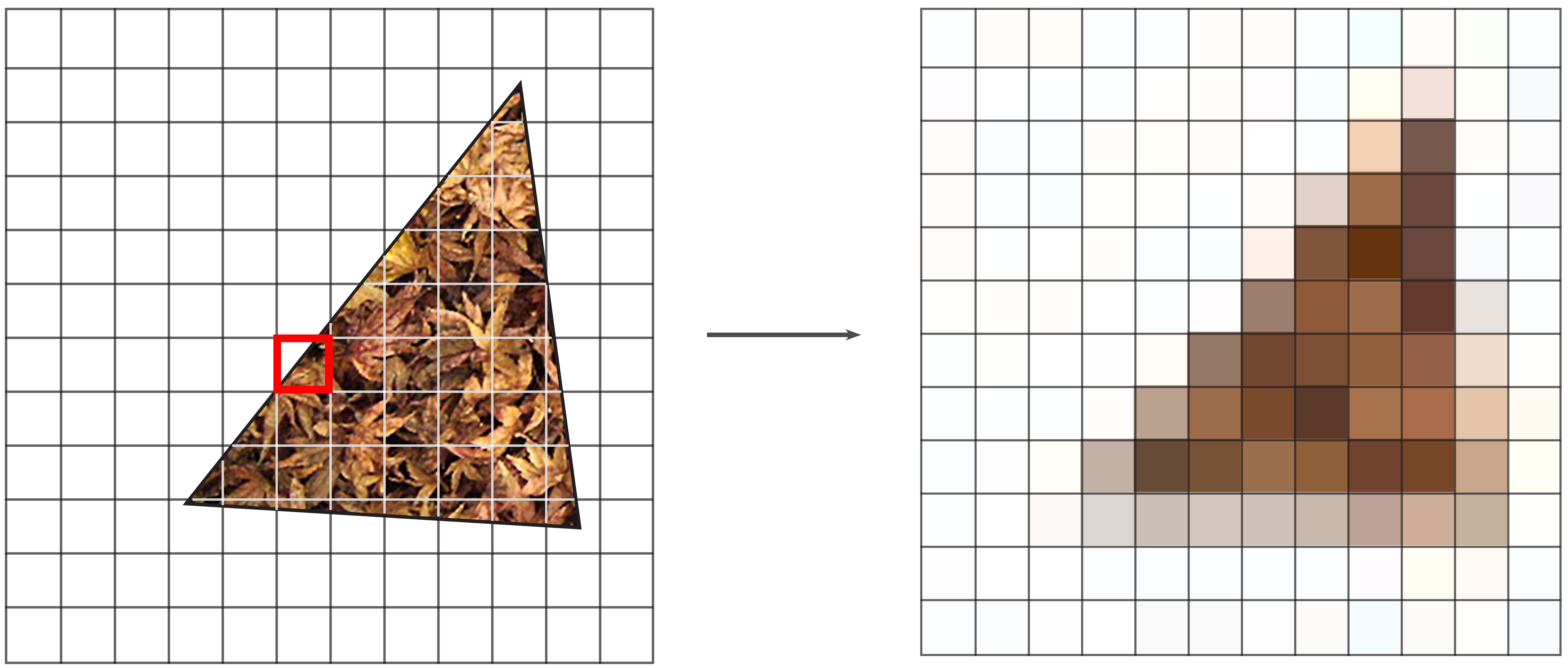

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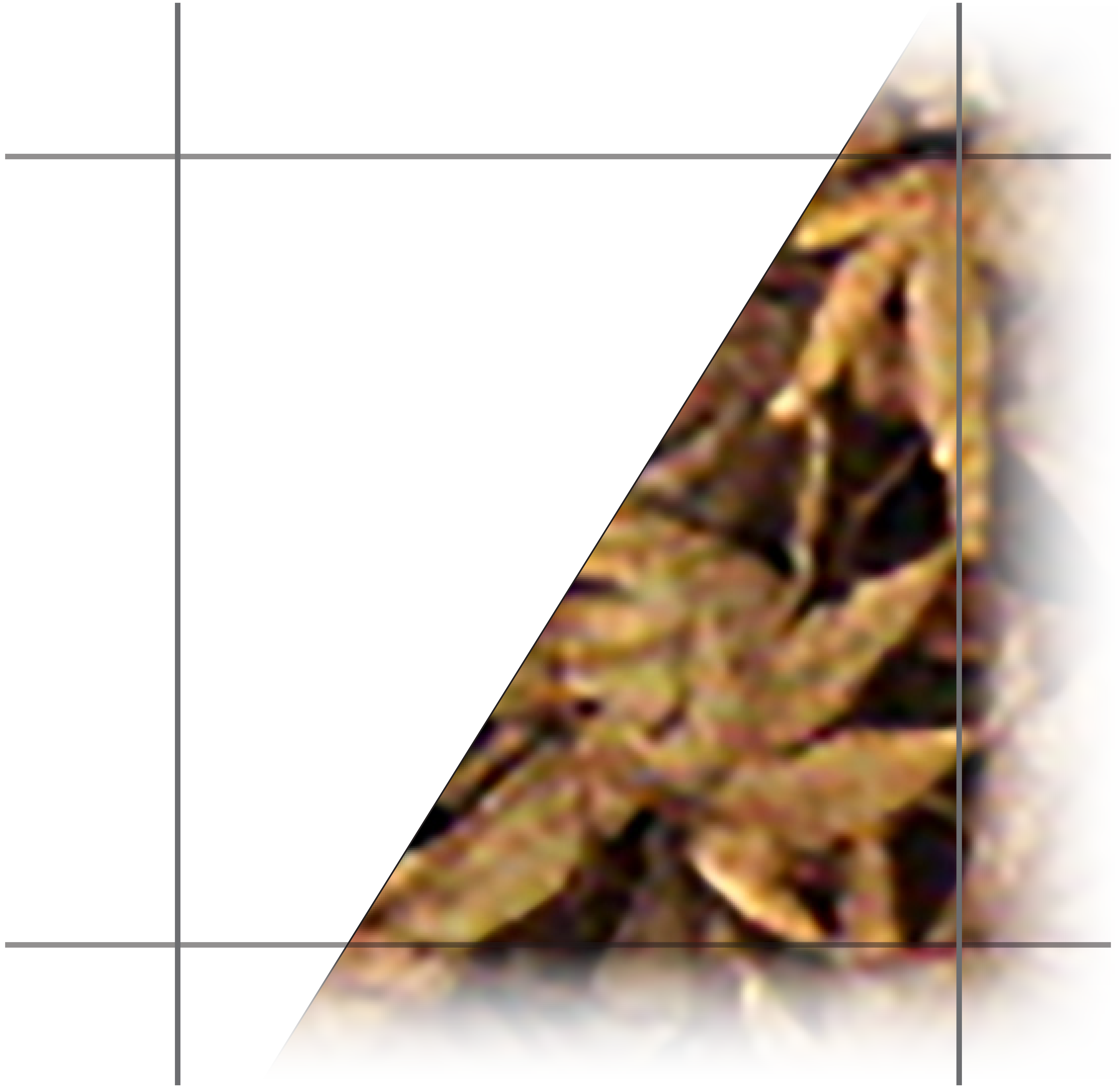

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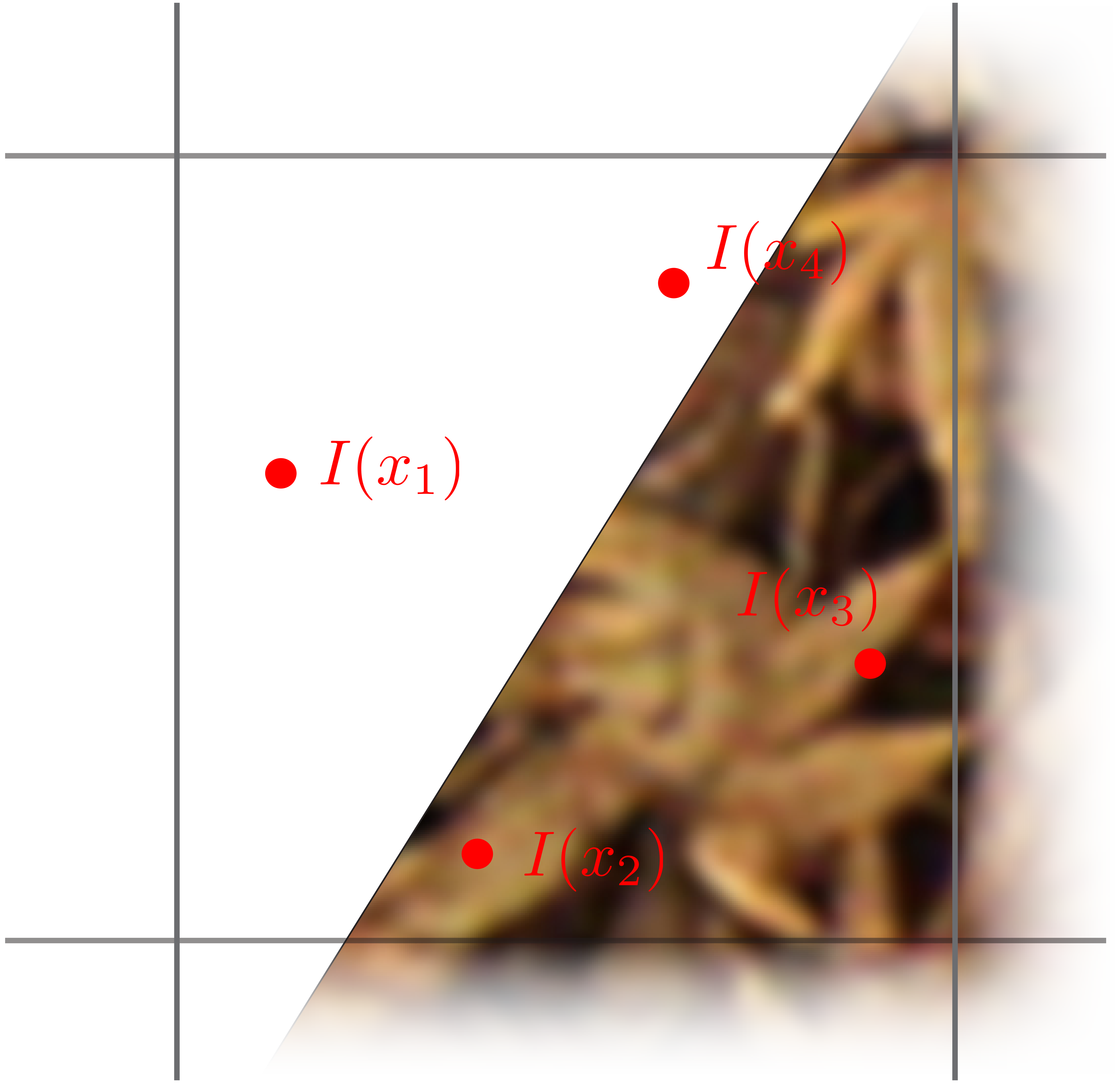

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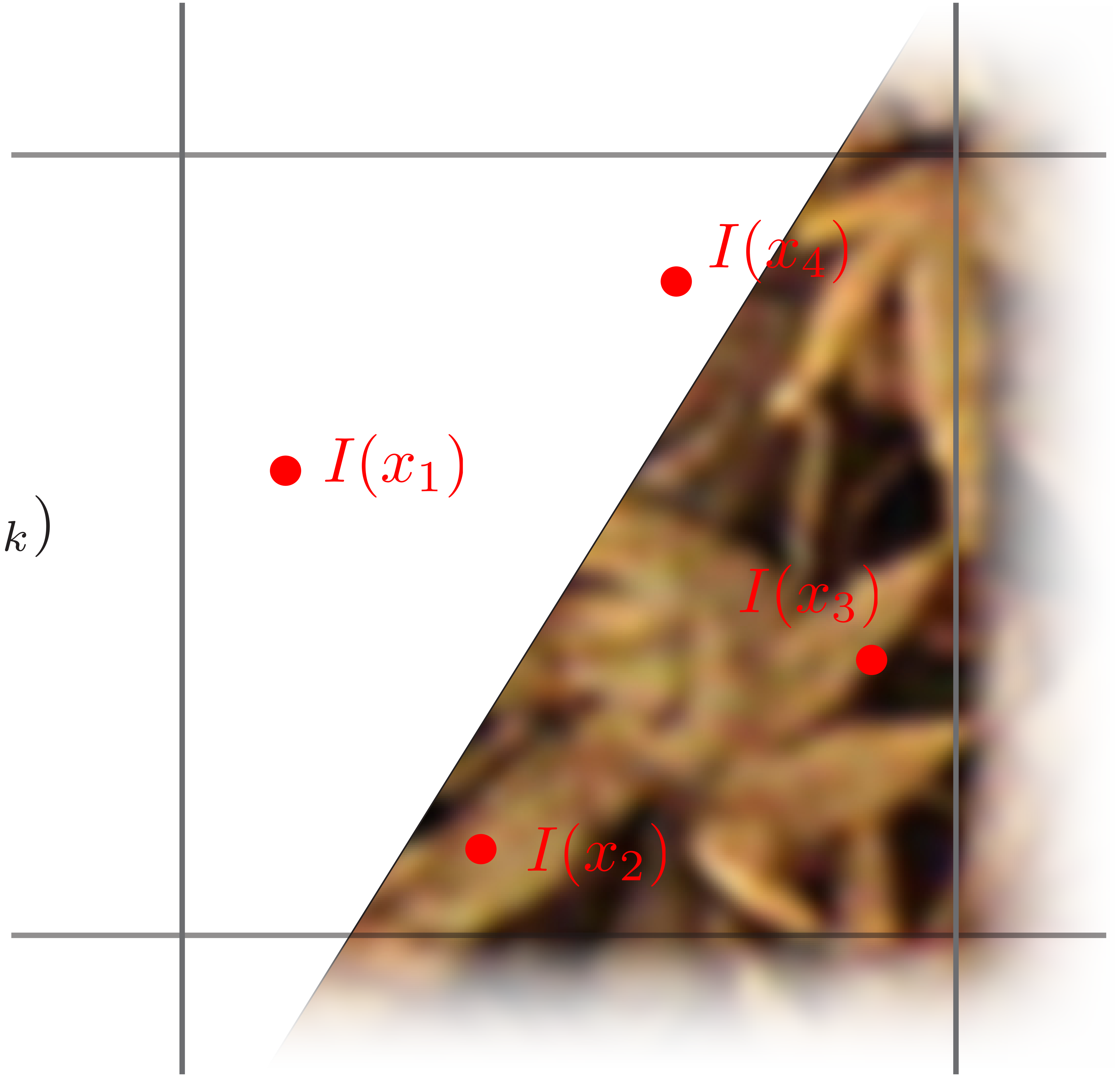
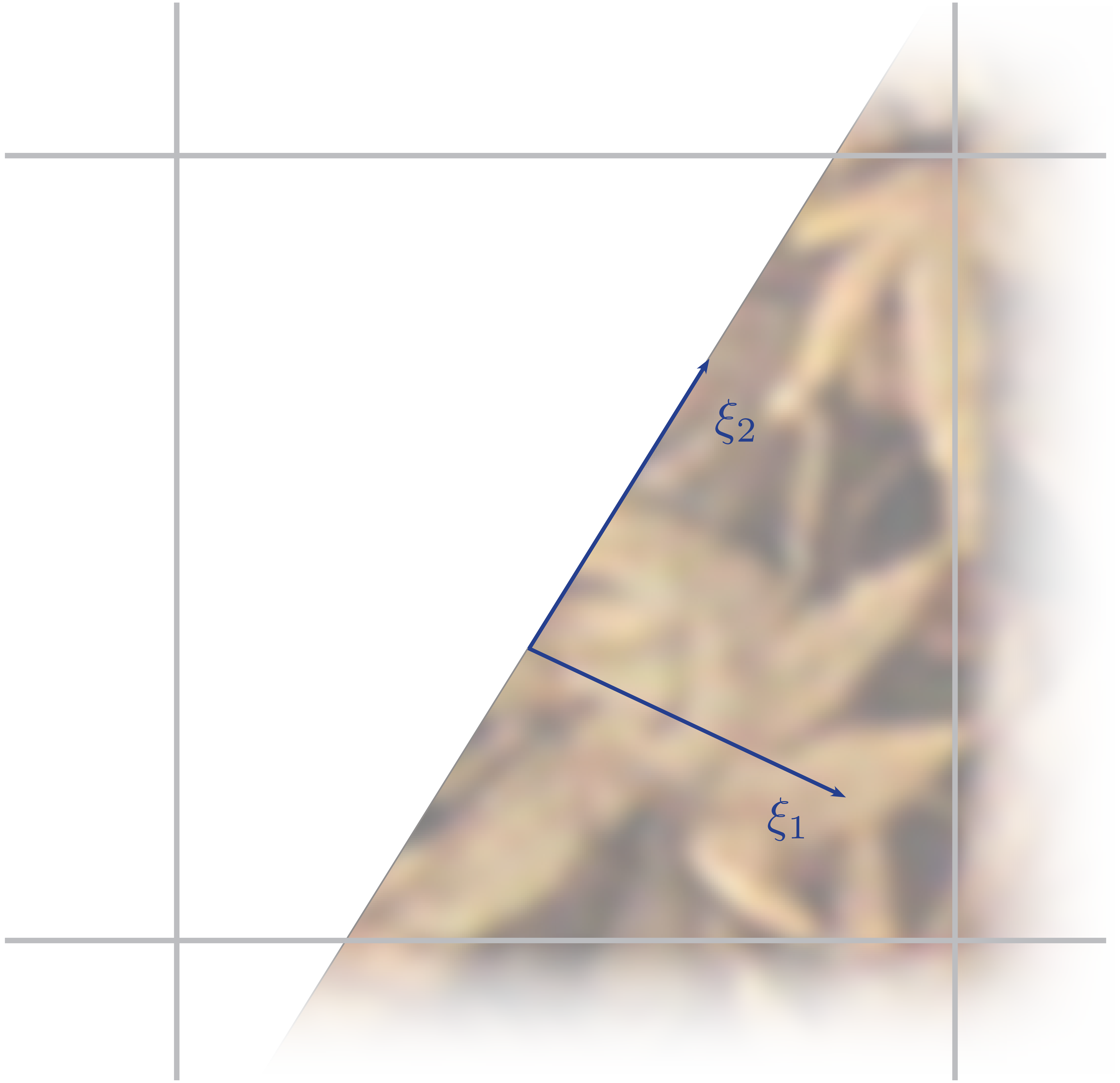
$$I_{ij} = \frac{|P_{ij}|}{4} \sum_{k=1}^{4} I(x_k)$$



Idea

Localized Fourier transform



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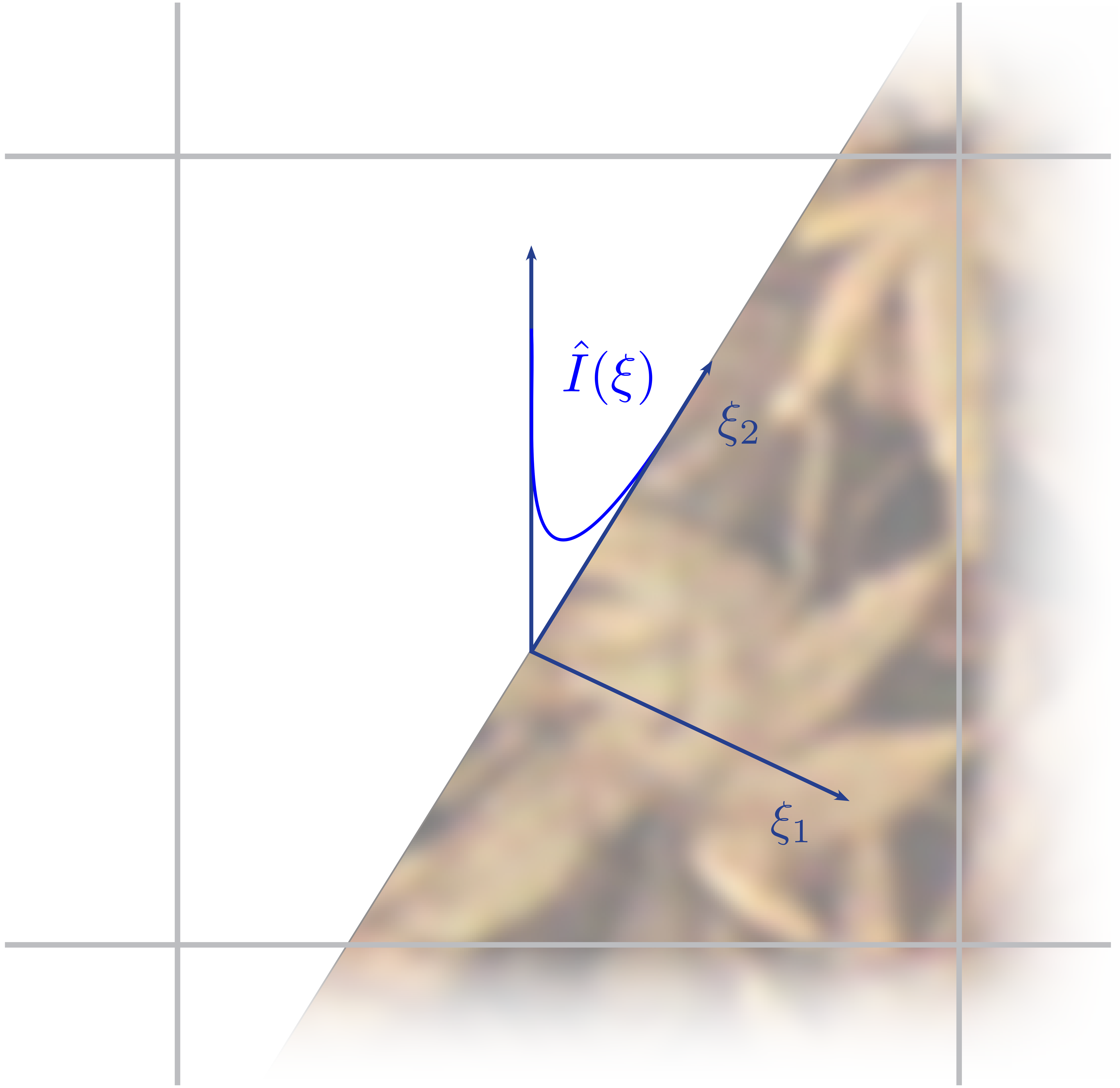

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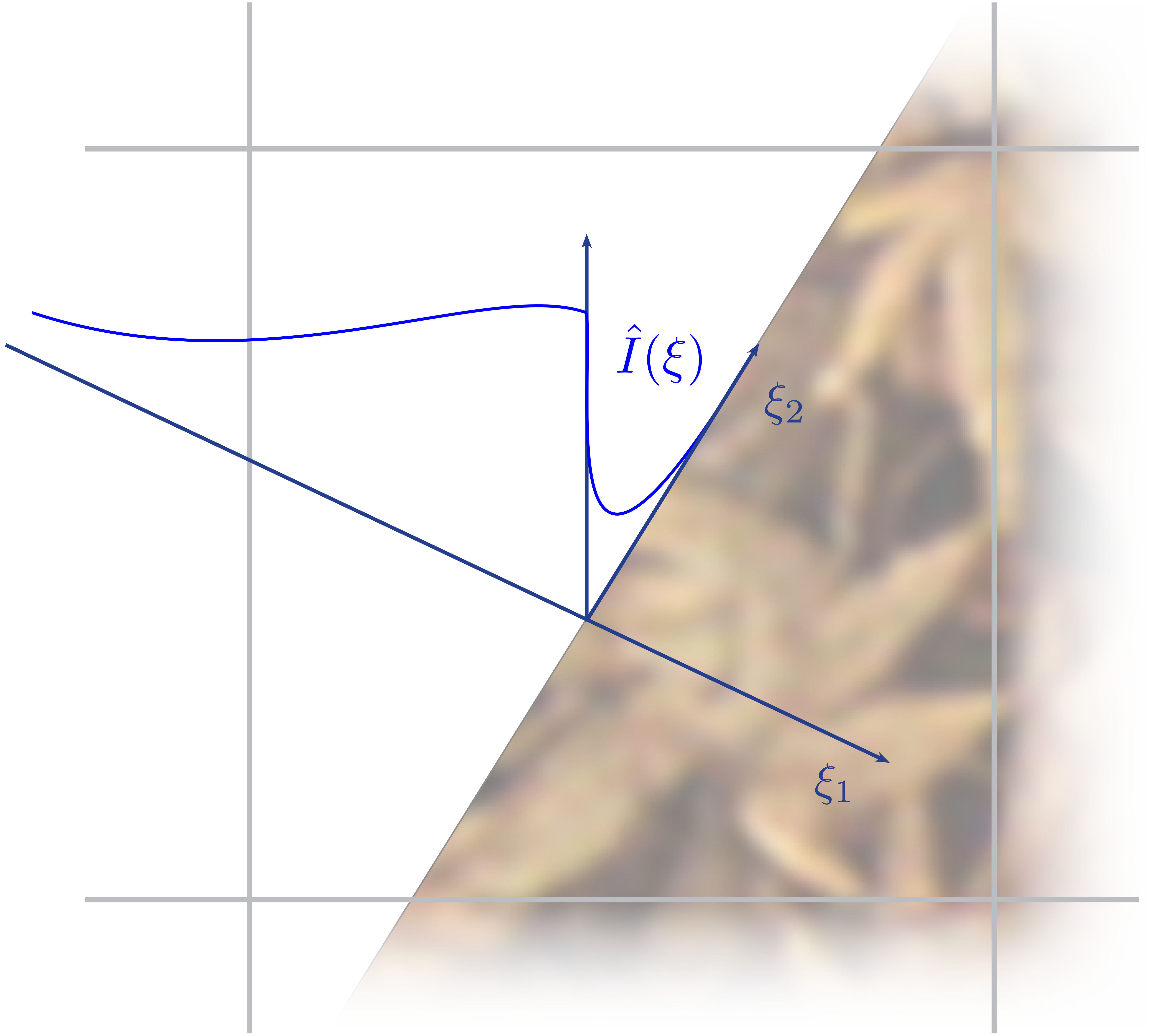

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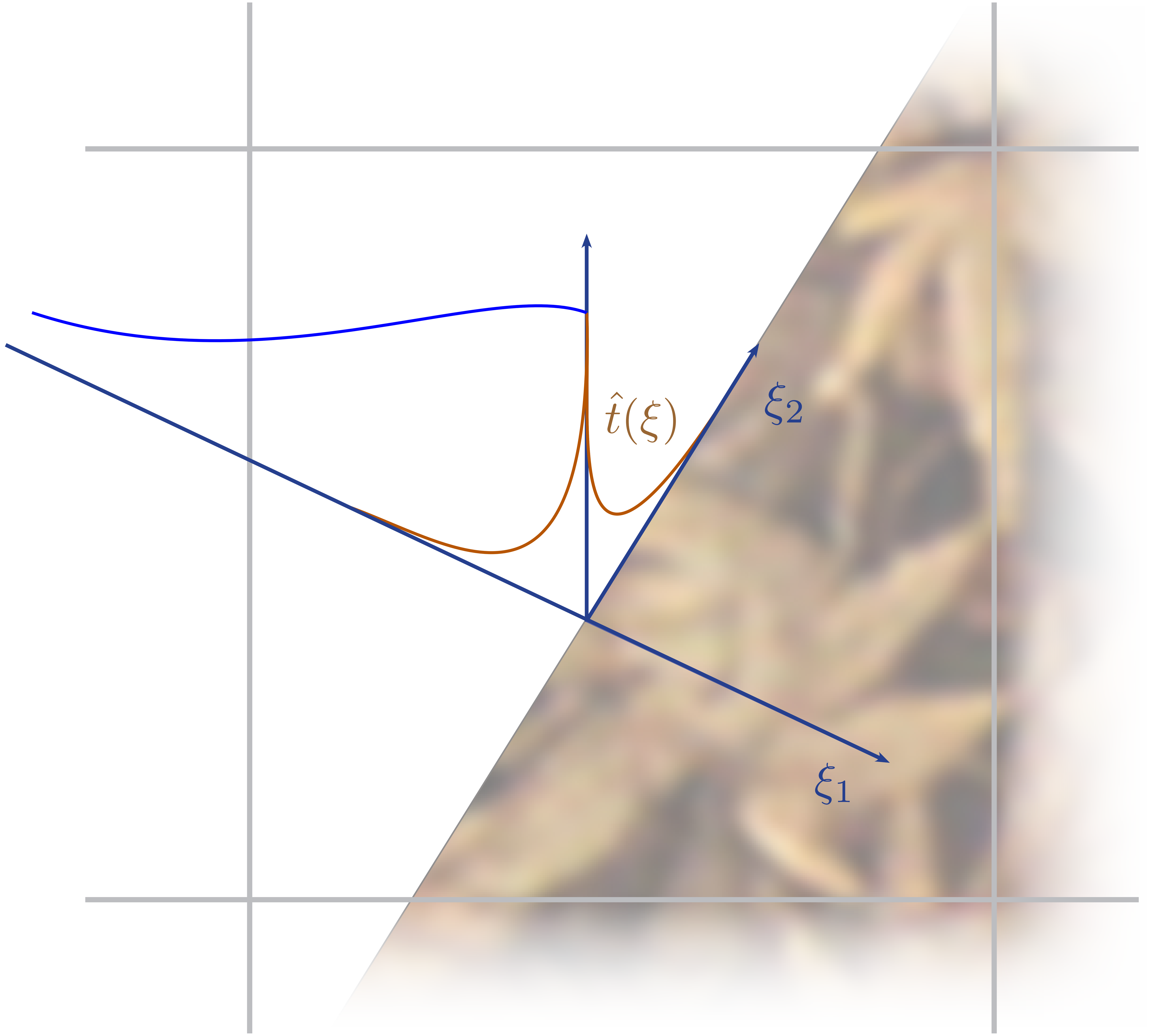

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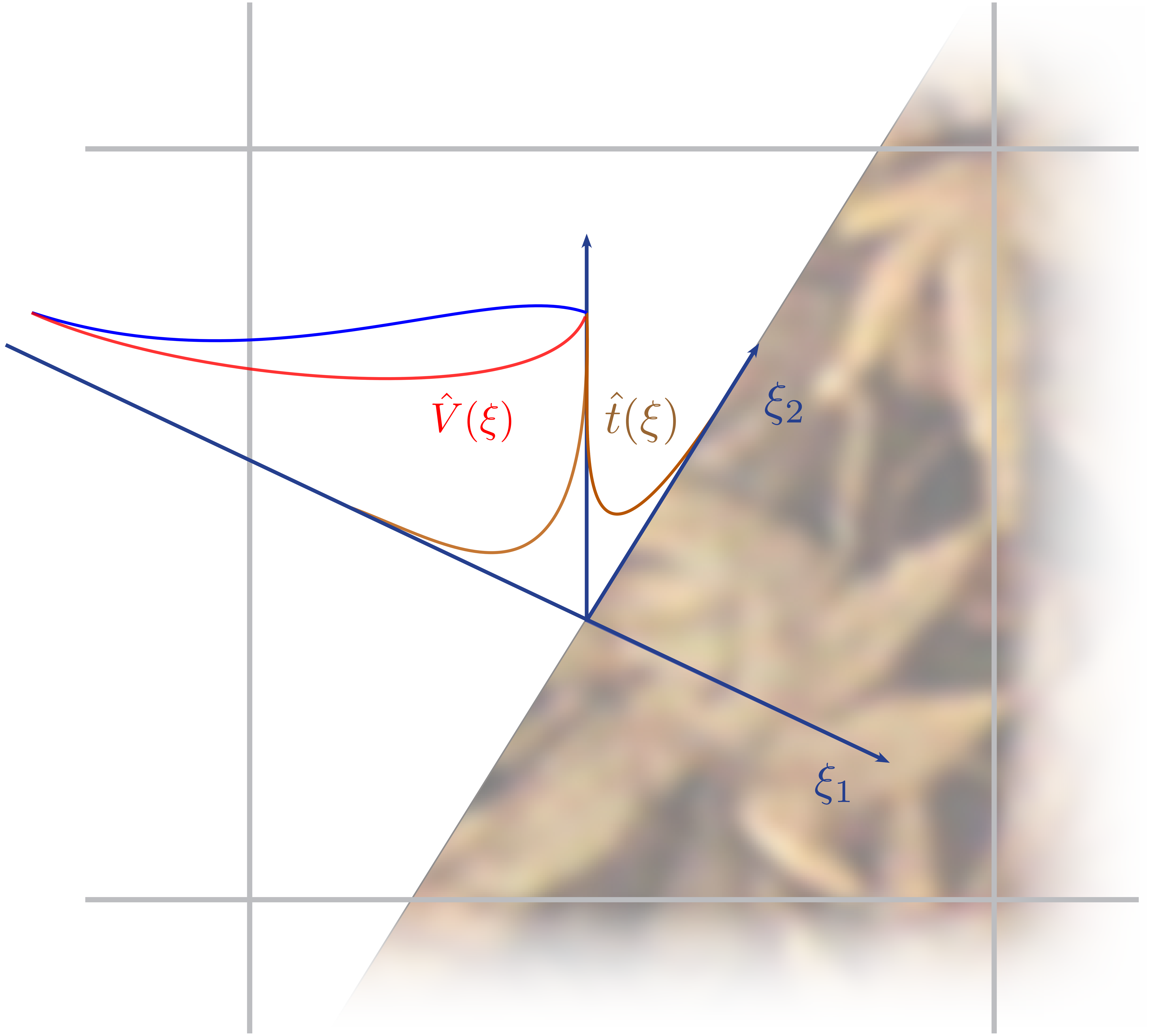

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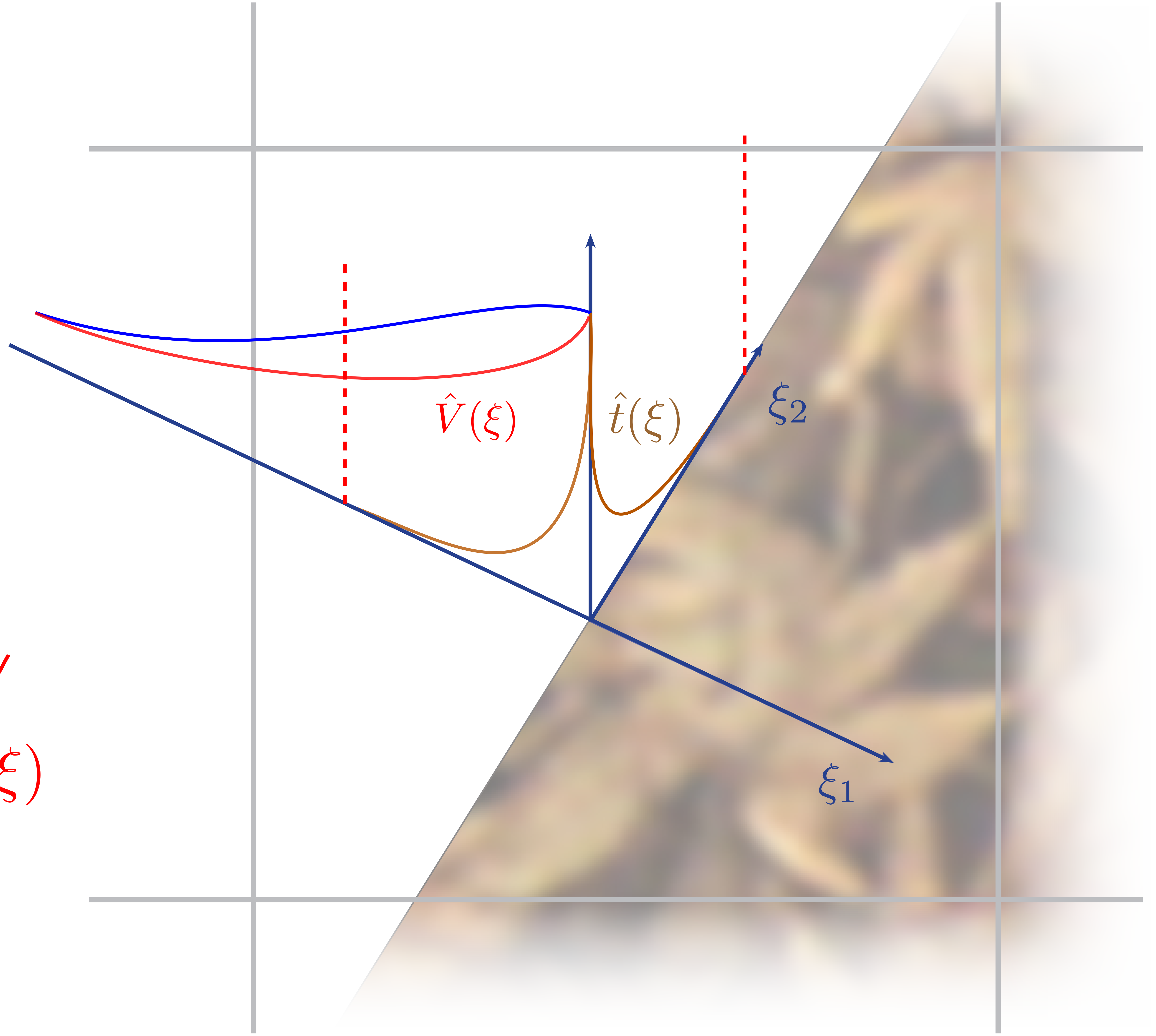

Sampling: regularize / pre-filter $\hat{I}(\xi)$

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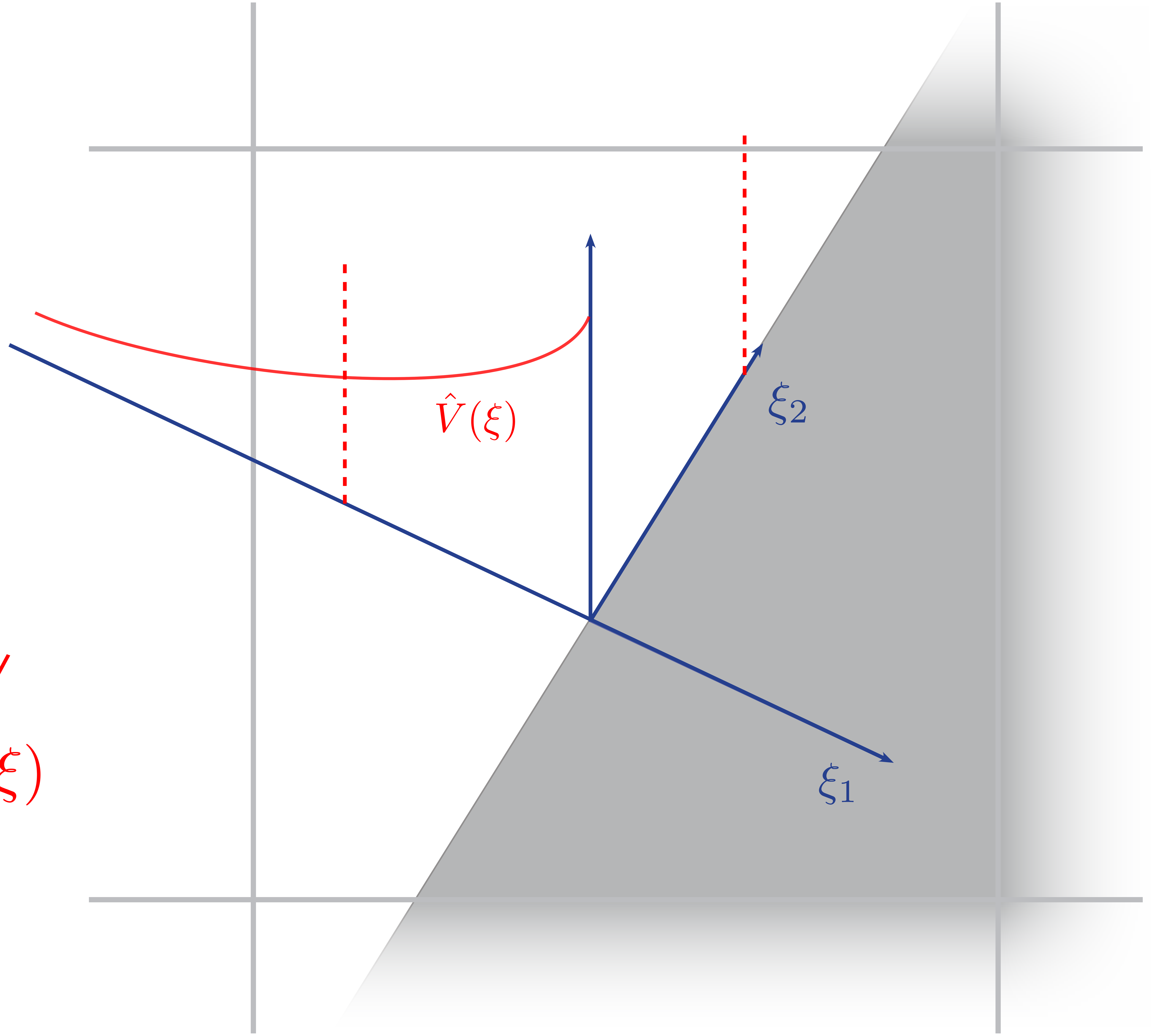

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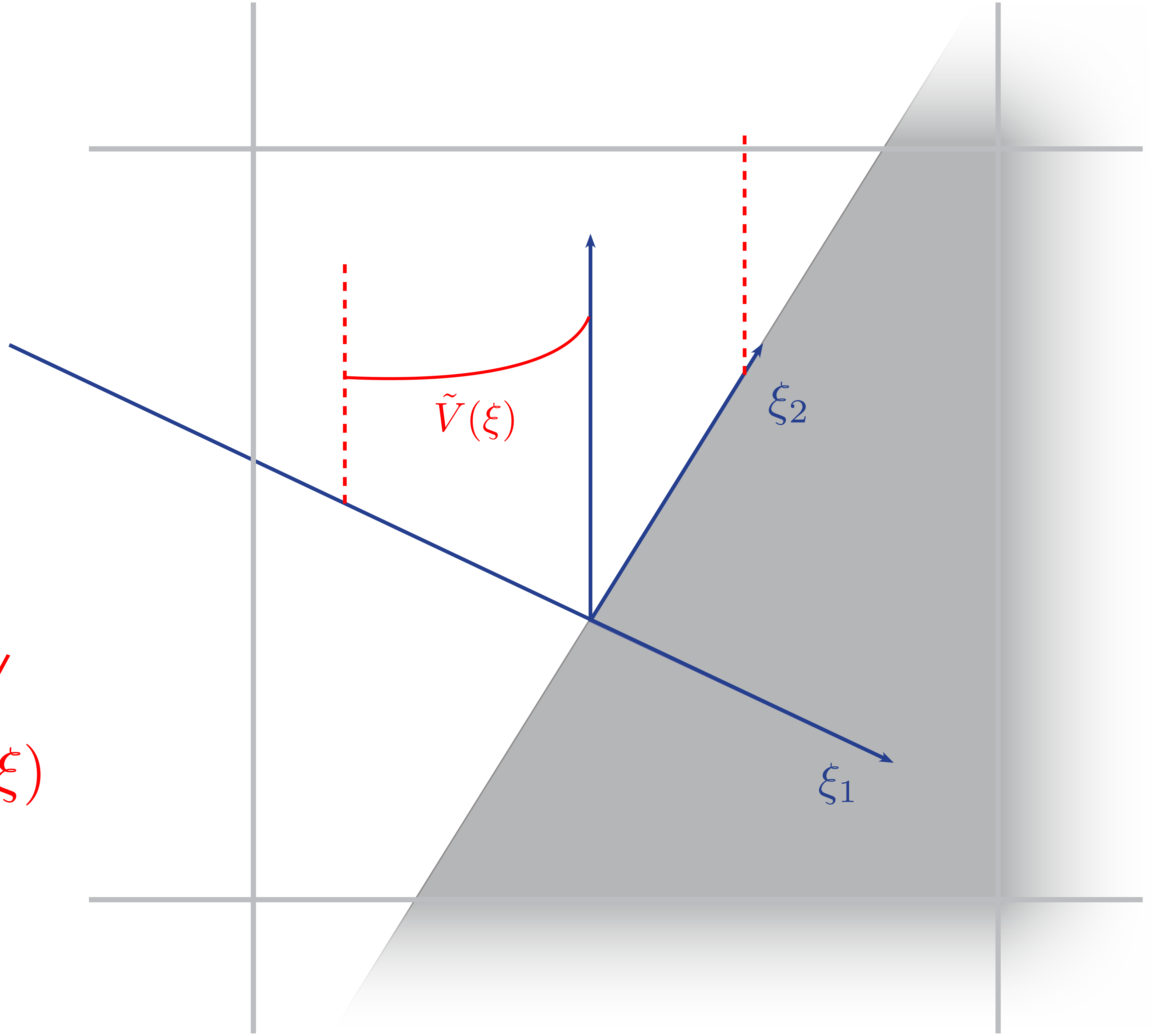

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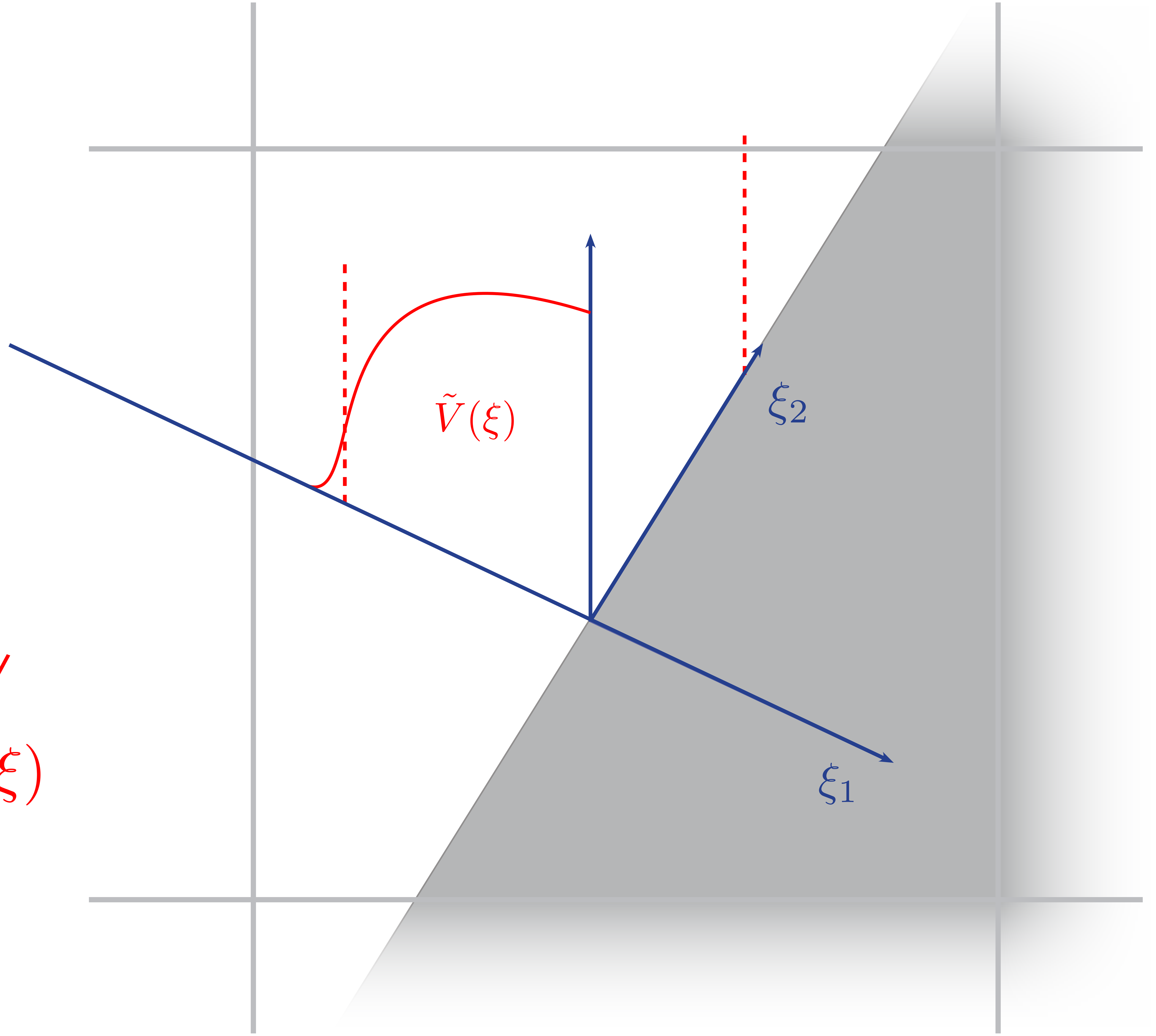

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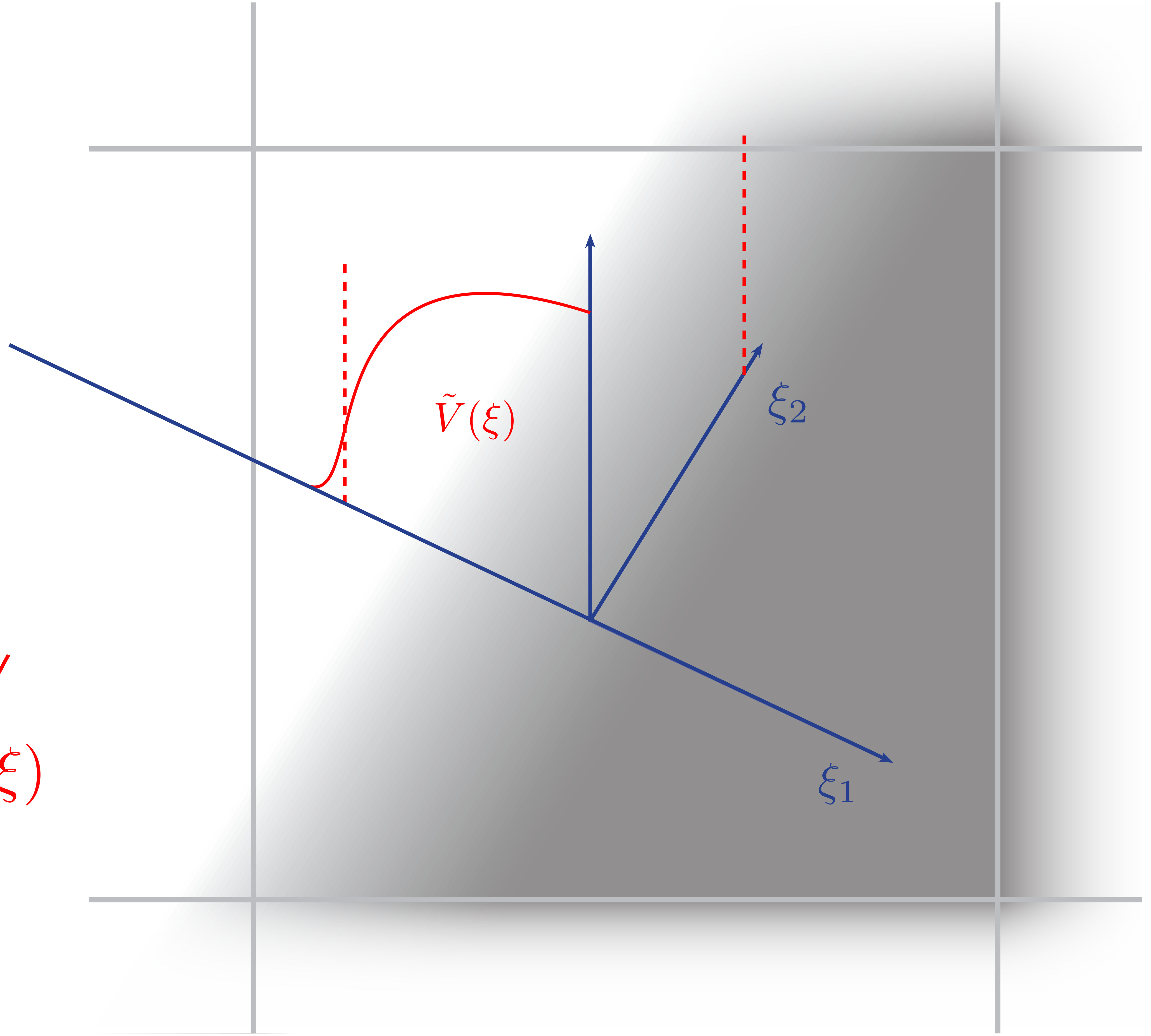

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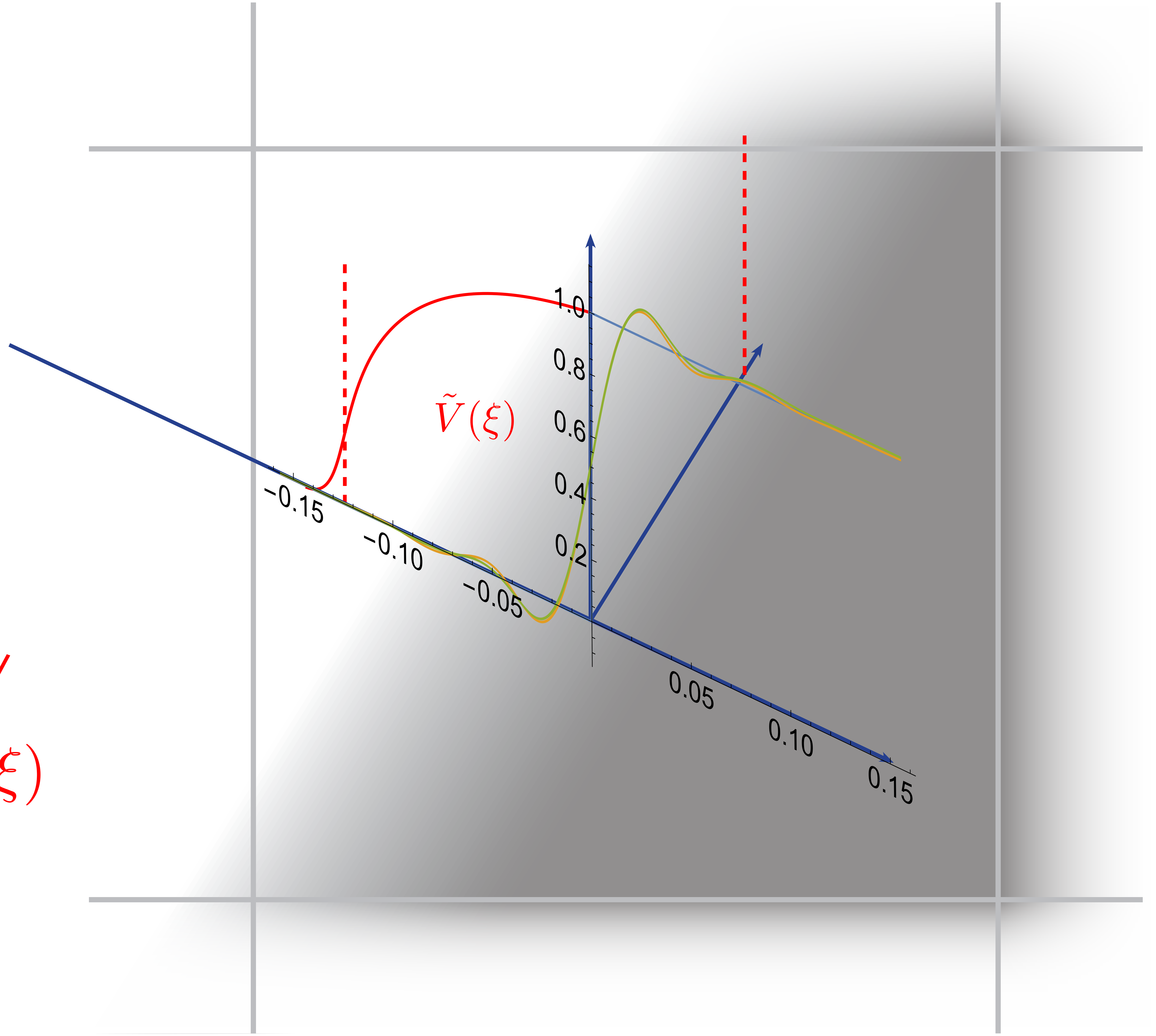

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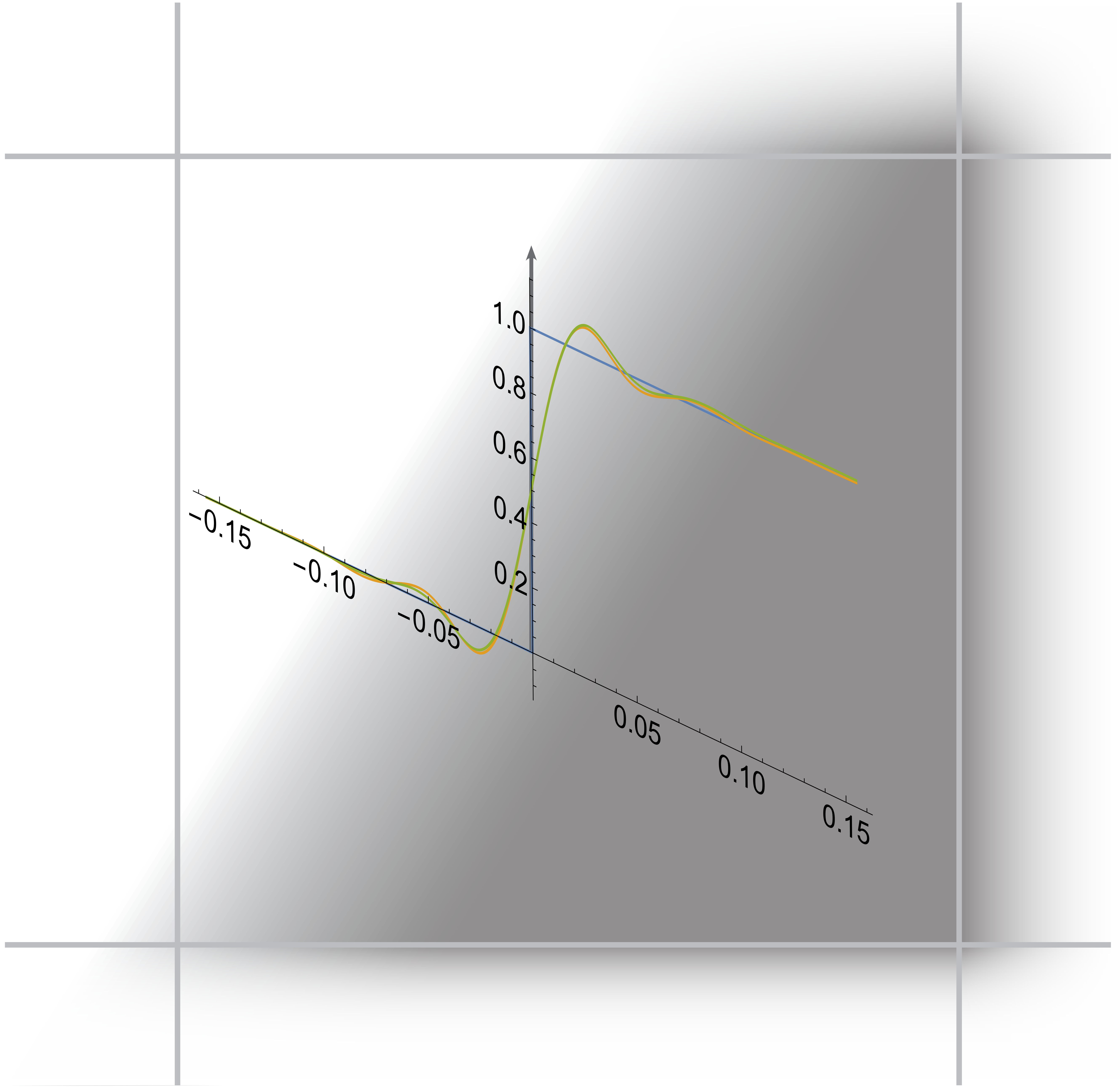

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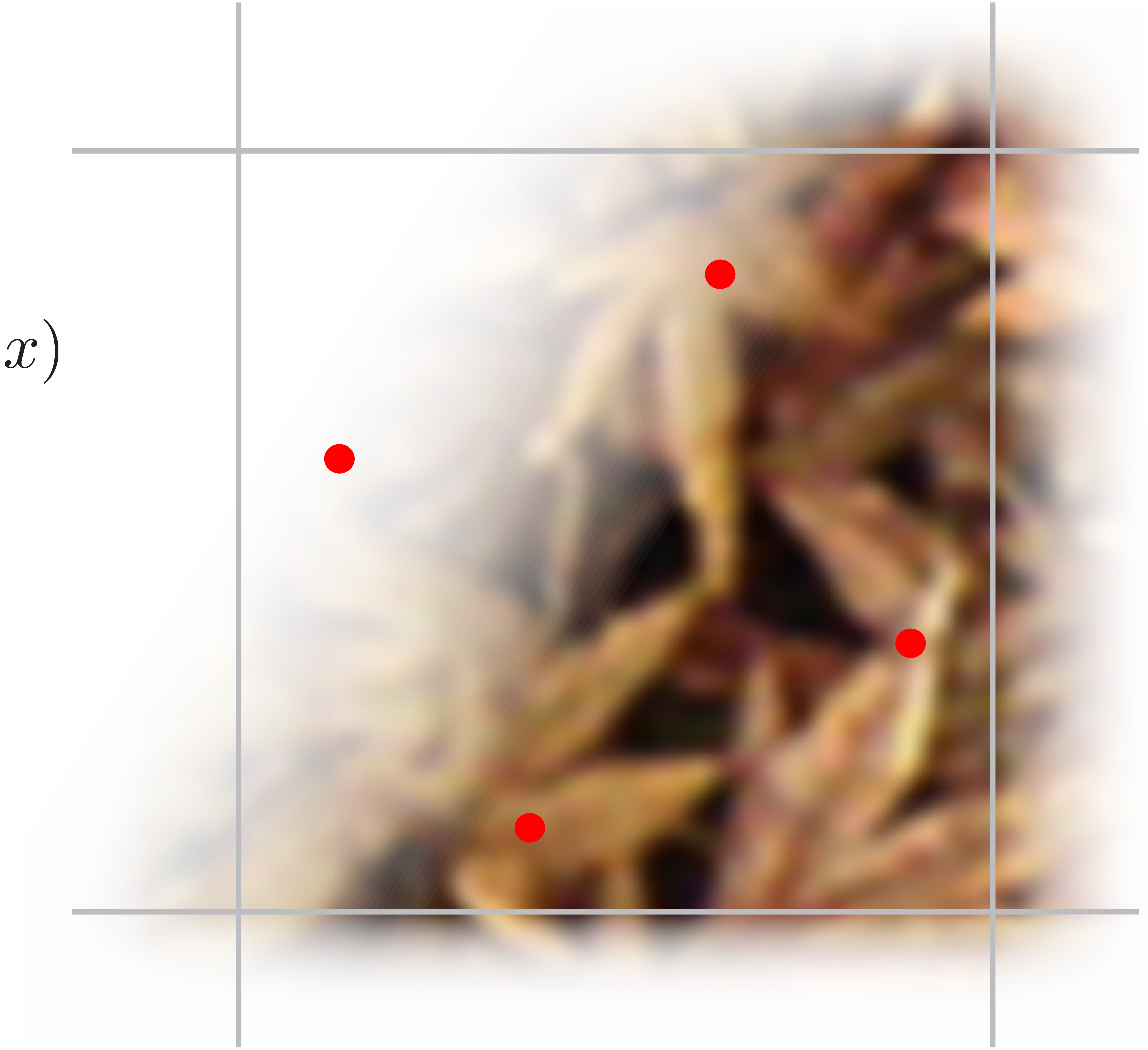

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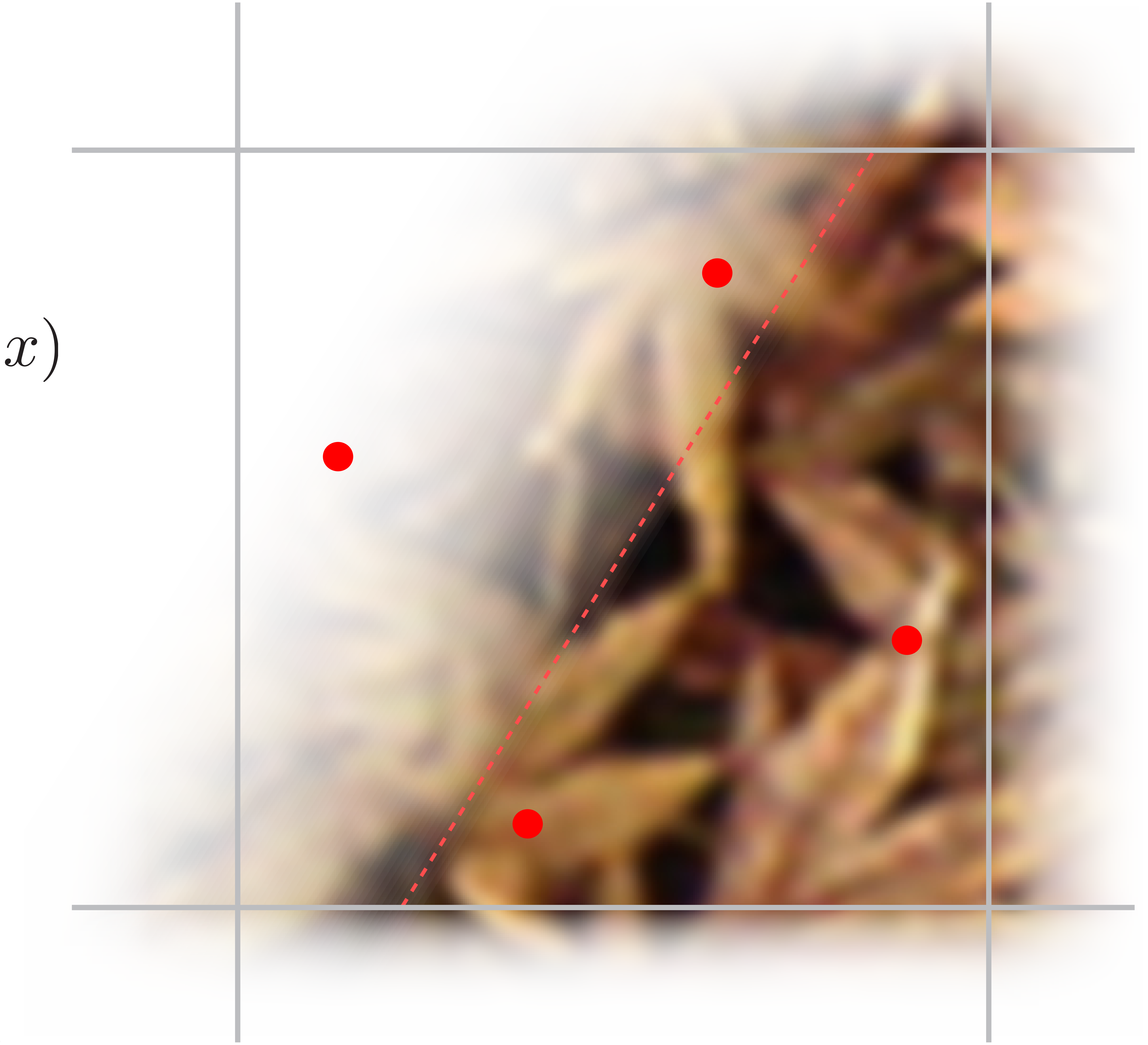

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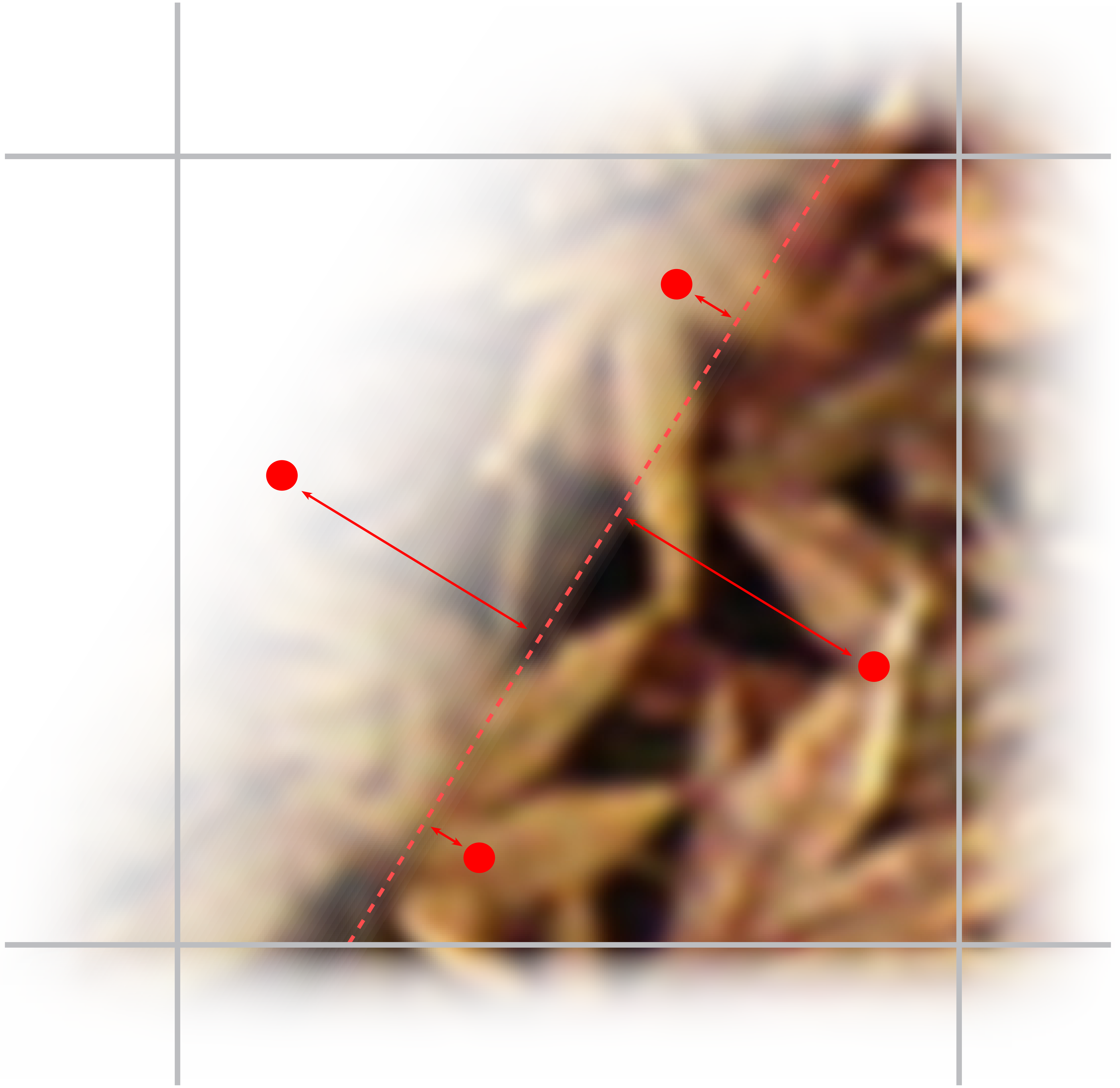

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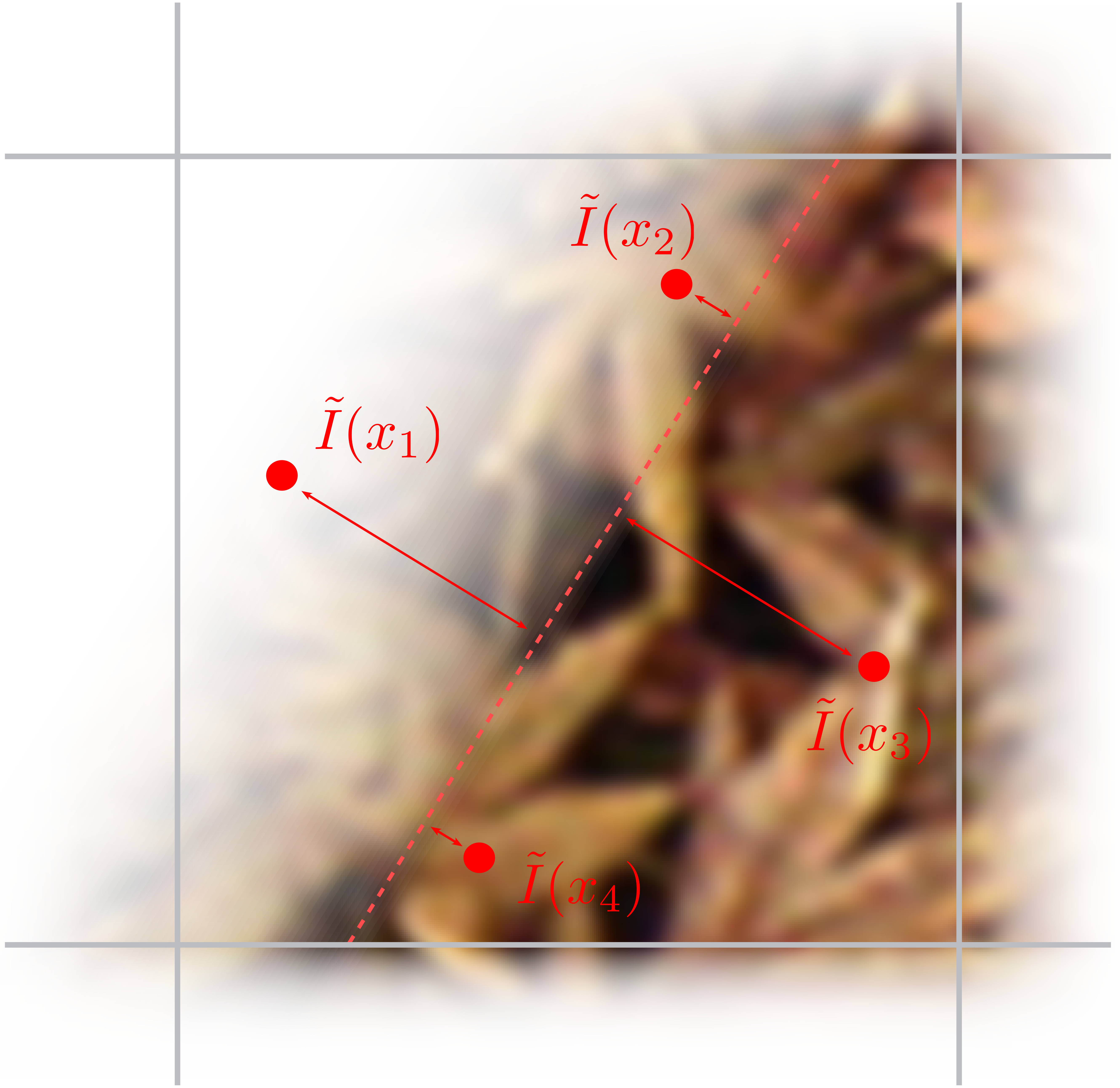

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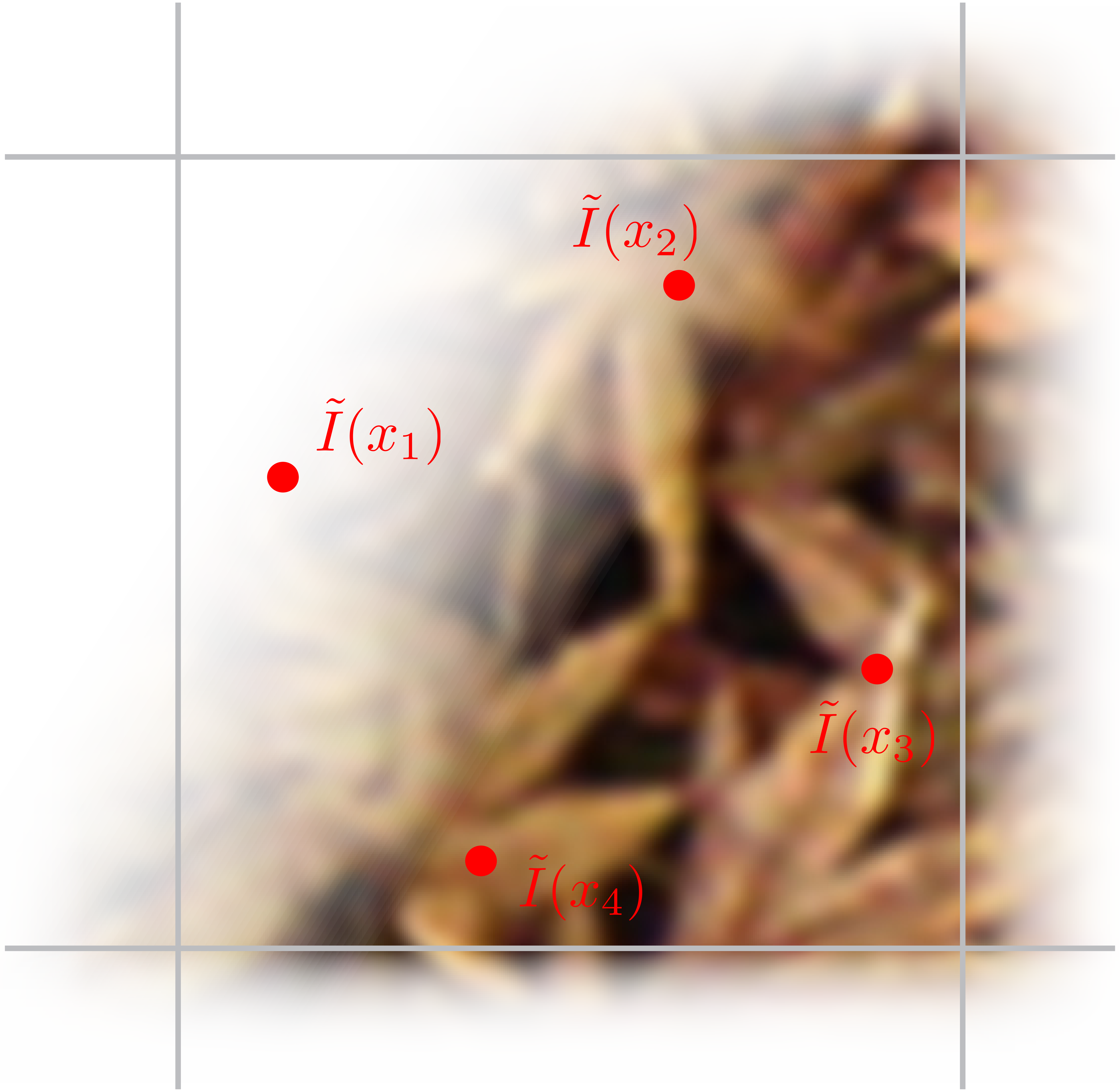

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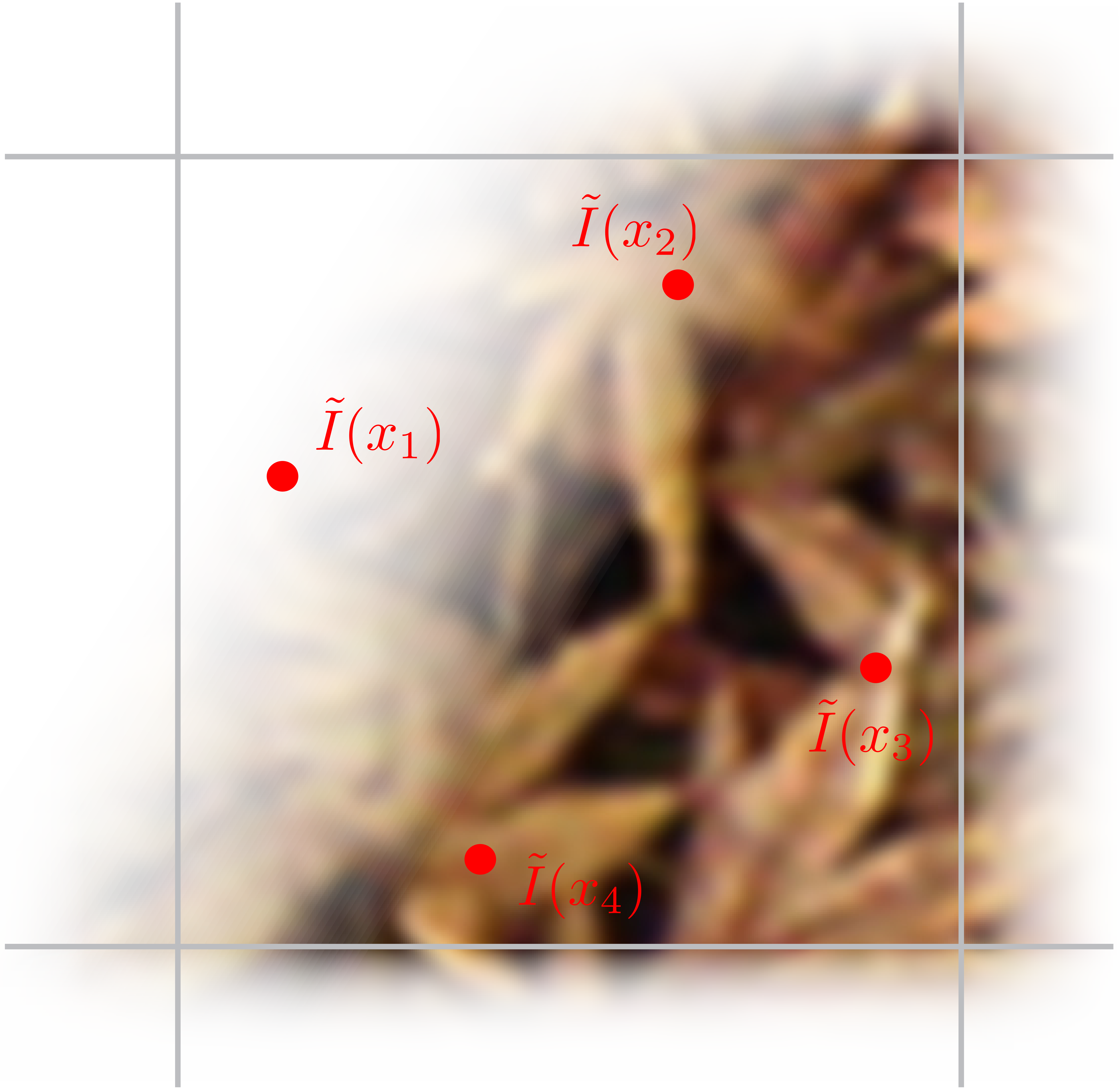

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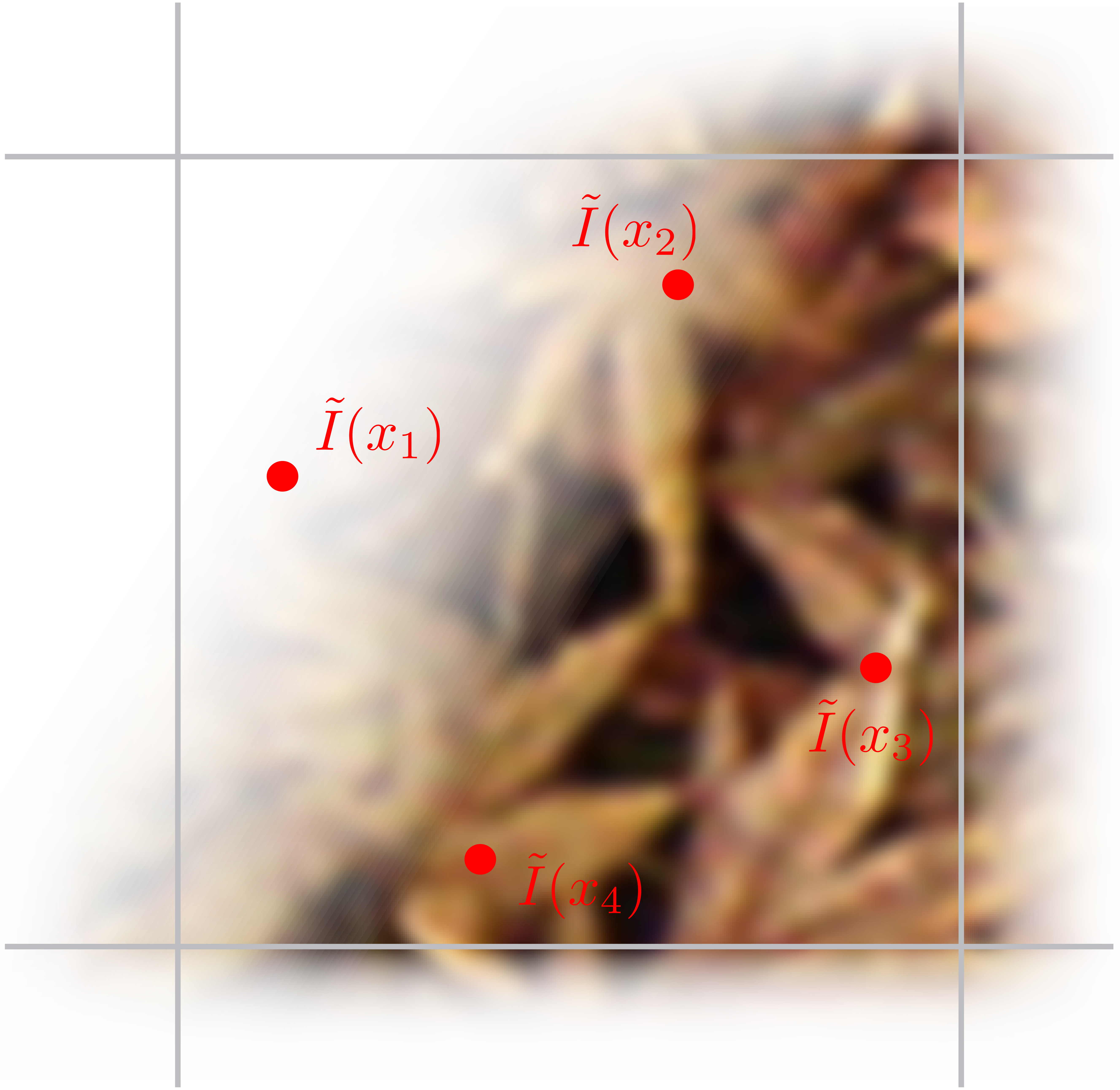

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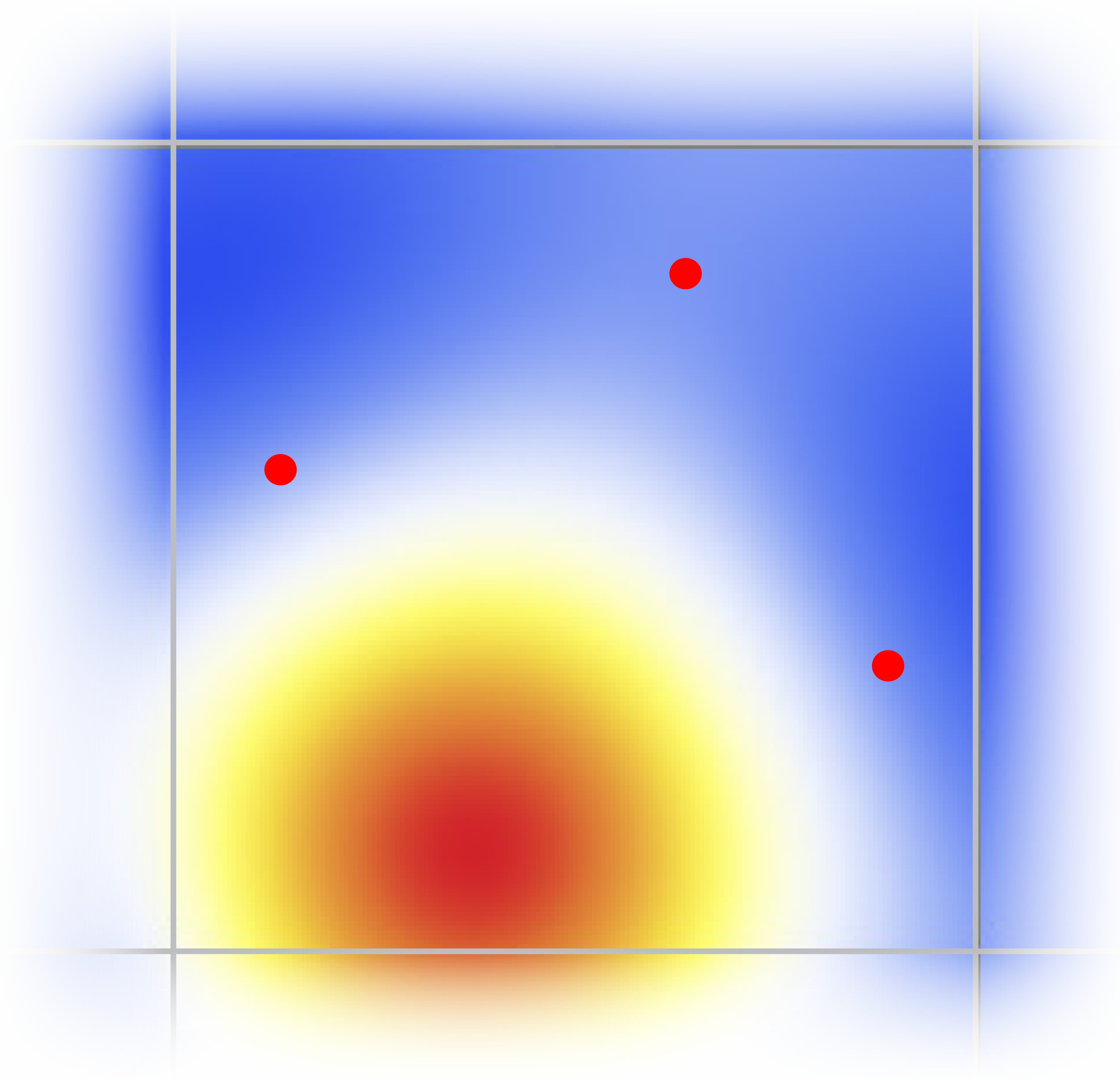

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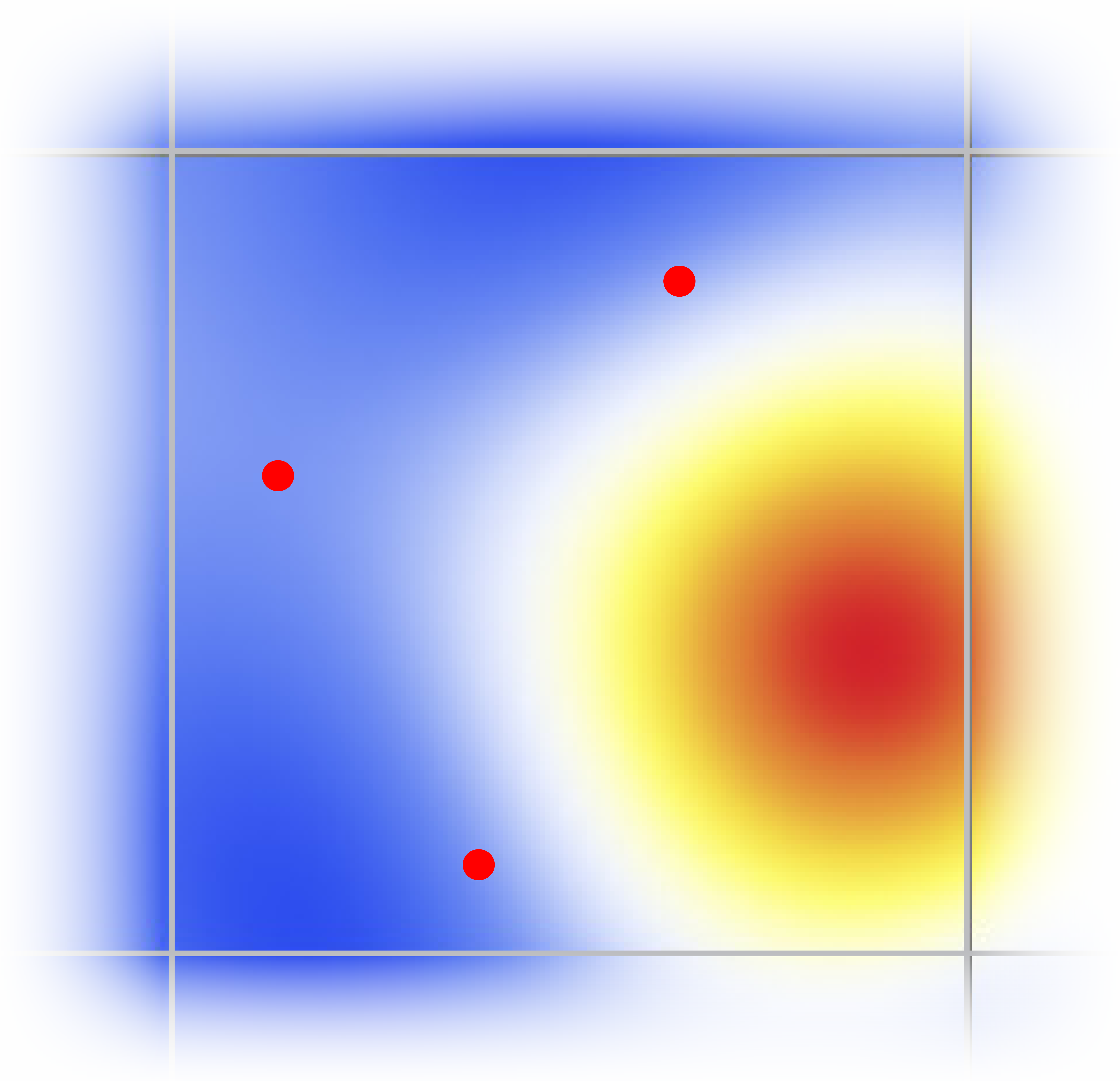

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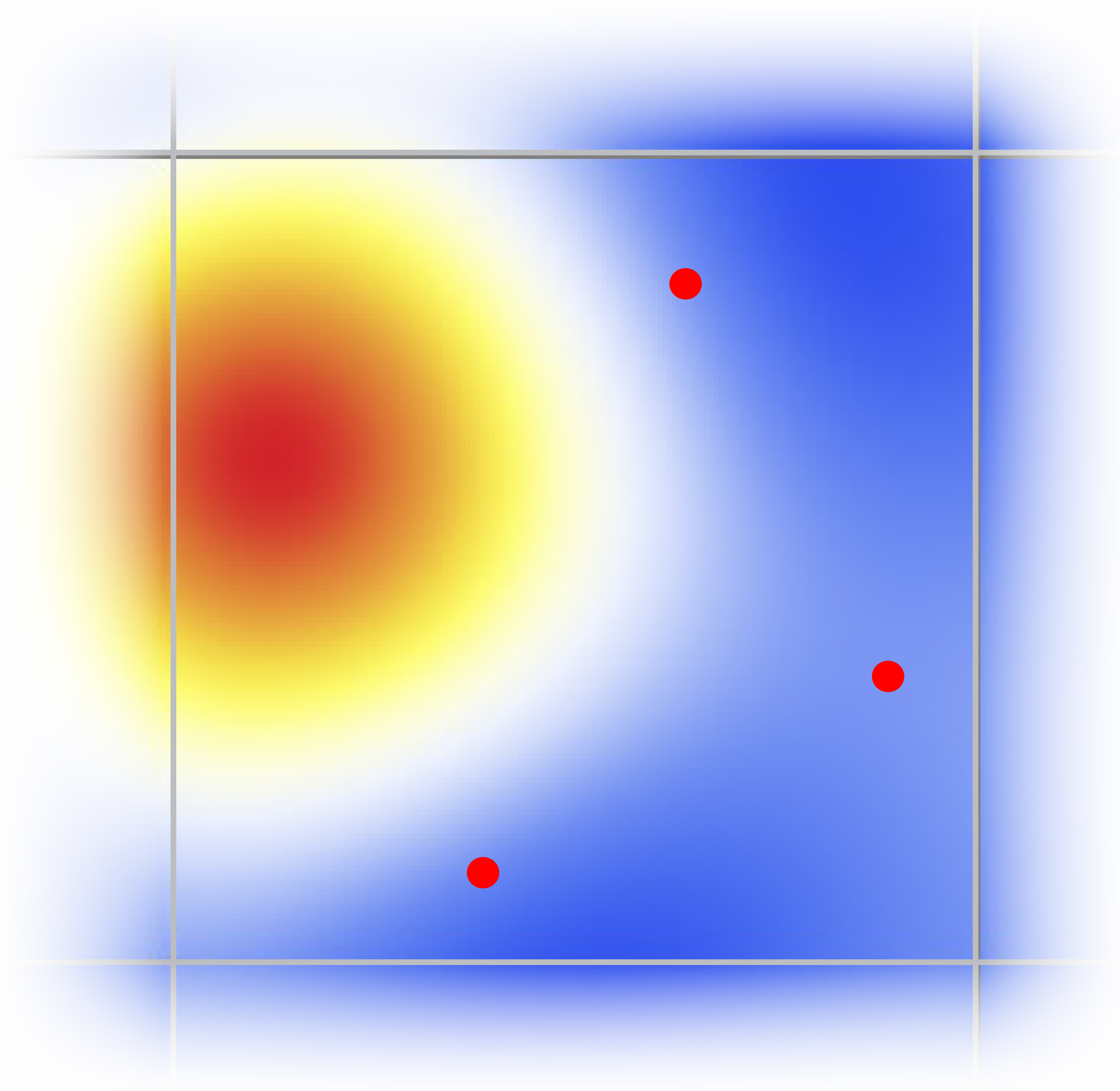

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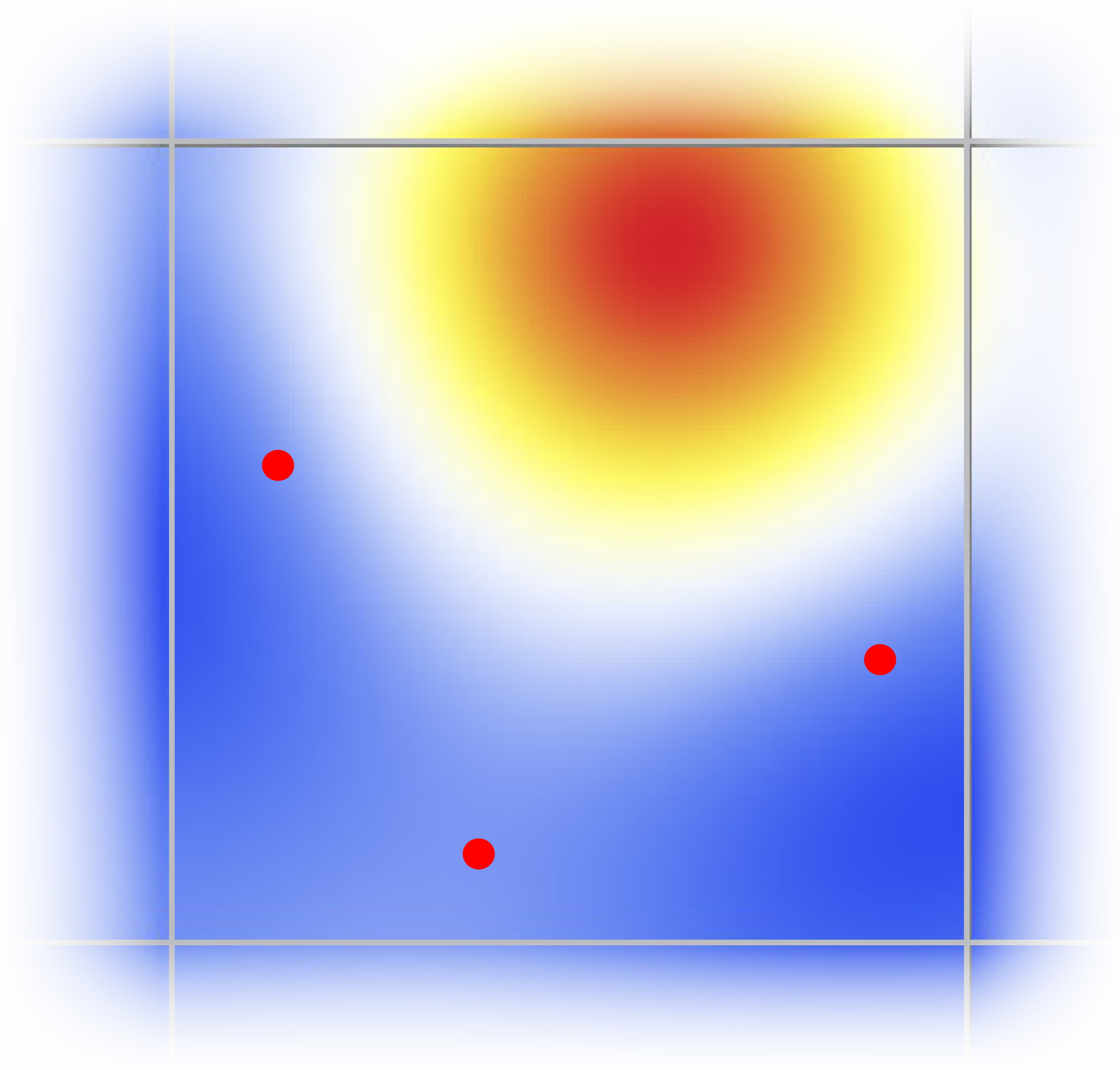

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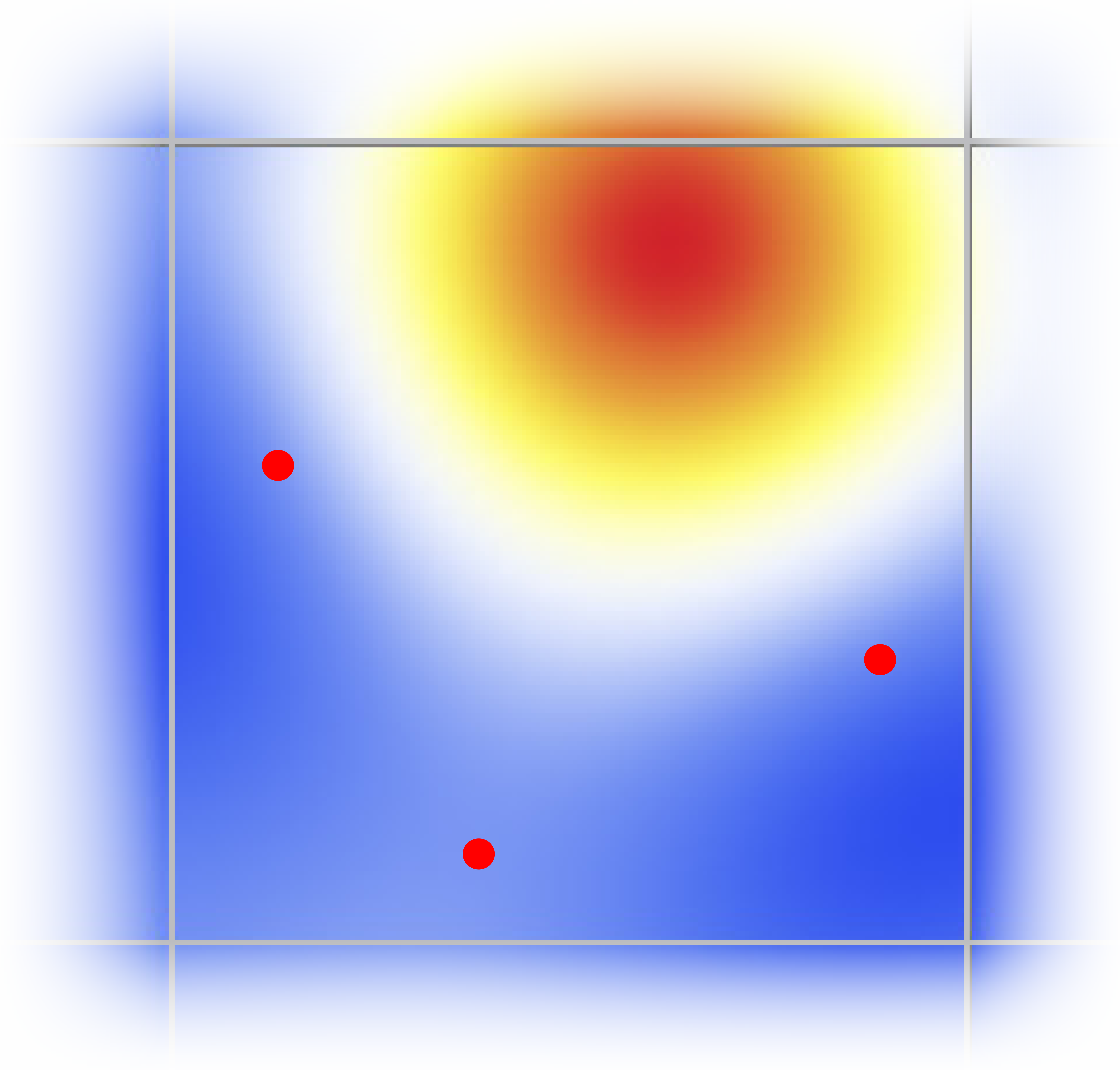
Depend on / coupled to pre-filtering



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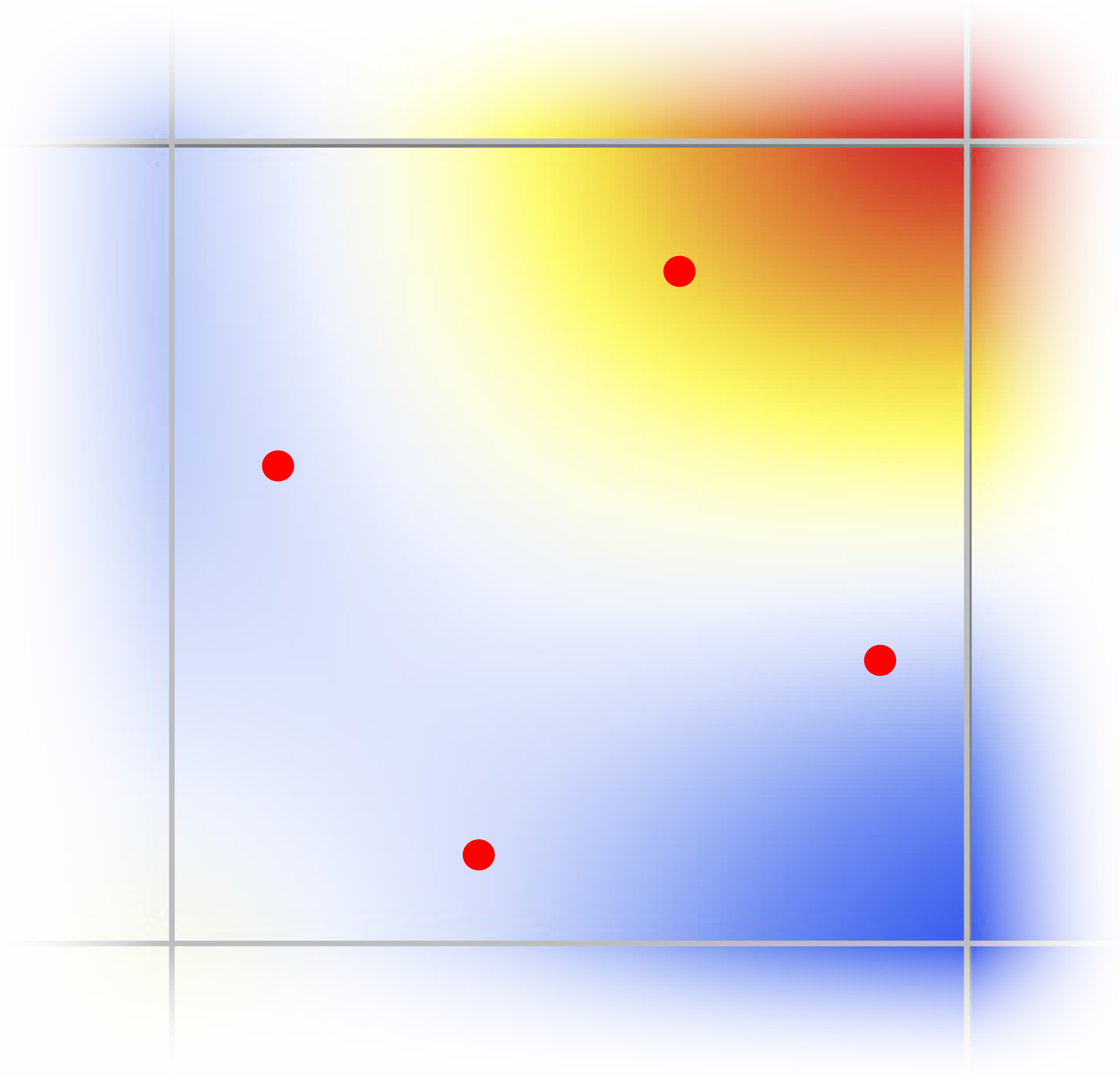

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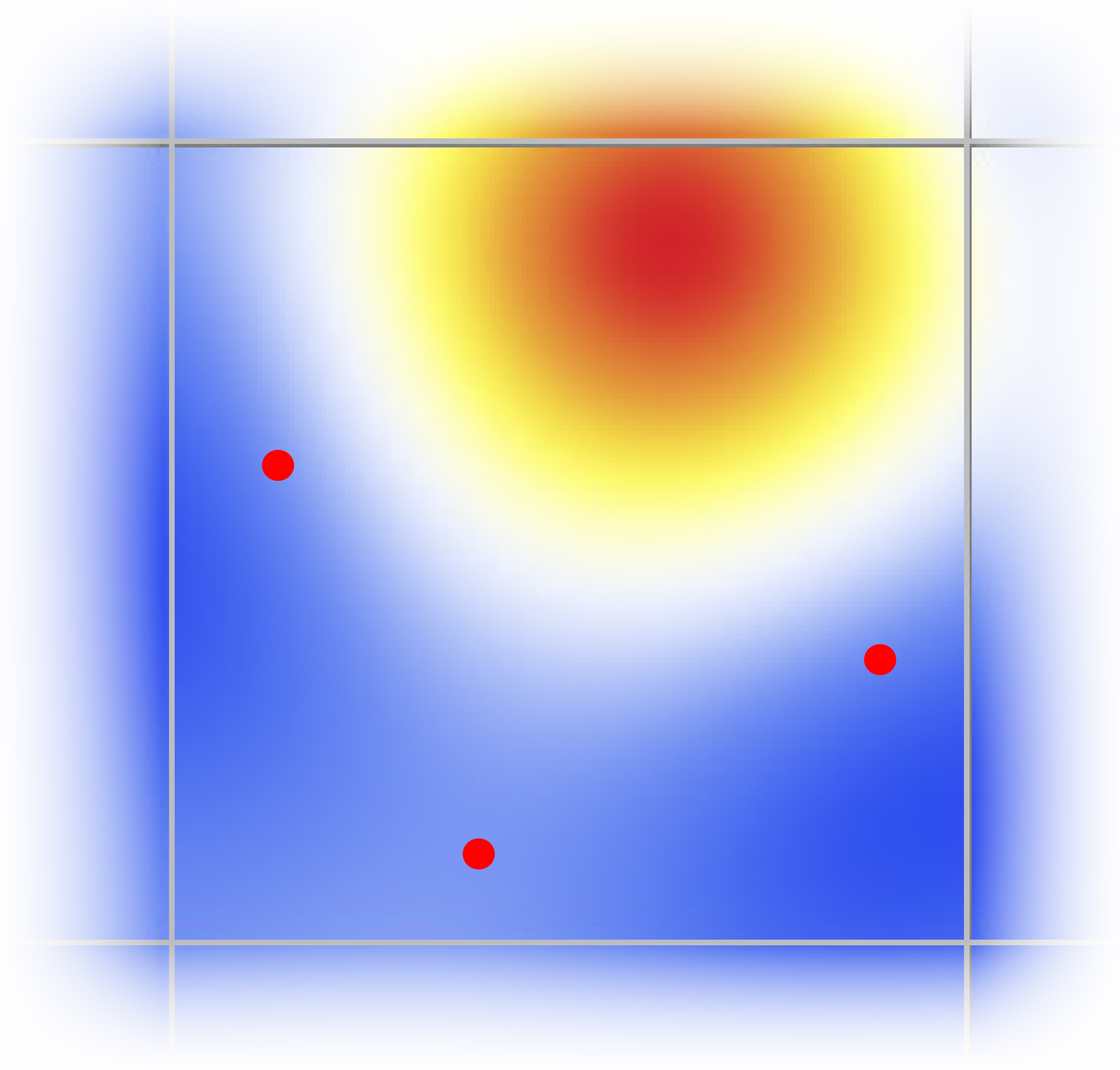

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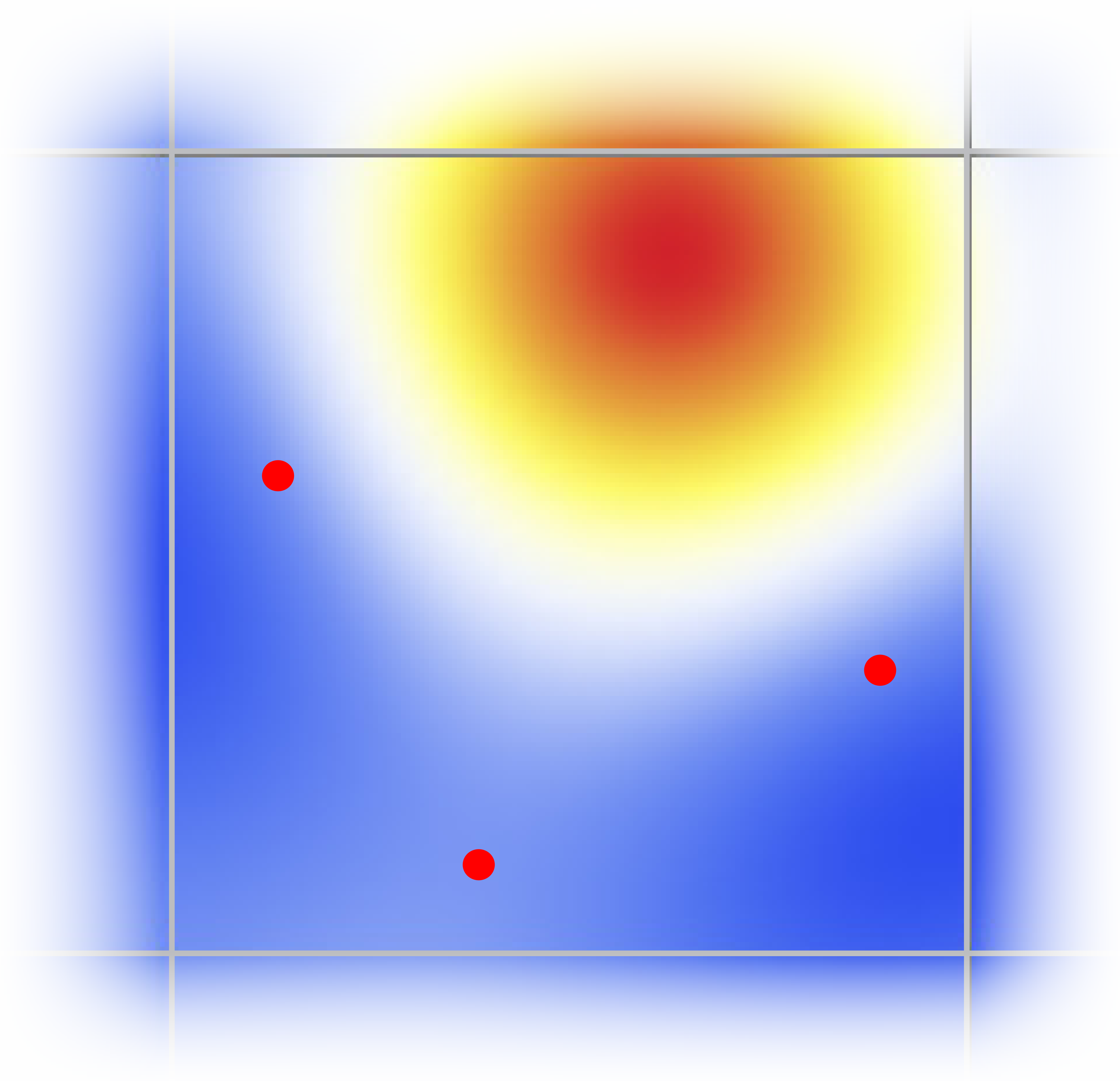
$$I_{ij} \approx \int_{P_{ij}} \tilde{I}(x)\, dx$$



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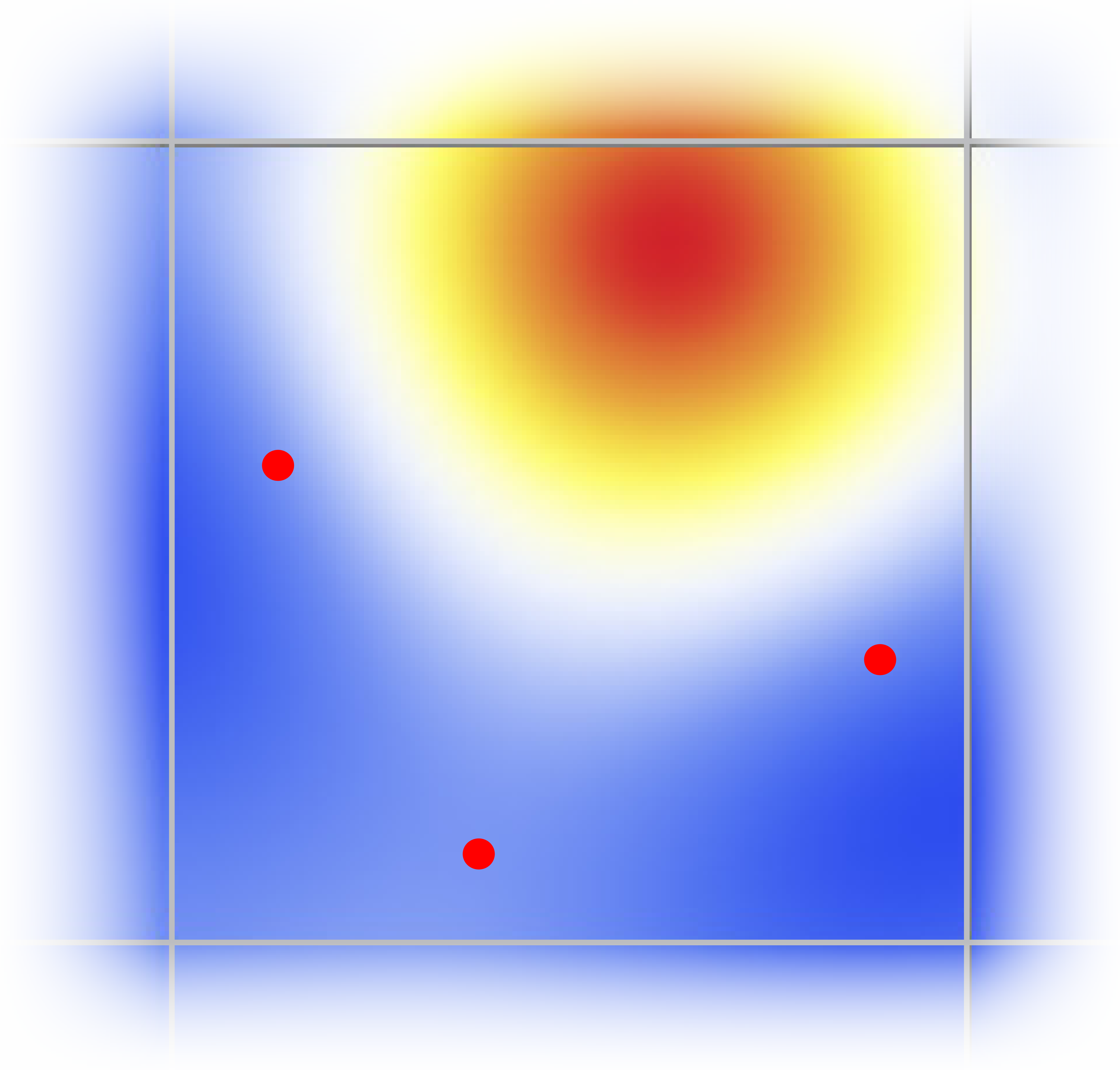

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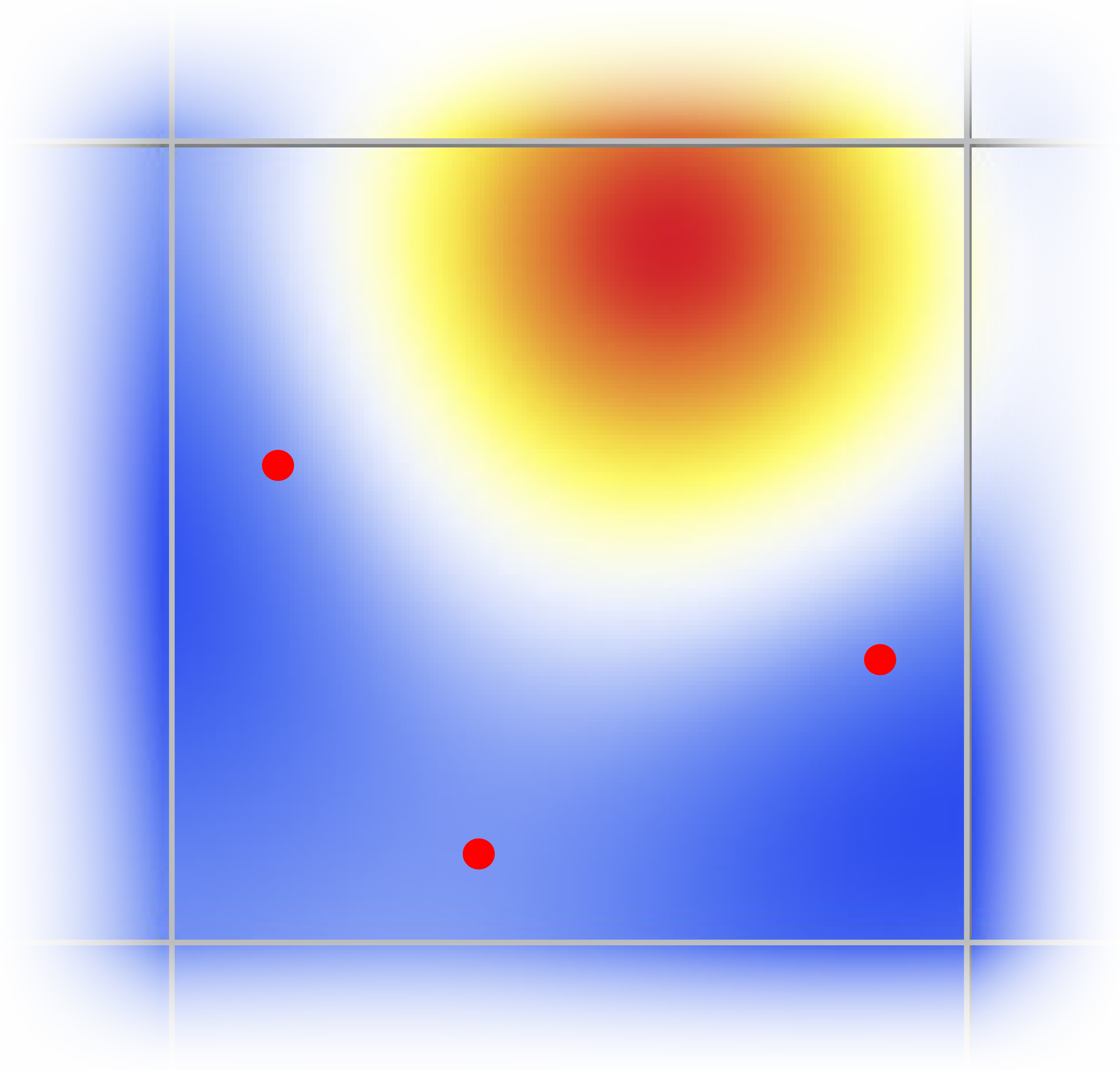

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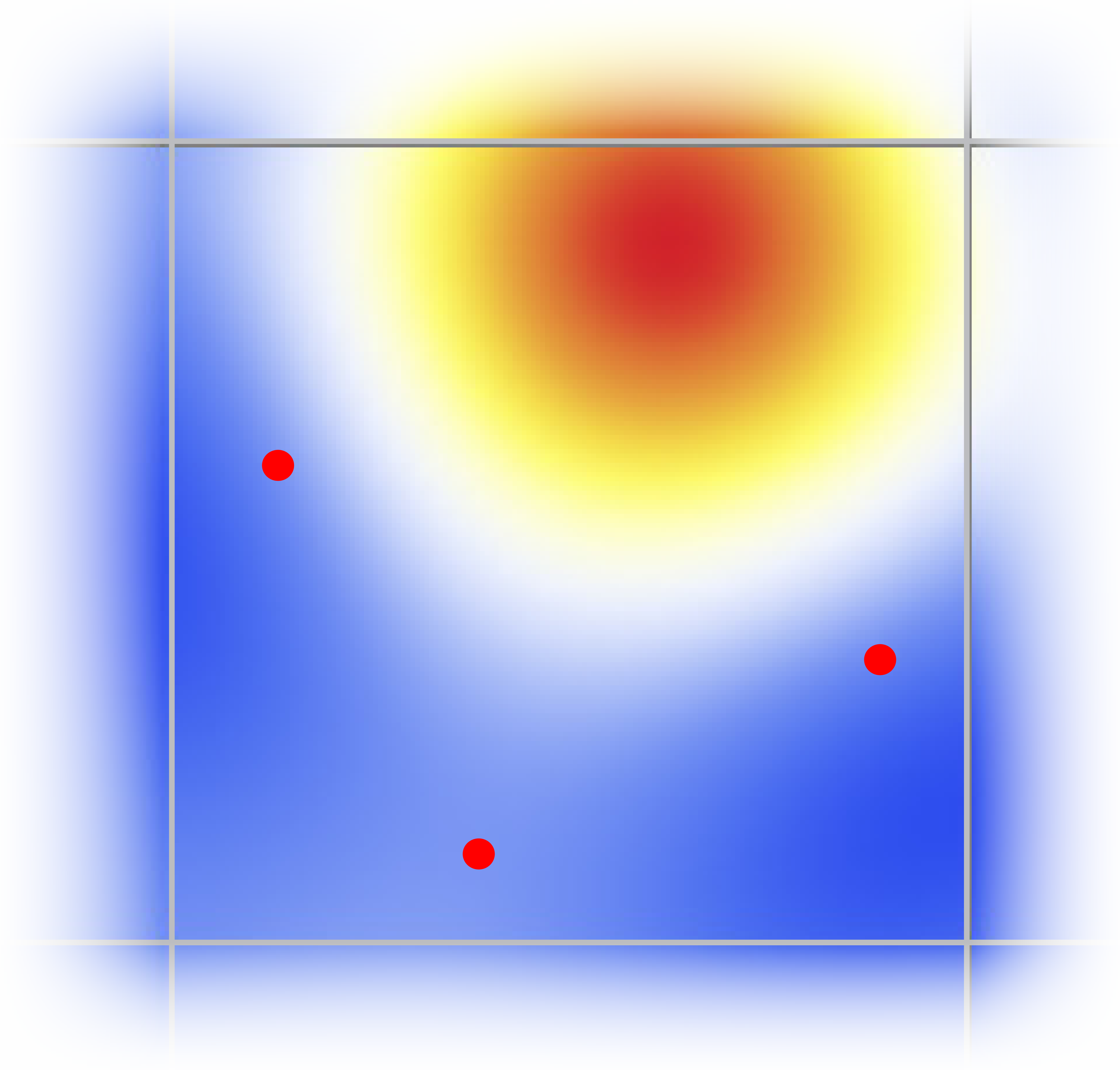

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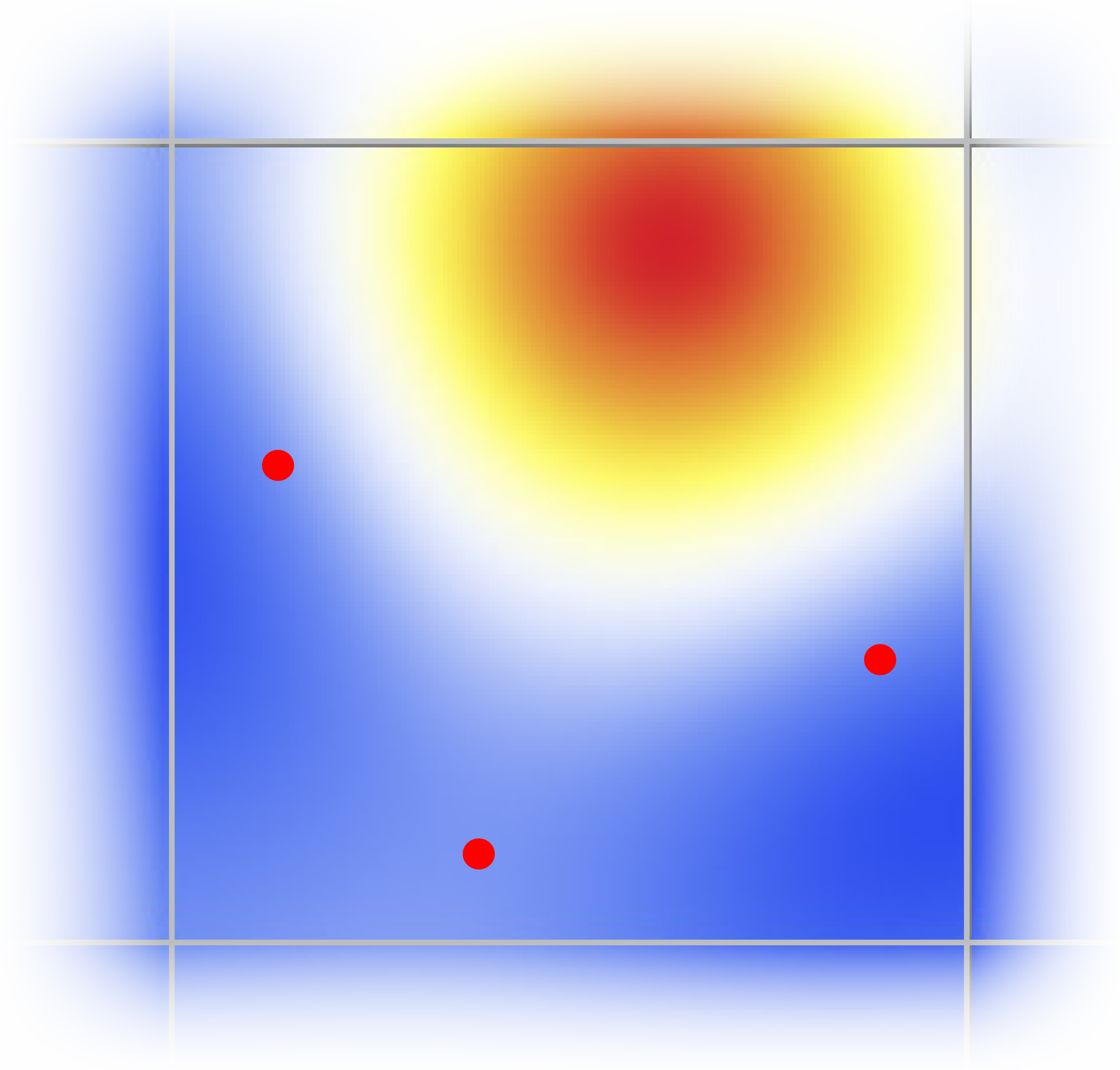

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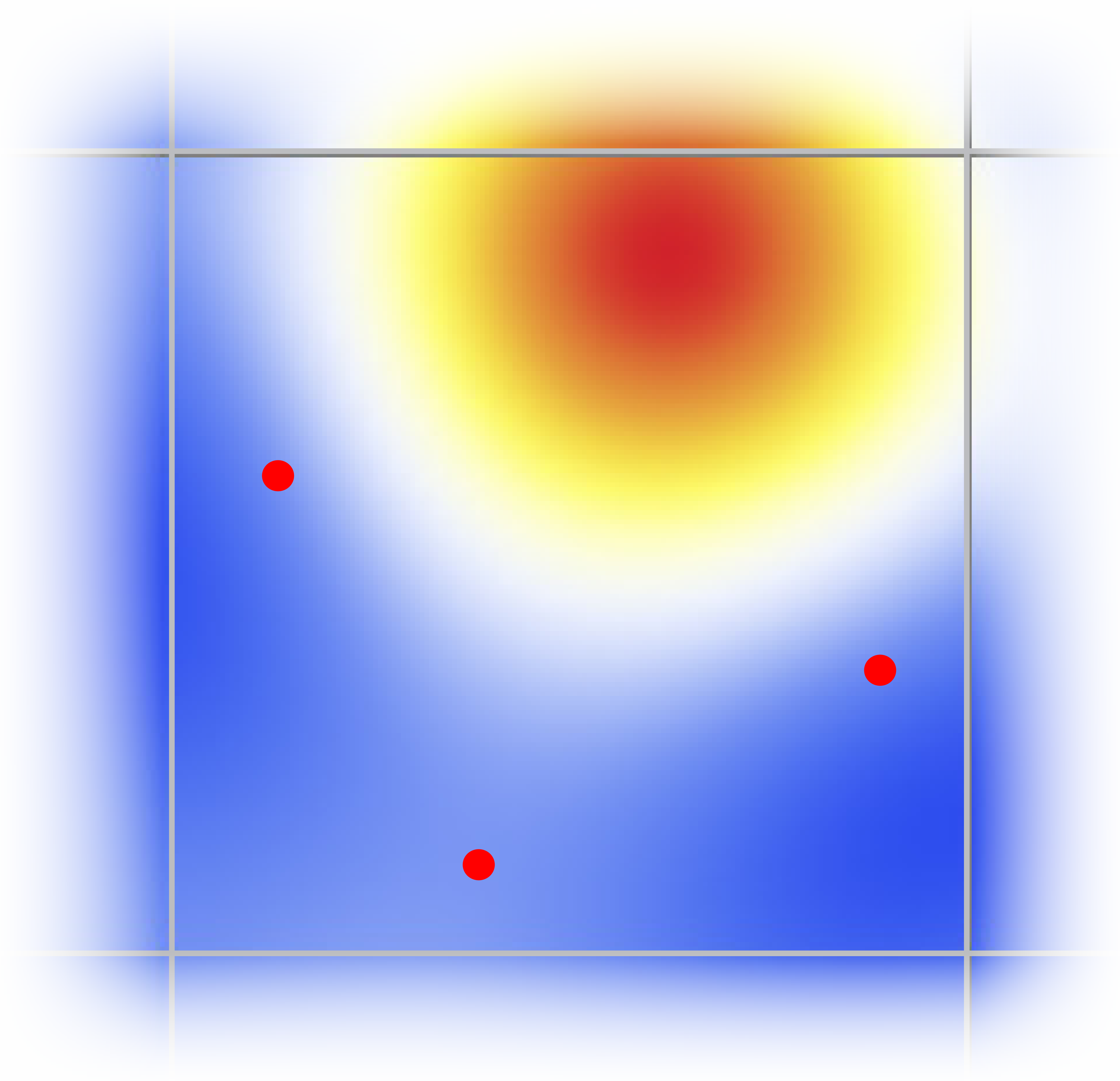

Quadrature weights

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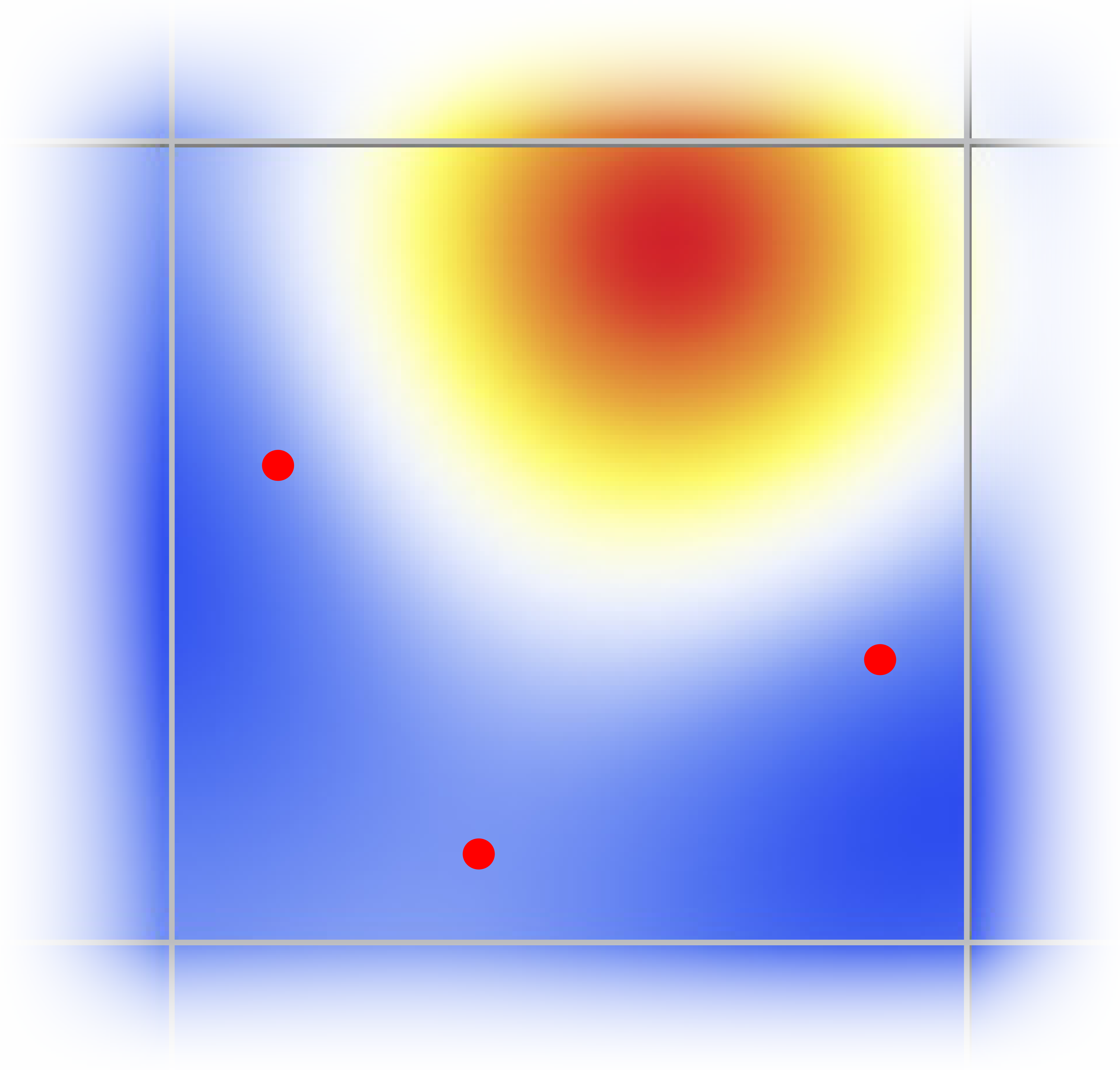

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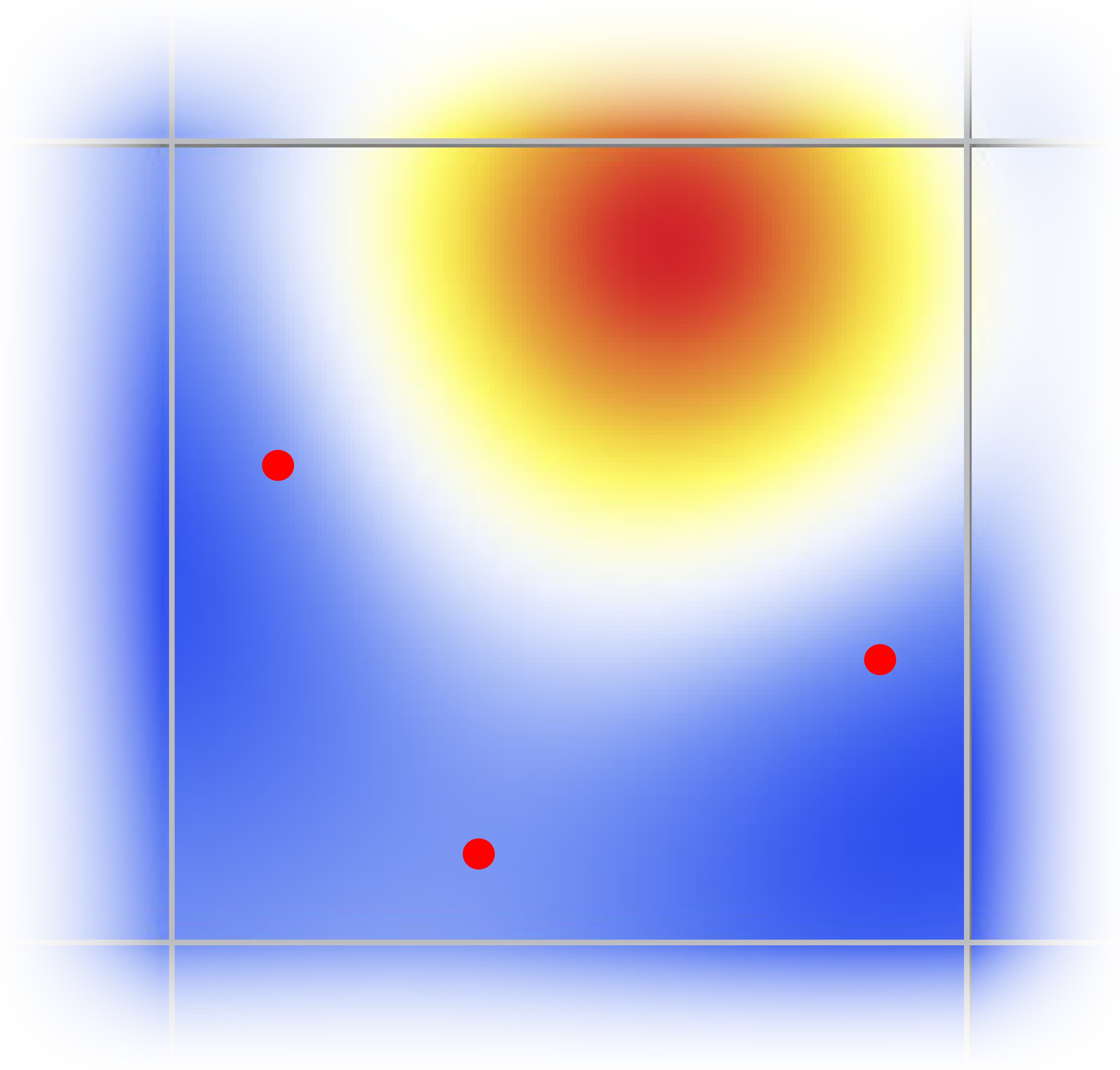

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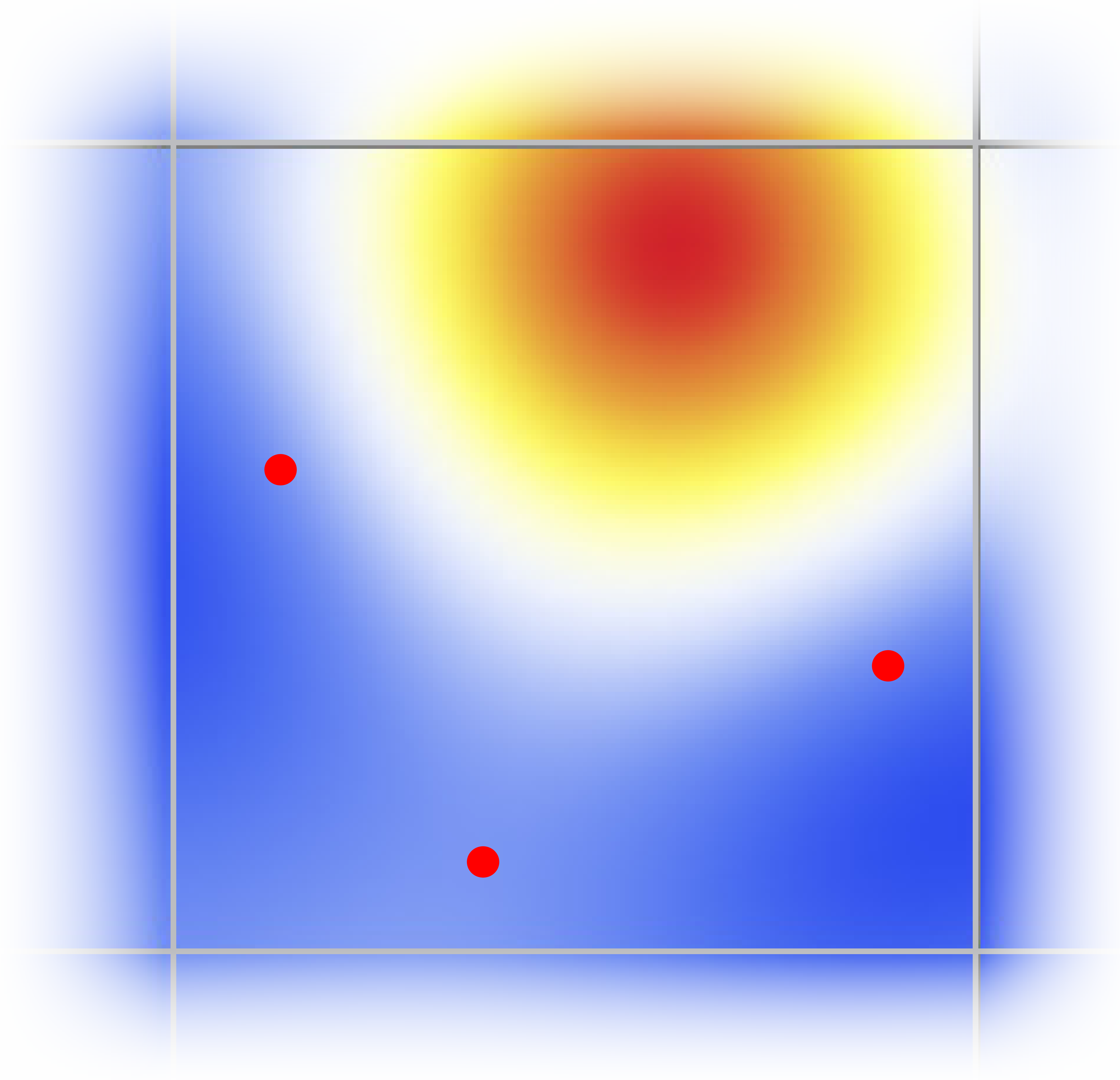

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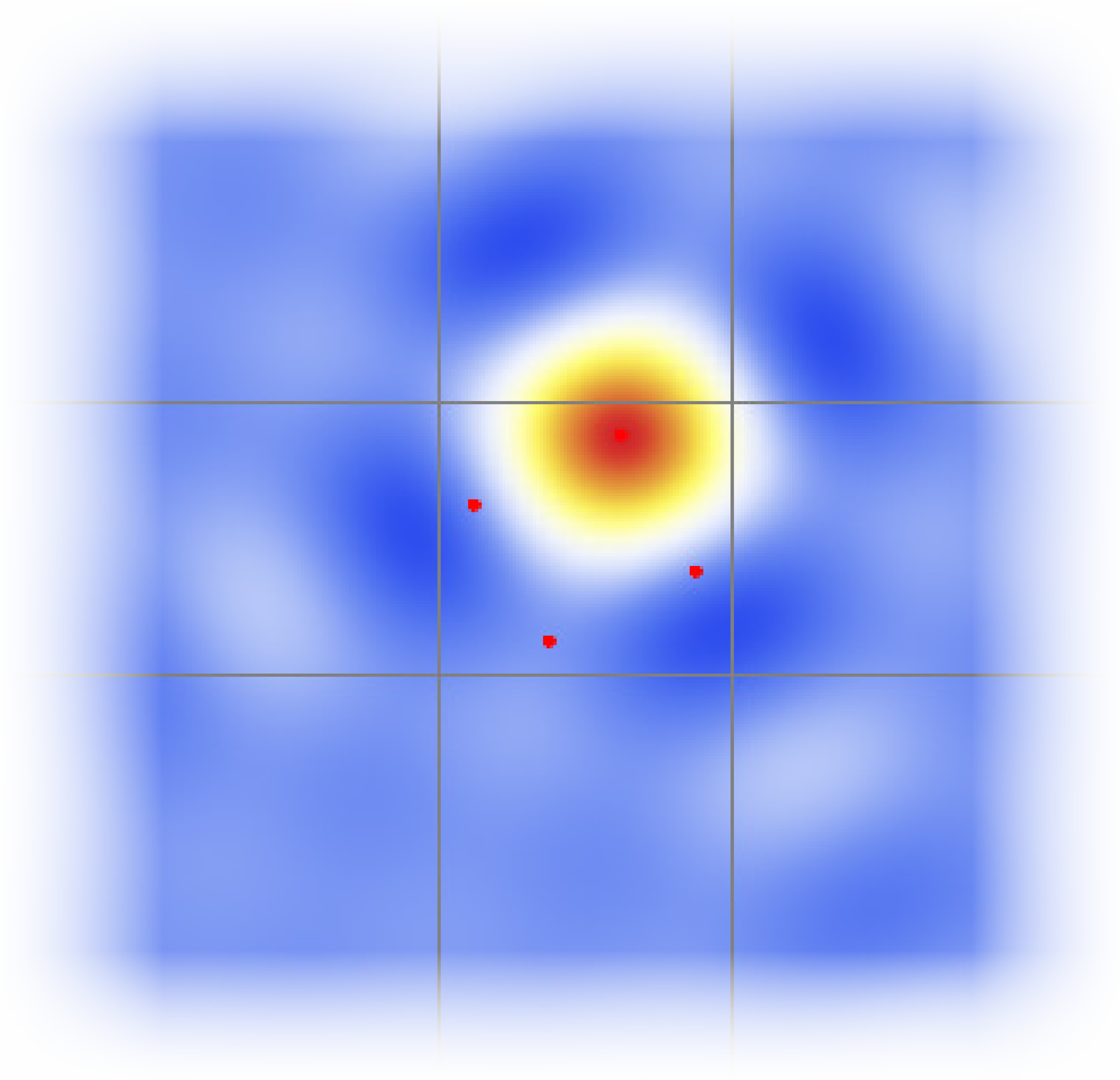

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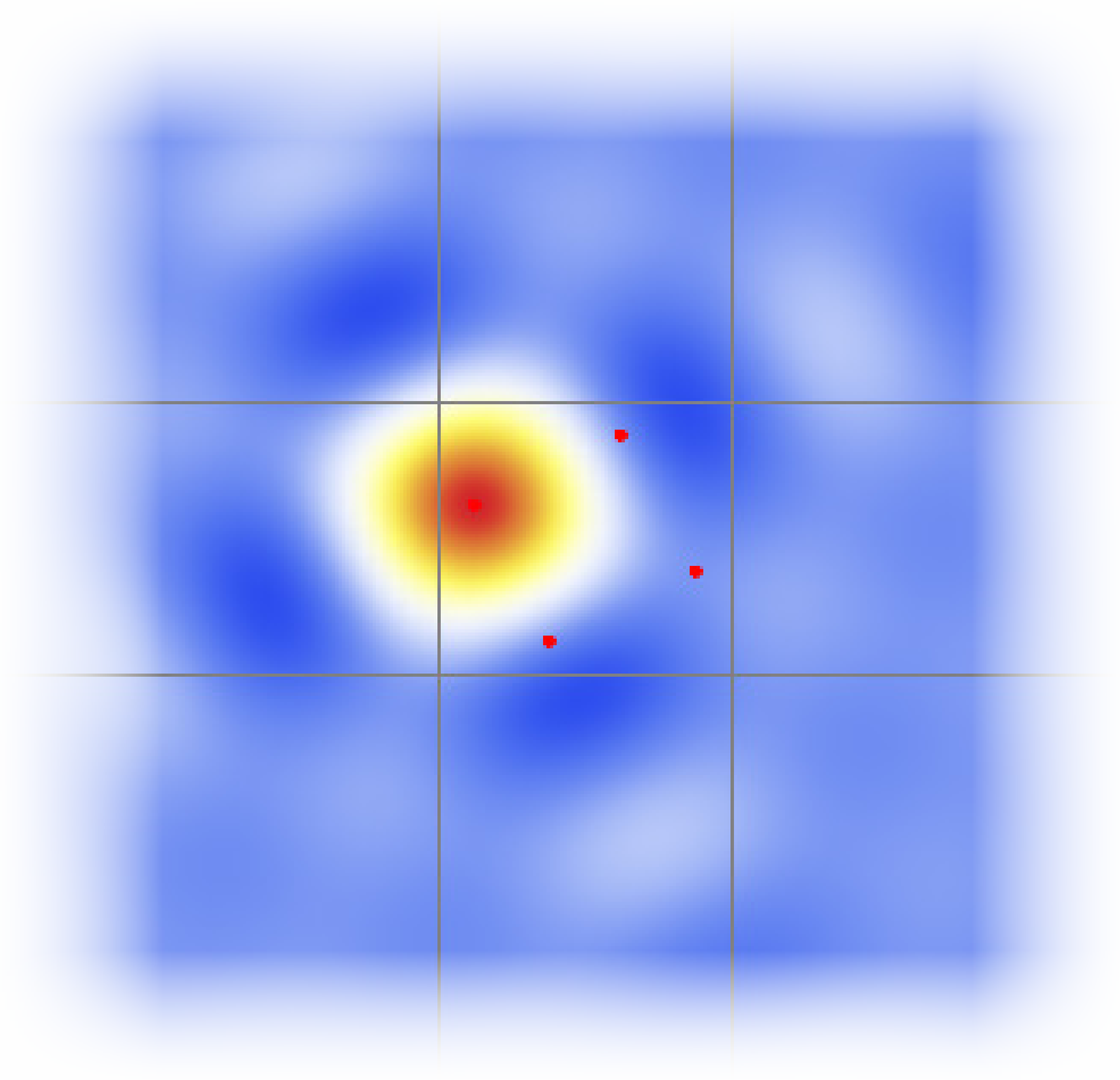

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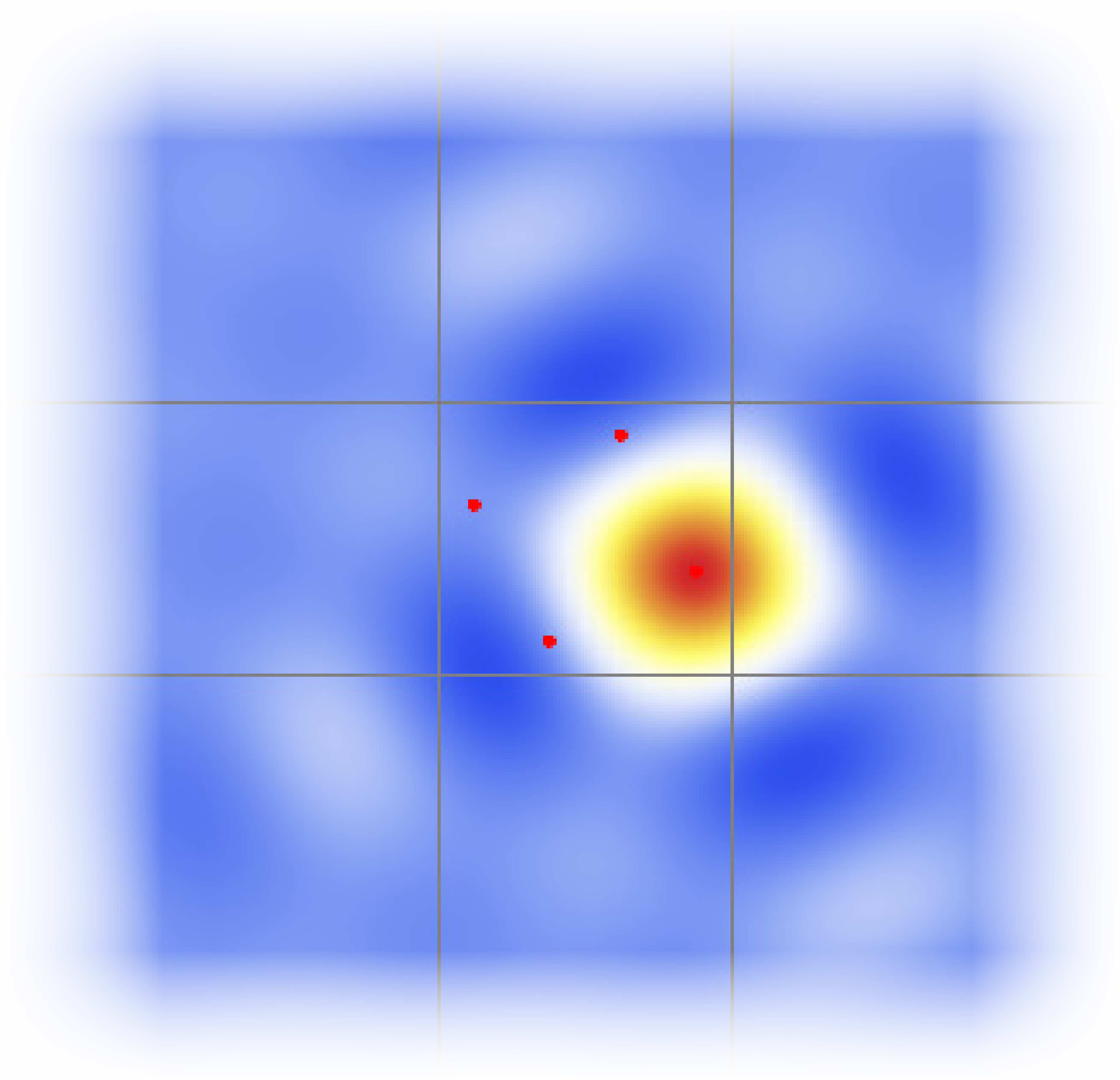

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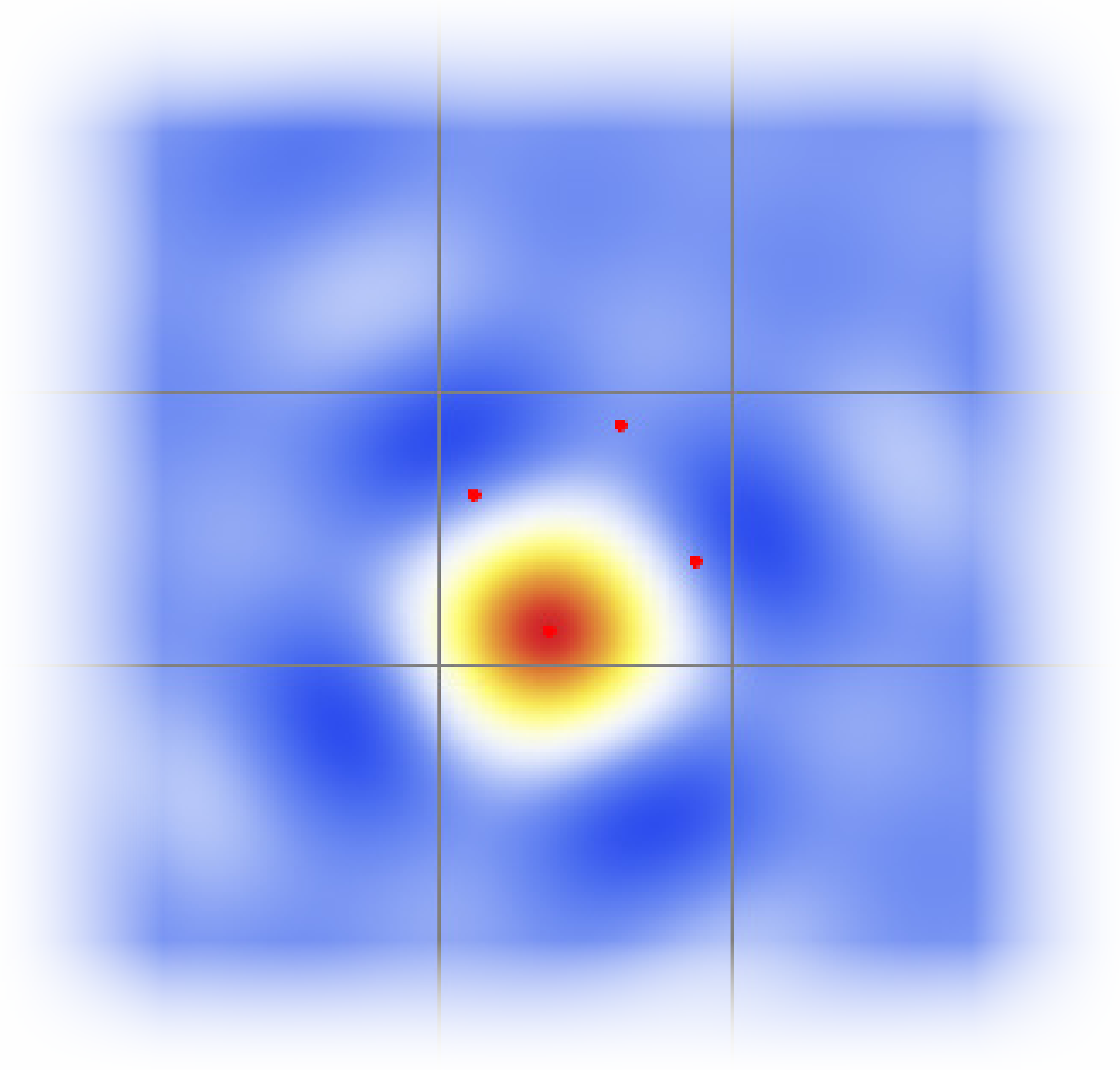
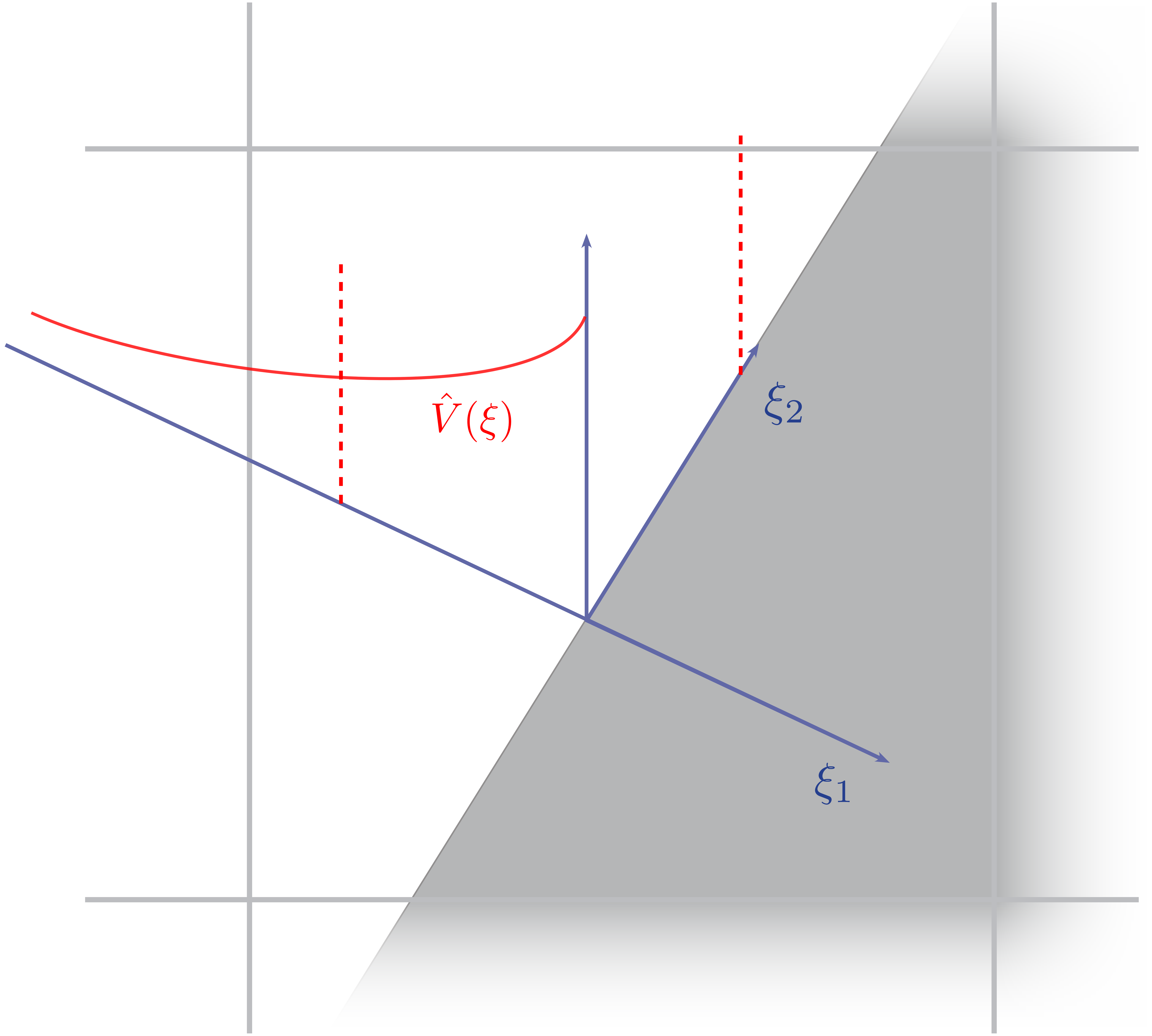
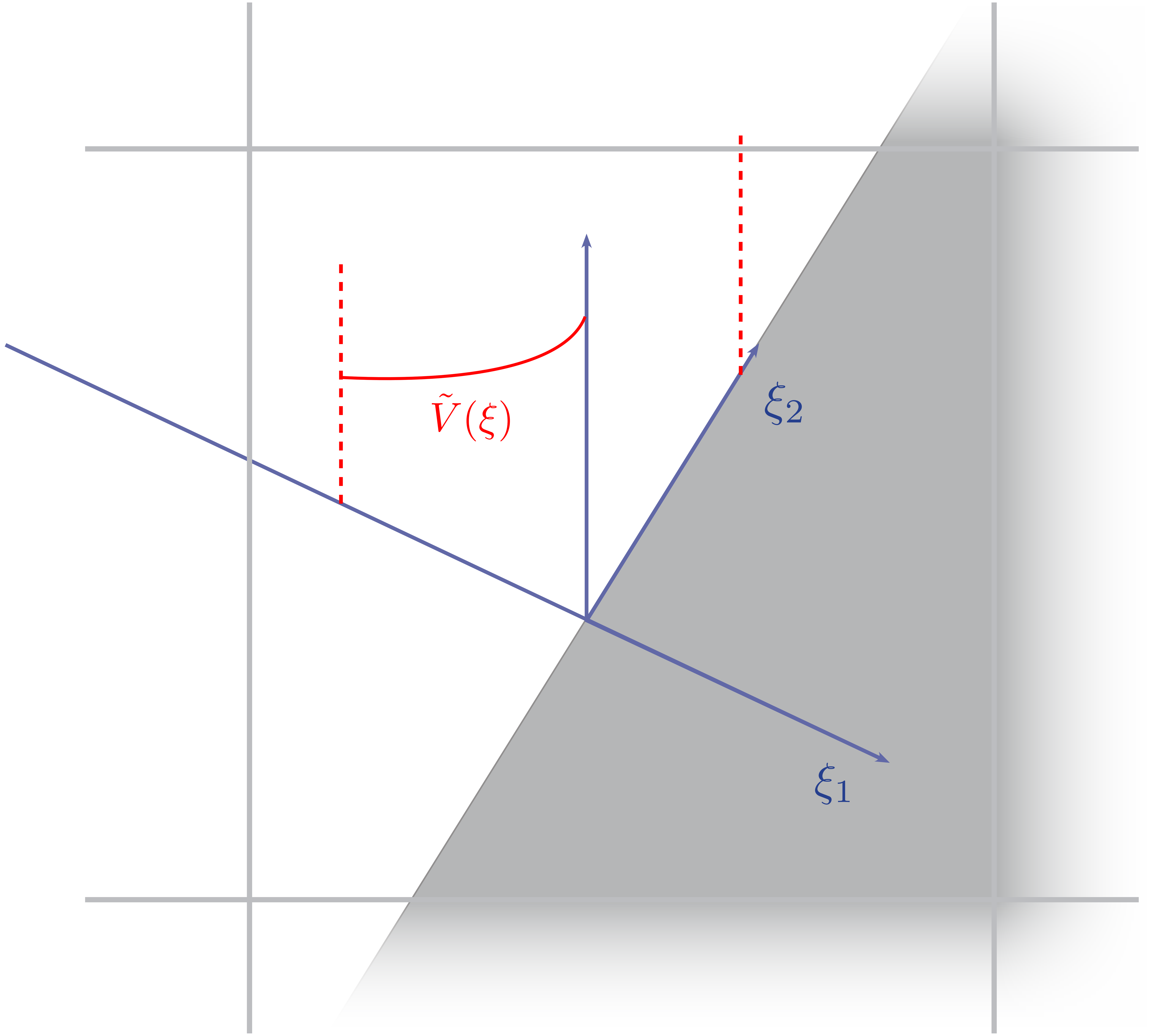



$\hat{V}(\xi)$
ξ_2
ξ_1



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ξ_2
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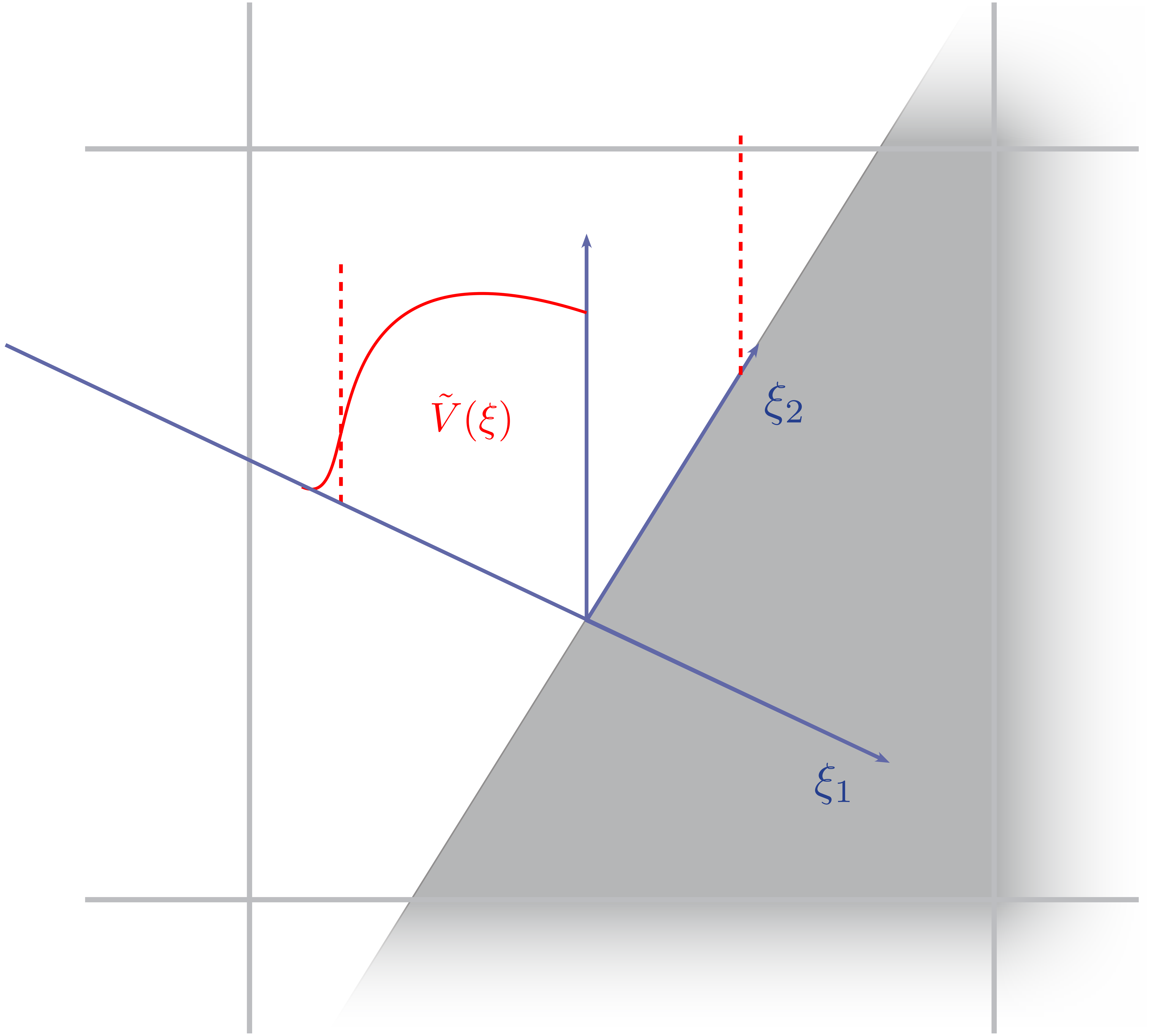

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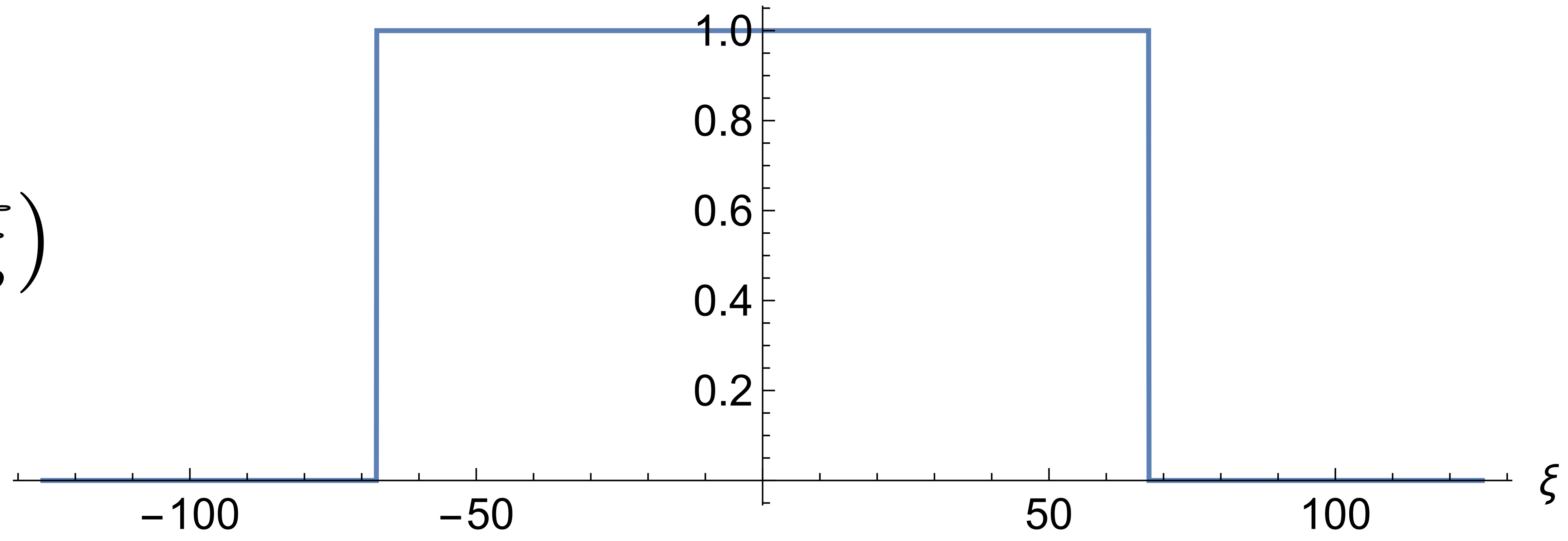

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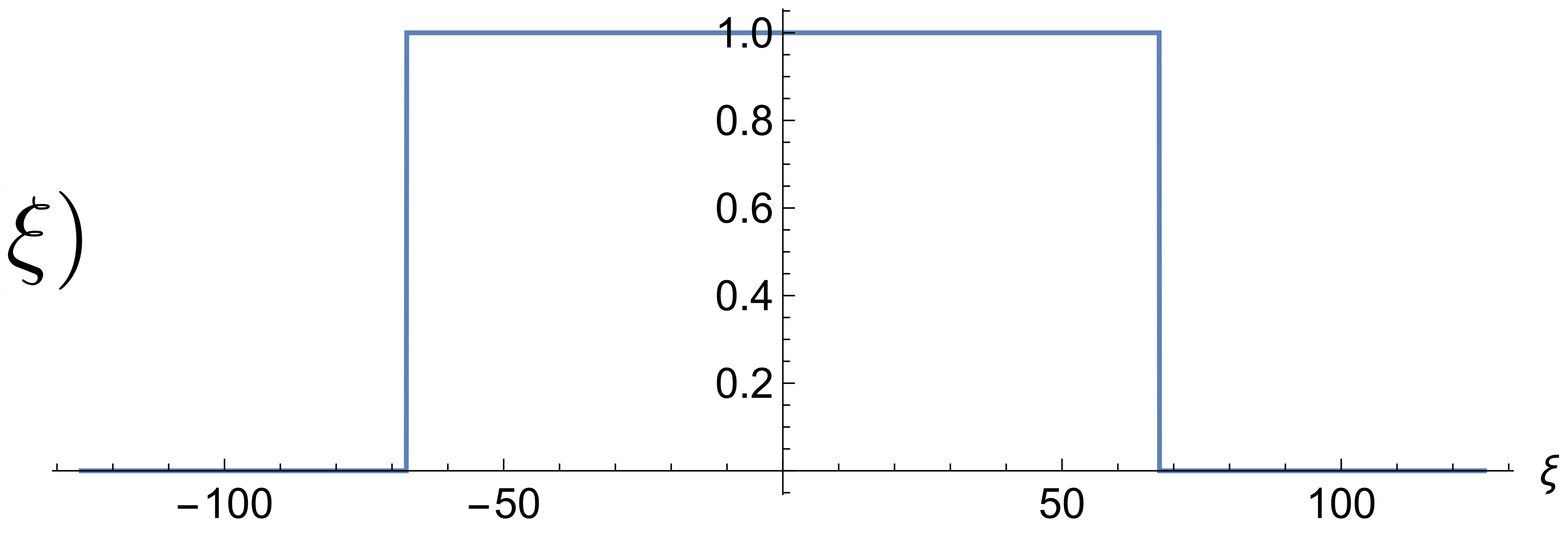

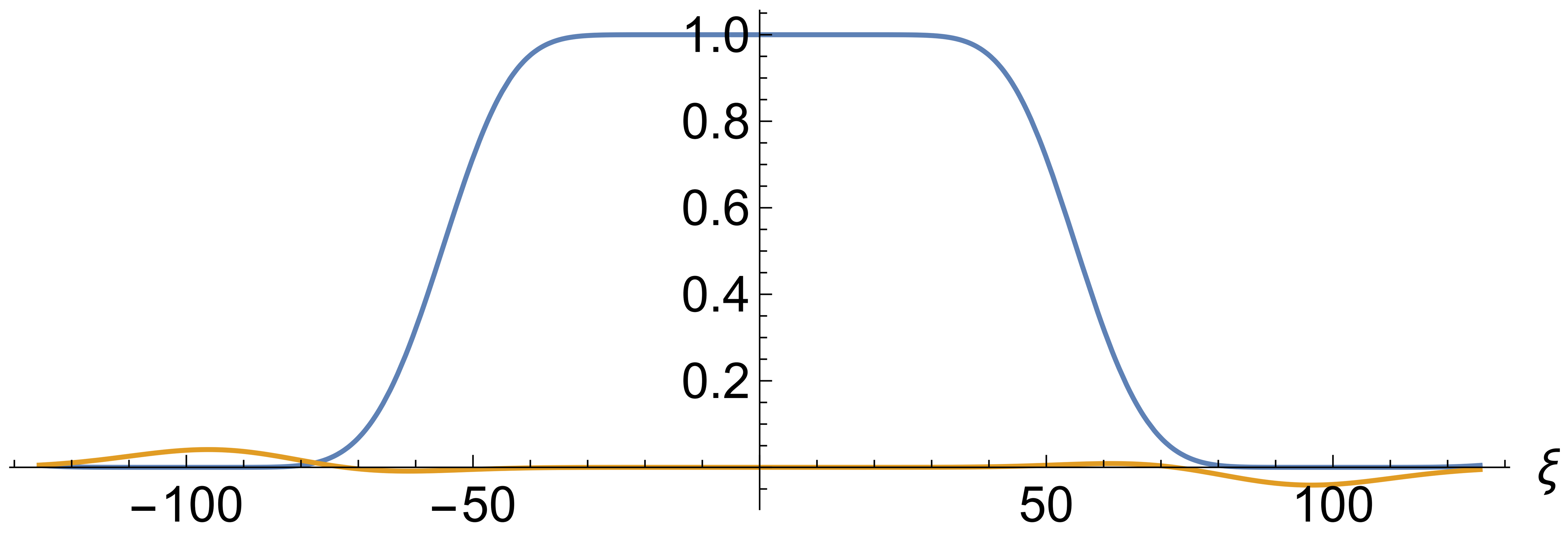

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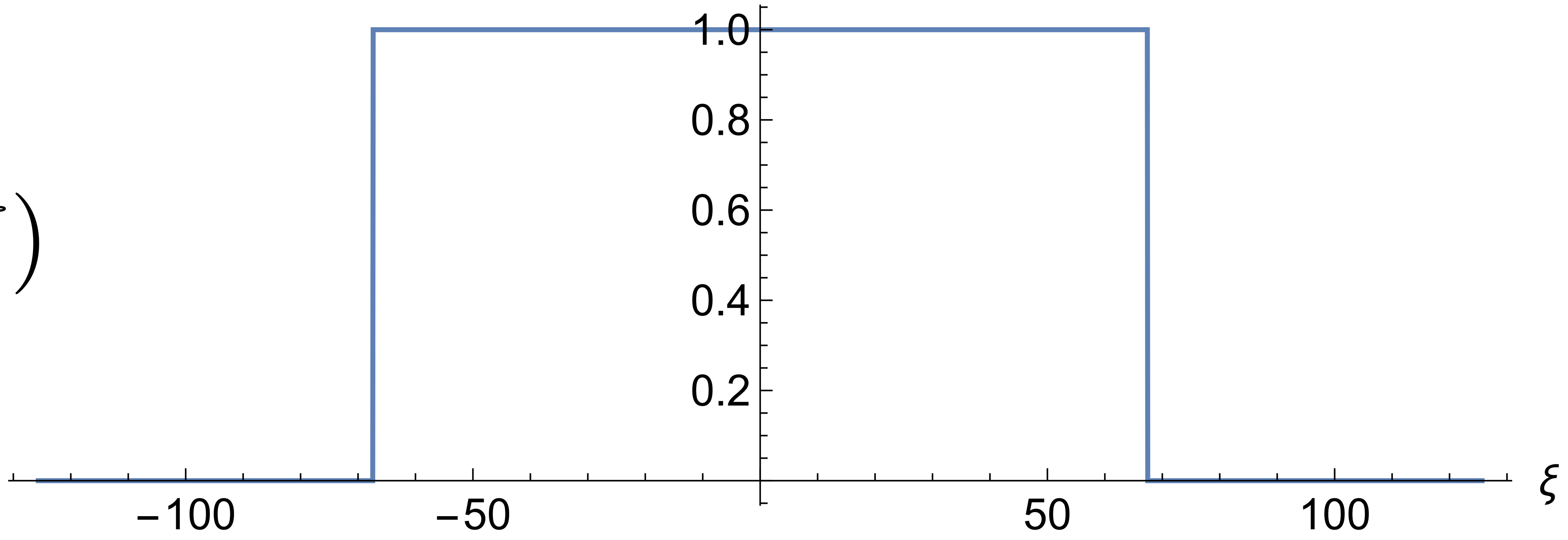
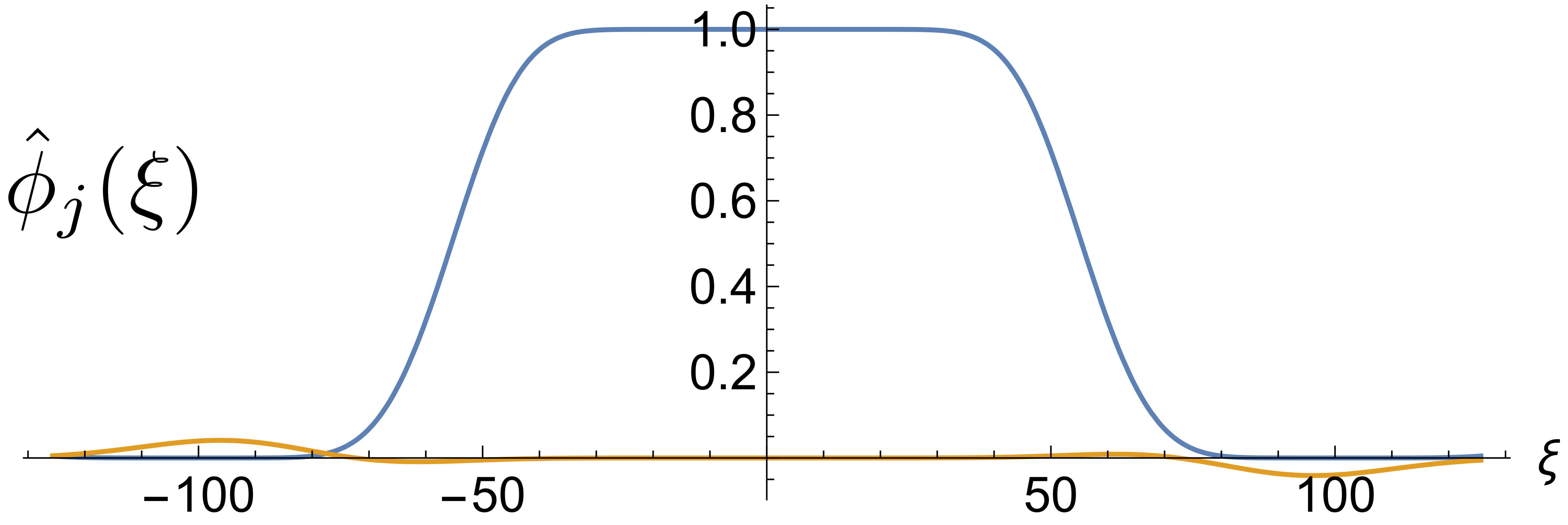
$$w_n = \int_{P_{ij}} \tilde{k}_n(x) \, dx$$

$\widehat{\mathrm{sinc}_{\mathrm{B}}}(\xi)$



coiflet-$\hat{\phi}_j(\xi)$

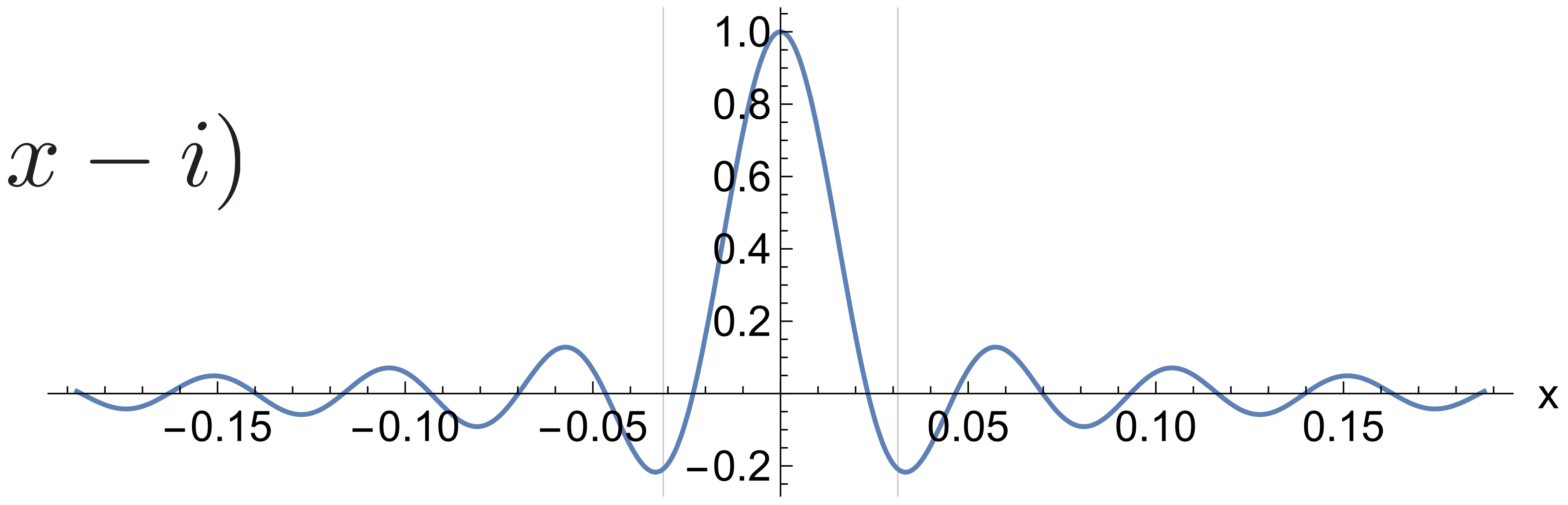


Quadrature weights

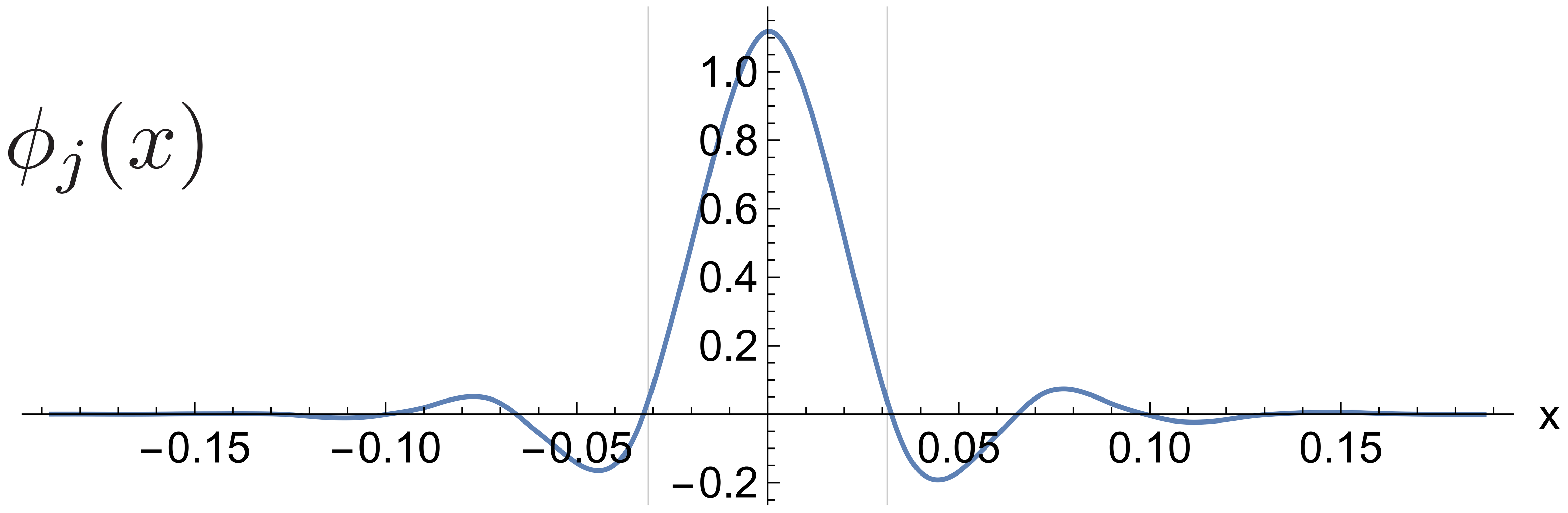
$$I_{ij} = \sum_{n=1}^{4} w_n \, \tilde{I}(x_n)$$

$$w_n = \int_{P_{ij}} \tilde{k}_n(x) \, dx$$

$\mathrm{sinc}_B(x - i)$



coiflet-$\phi_j(x)$

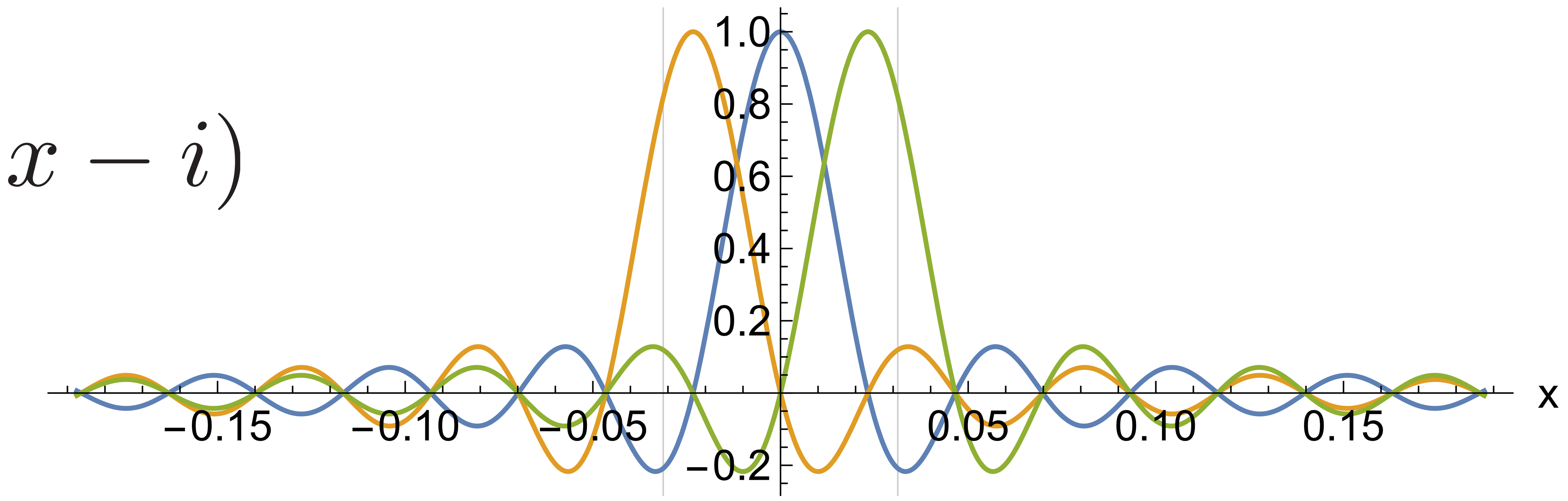


Quadrature weights

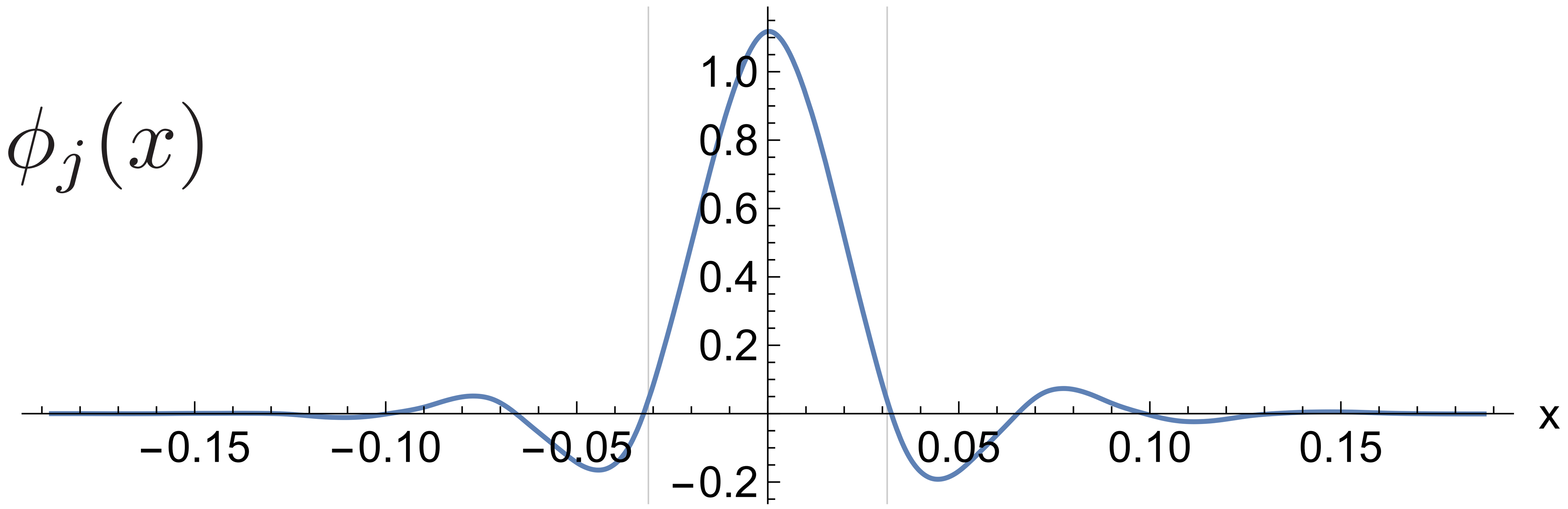
$$I_{ij} = \sum_{n=1}^{4} w_n \, \tilde{I}(x_n)$$

$$w_n = \int_{P_{ij}} \tilde{k}_n(x) \, dx$$

$\mathrm{sinc}_B(x - i)$



coiflet-$\phi_j(x)$

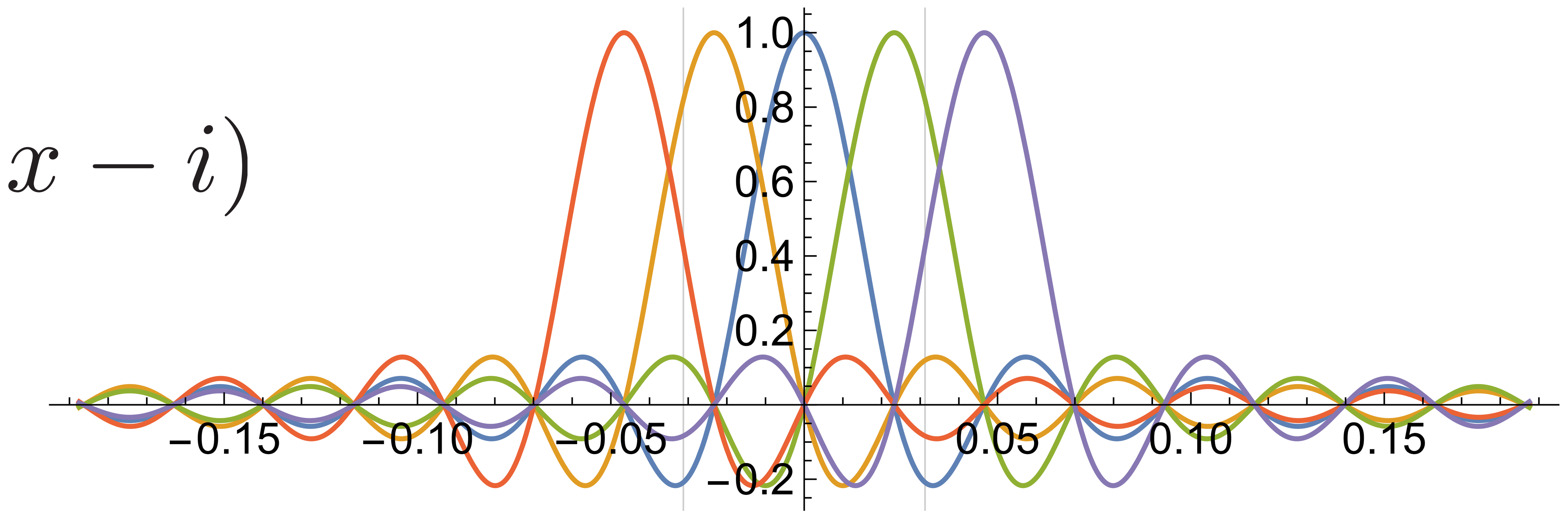


Quadrature weights

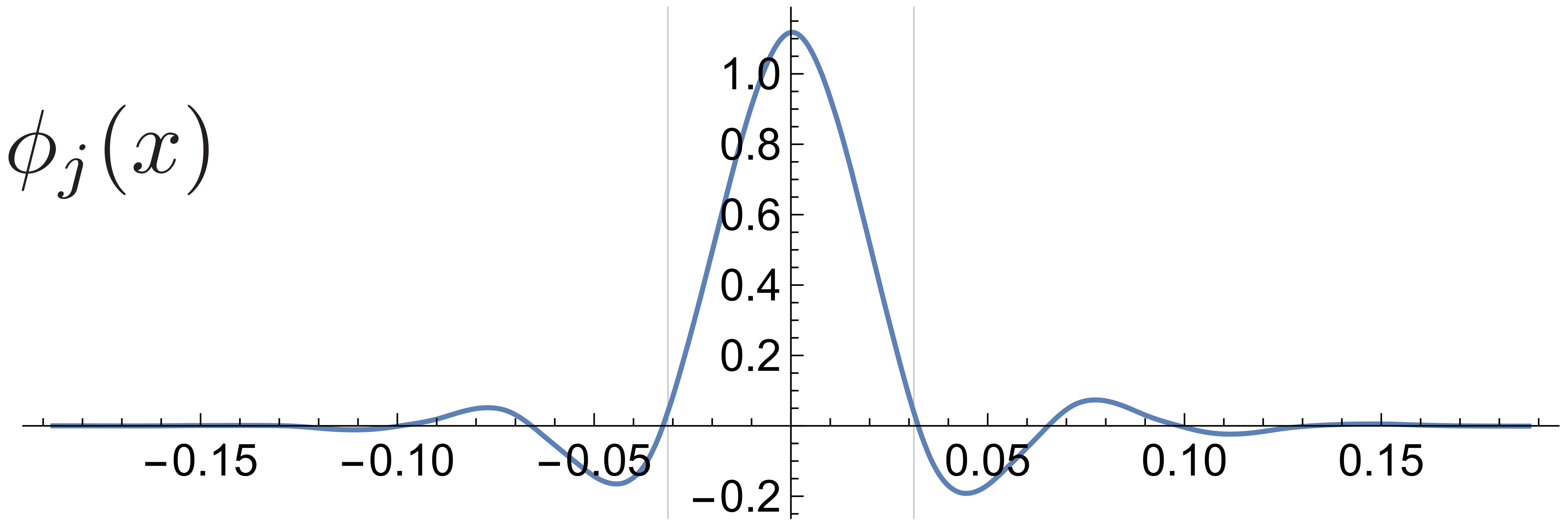
$$I_{ij} = \sum_{n=1}^{4} w_n \, \tilde{I}(x_n)$$

$$w_n = \int_{P_{ij}} \tilde{k}_n(x) \, dx$$

$\mathrm{sinc}_B(x - i)$



coiflet-$\phi_j(x)$

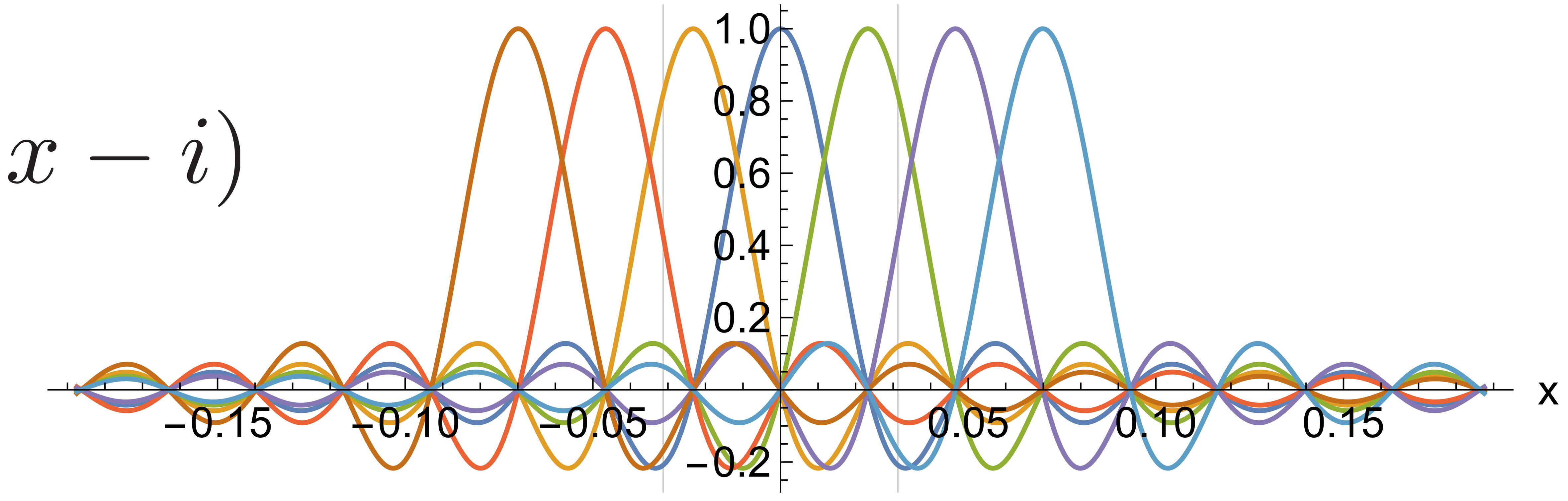


Quadrature weights

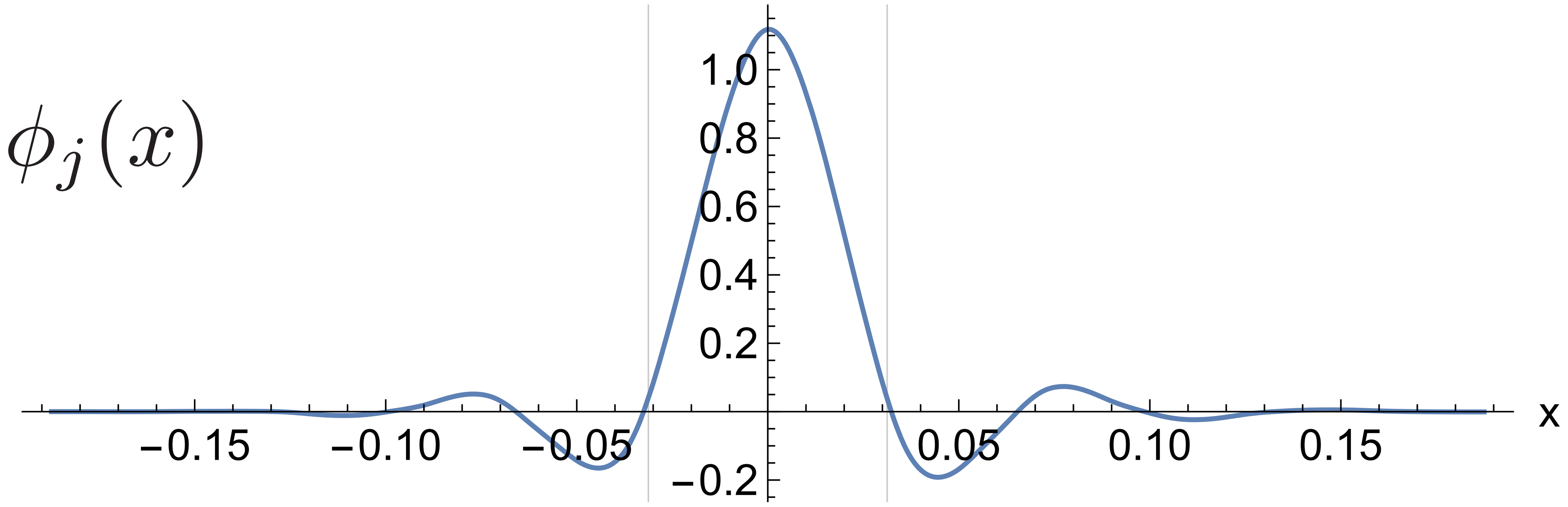
$$I_{ij} = \sum_{n=1}^{4} w_n \, \tilde{I}(x_n)$$

$$w_n = \int_{P_{ij}} \tilde{k}_n(x) \, dx$$

$\mathrm{sinc}_B(x - i)$



coiflet-$\phi_j(x)$



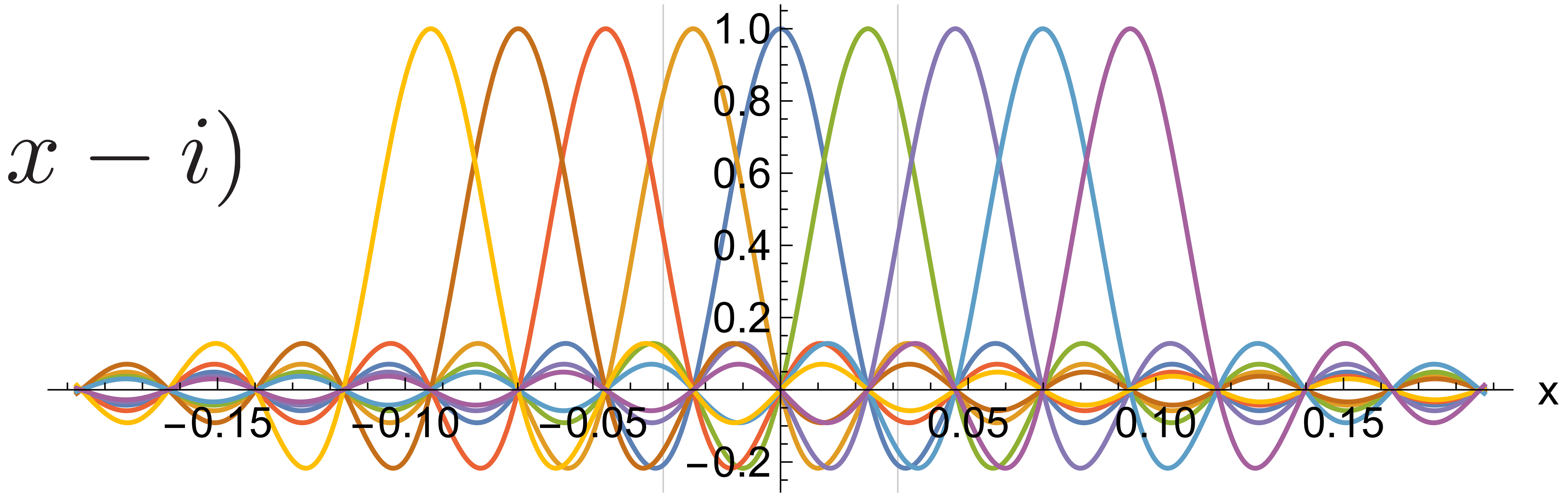
Quadrature weights

$$\tilde{I}(x) = \sum_{n=1}^{4} \tilde{I}(x_n)\,\mathrm{sinc}_B(x-n)$$

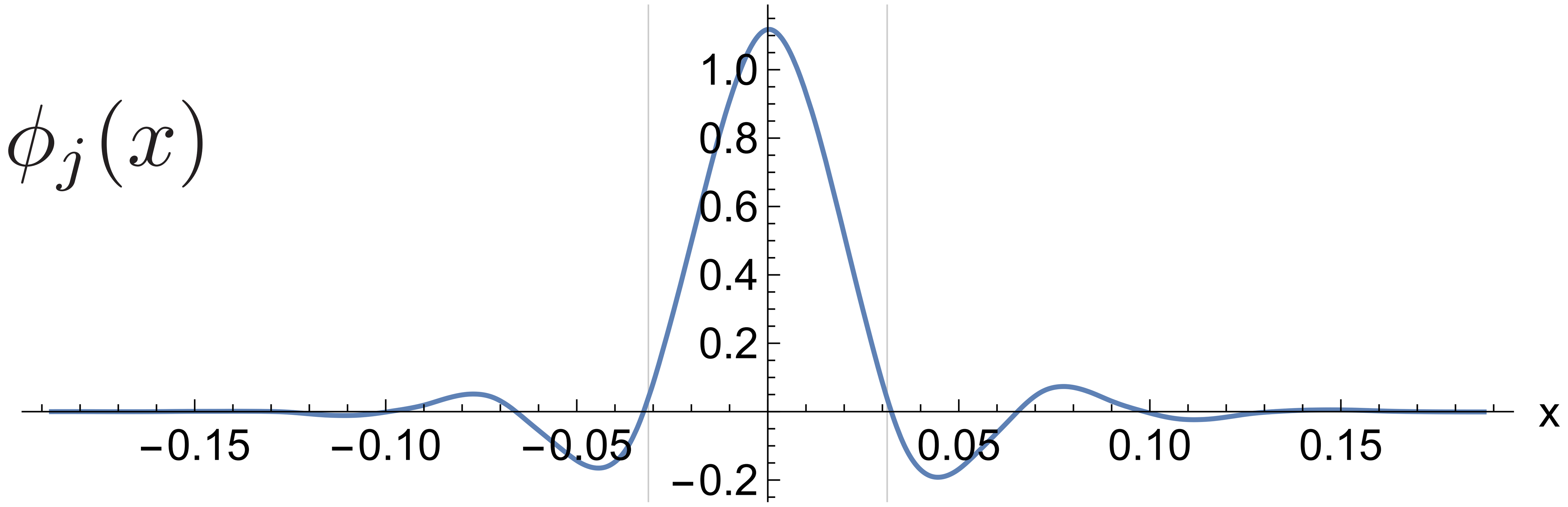
$$I_{ij} = \sum_{n=1}^{4} w_n\,\tilde{I}(x_n)$$

$$w_n = \int_{P_{ij}} \tilde{k}_n(x)\,dx$$

$\mathrm{sinc}_B(x-i)$



coiflet-$\phi_j(x)$



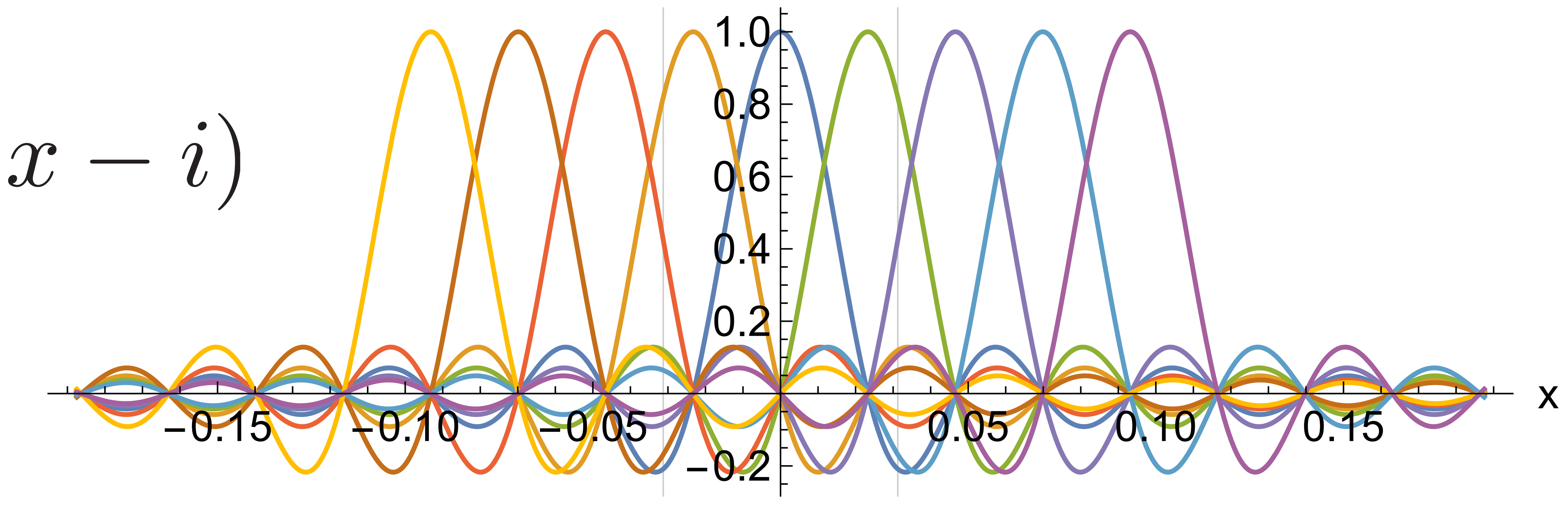
Quadrature weights

$$\tilde{I}(x) = \sum_{n=1}^{4} \tilde{I}(x_n)\, \mathrm{sinc}_B(x - n)$$

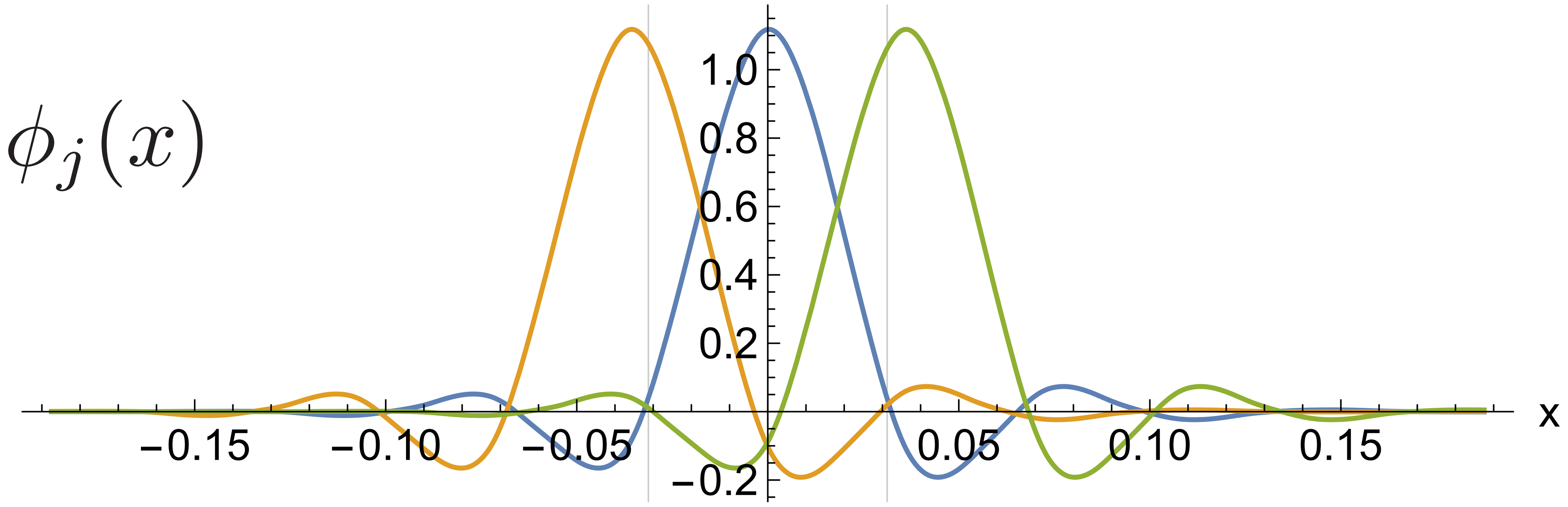
$$I_{ij} = \sum_{n=1}^{4} w_n\, \tilde{I}(x_n)$$

$$w_n = \int_{P_{ij}} \tilde{k}_n(x)\, dx$$

$\mathrm{sinc}_B(x - i)$



coiflet-$\phi_j(x)$



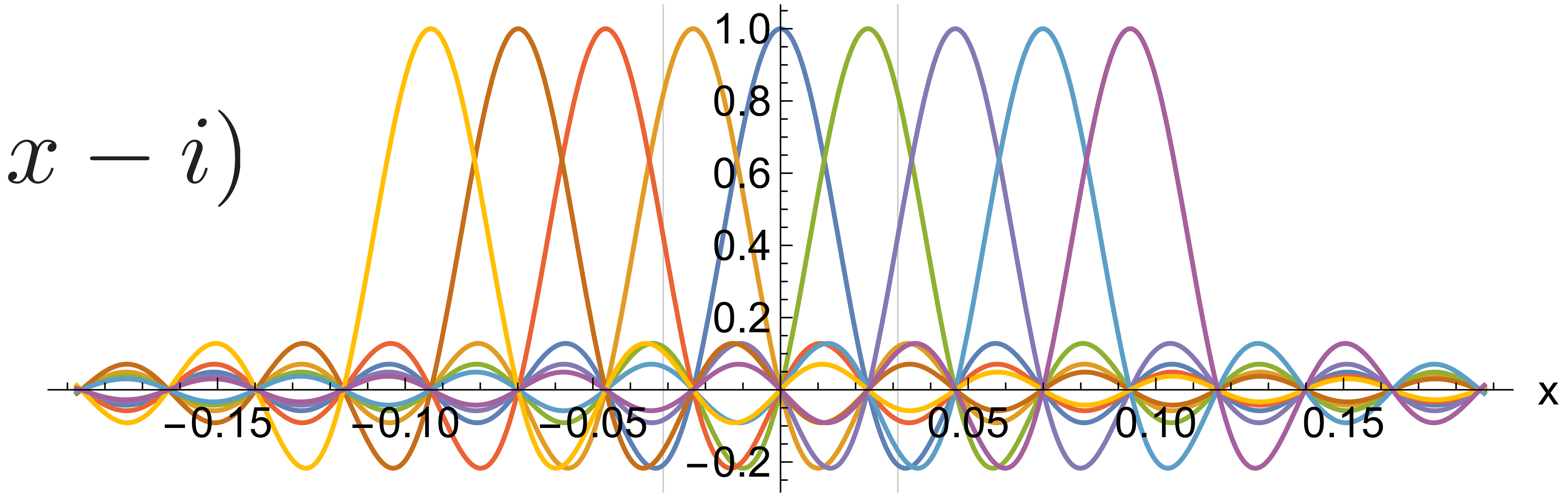
Quadrature weights

$$\tilde{I}(x) = \sum_{n=1}^{4} \tilde{I}(x_n)\,\mathrm{sinc}_B(x-n)$$

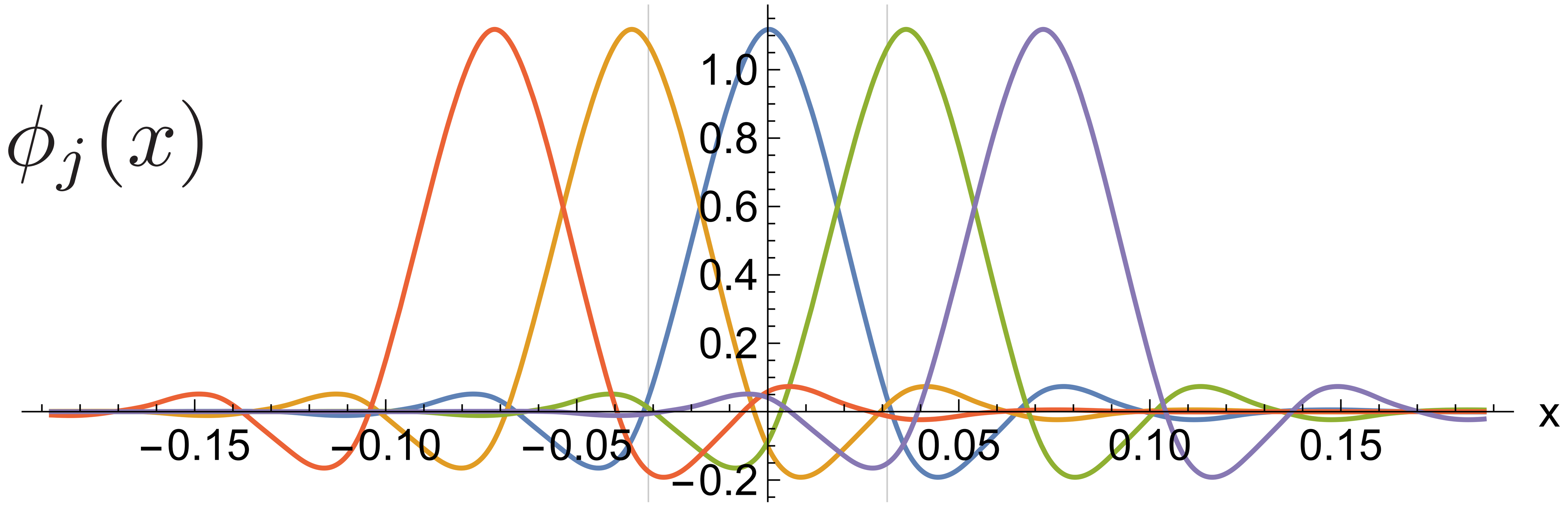
$$I_{ij} = \sum_{n=1}^{4} w_n\,\tilde{I}(x_n)$$

$$w_n = \int_{P_{ij}} \tilde{k}_n(x)\,dx$$

$\mathrm{sinc}_B(x-i)$



coiflet-$\phi_j(x)$



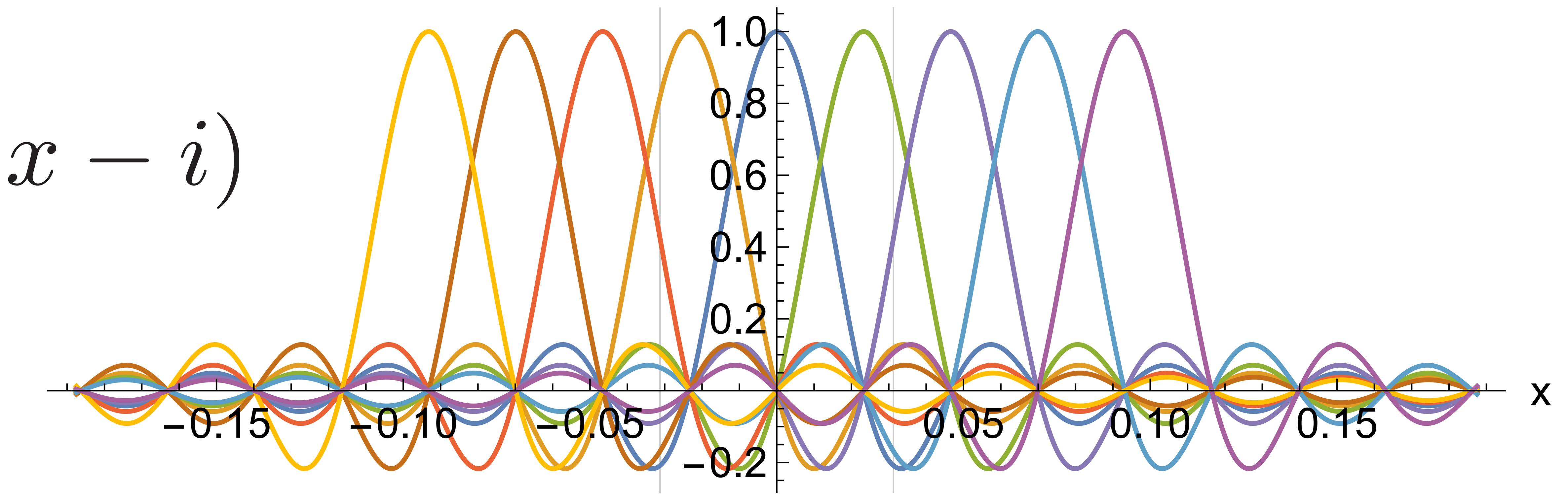
Quadrature weights

$$\tilde{I}(x) = \sum_{n=1}^{4} \tilde{I}(x_n)\,\mathrm{sinc}_B(x-n)$$

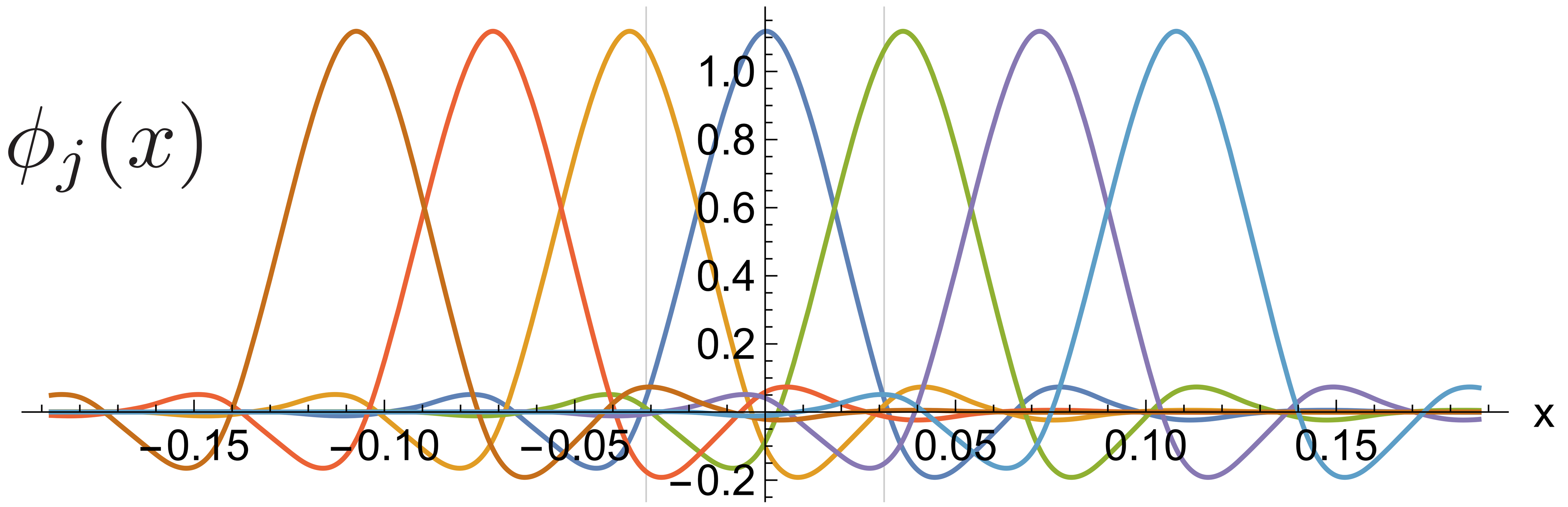
$$I_{ij} = \sum_{n=1}^{4} w_n\,\tilde{I}(x_n)$$

$$w_n = \int_{P_{ij}} \tilde{k}_n(x)\,dx$$

$\mathrm{sinc}_B(x-i)$



coiflet-$\phi_j(x)$



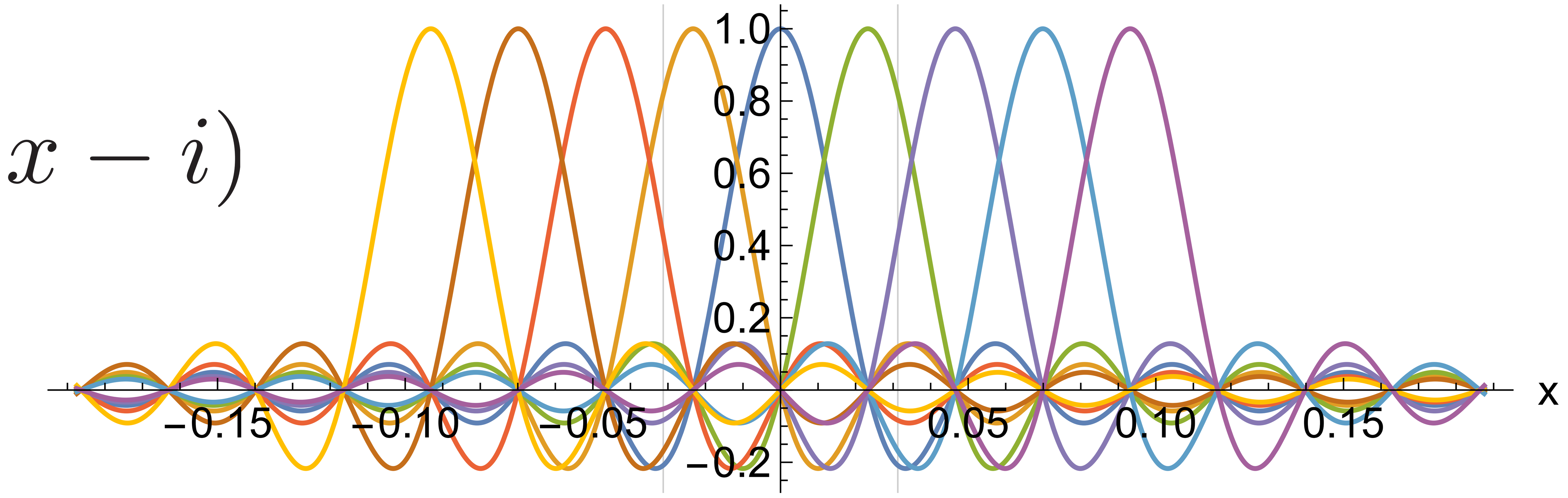
Quadrature weights

$$I_{ij} = \sum_{n=1}^{4} w_n \, \tilde{I}(x_n)$$

$$w_n = \int_{P_{ij}} \tilde{k}_n(x) \, dx$$

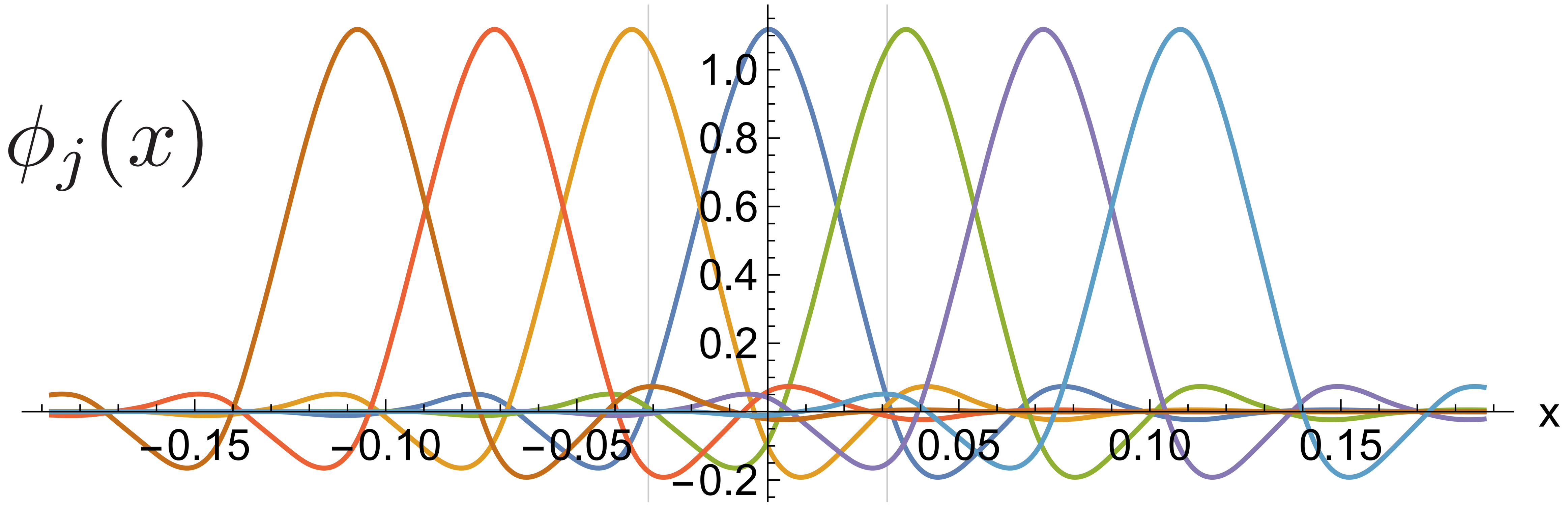
$$\tilde{I}(x) = \sum_{n=1}^{4} \tilde{I}(x_n) \, \mathrm{sinc}_B(x-n)$$

$\mathrm{sinc}_B(x-i)$



$$\tilde{I}(x) = \sum_{n=1}^{4} \tilde{I}_n \, \phi_j(x-n)$$

coiflet-$\phi_j(x)$

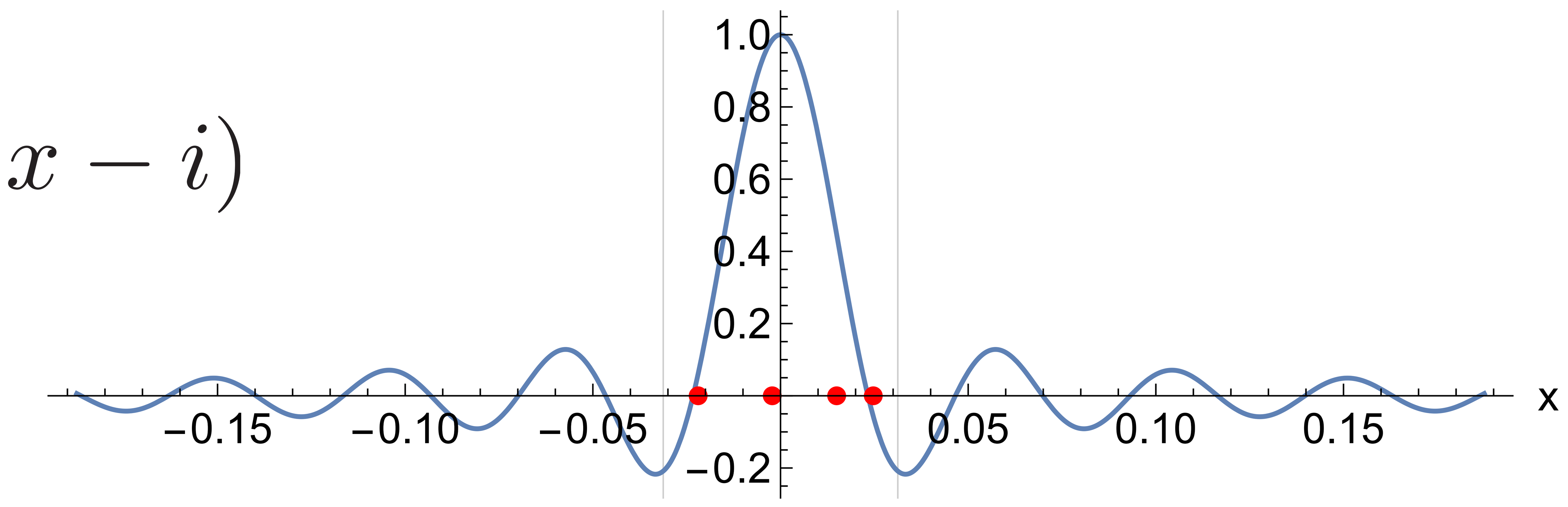


Quadrature weights

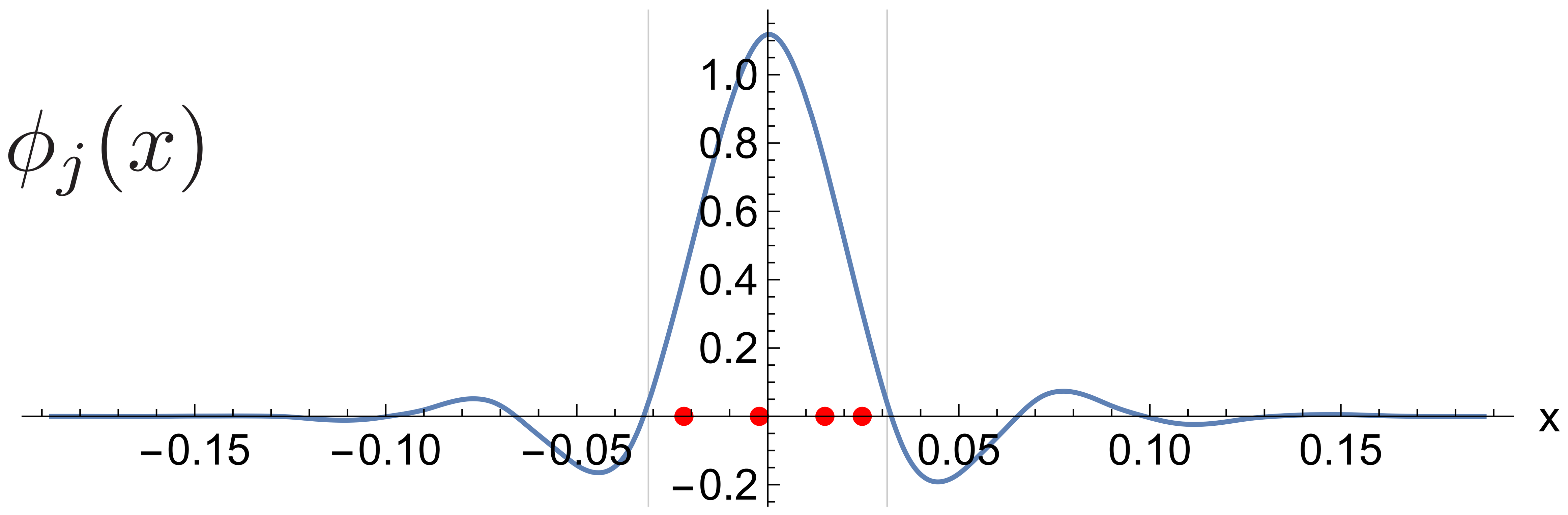
$$I_{ij} = \sum_{n=1}^{4} w_n \, \tilde{I}(x_n)$$

$$w_n = \int_{P_{ij}} \tilde{k}_n(x) \, dx$$

$\mathrm{sinc}_B(x - i)$



coiflet-$\phi_j(x)$

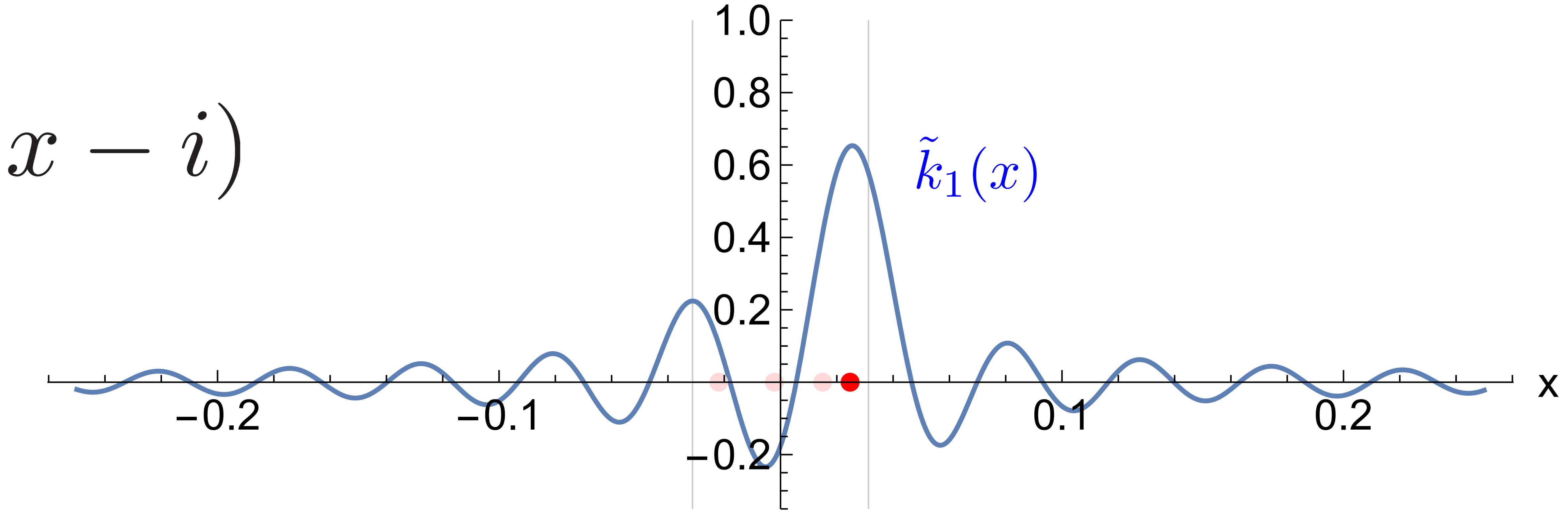


Quadrature weights

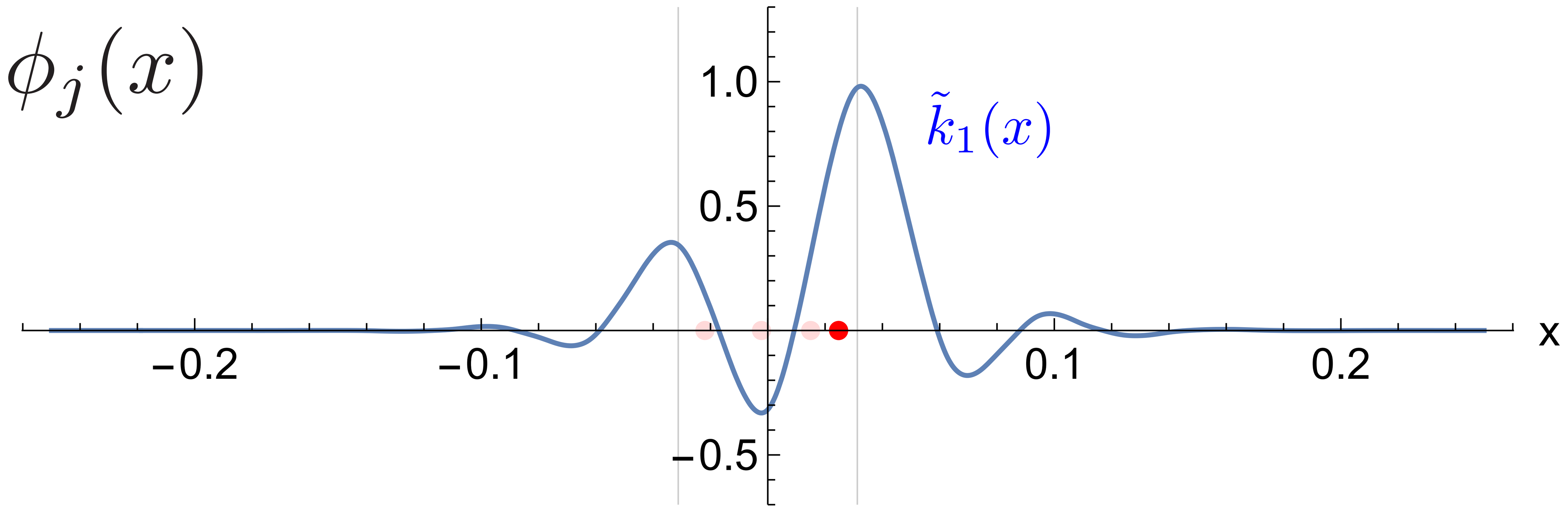
$$I_{ij} = \sum_{n=1}^{4} w_n \, \tilde{I}(x_n)$$

$$w_n = \int_{P_{ij}} \tilde{k}_n(x) \, dx$$

$\mathrm{sinc}_B(x - i)$



coiflet-$\phi_j(x)$

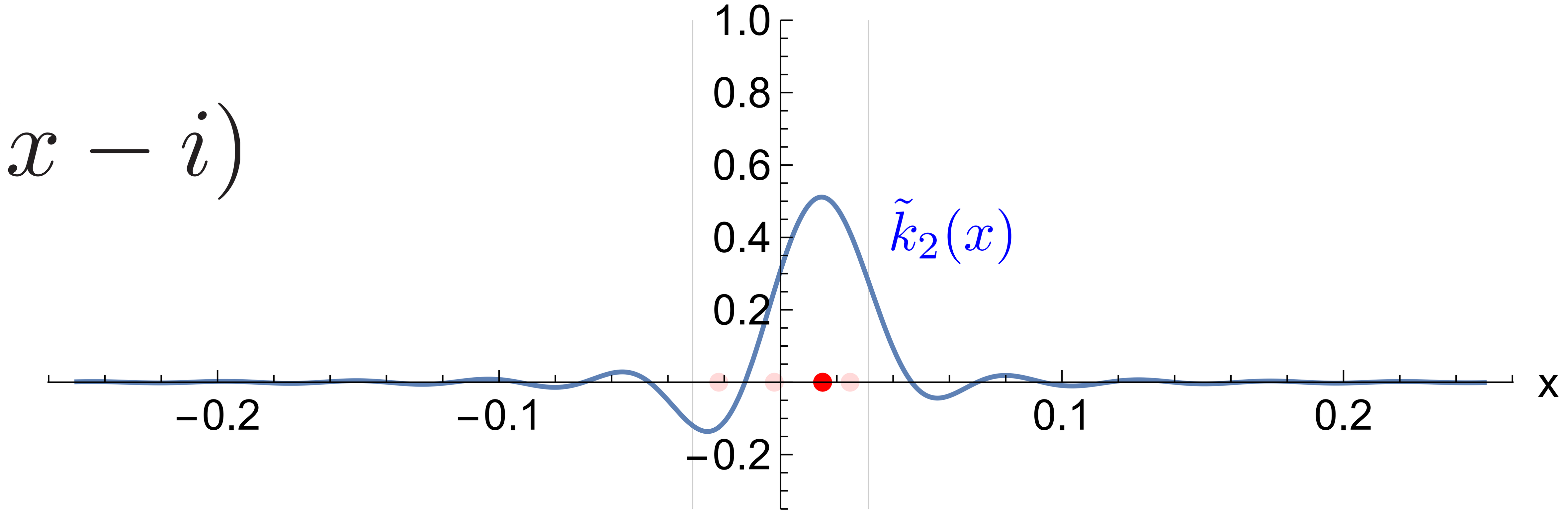


Quadrature weights

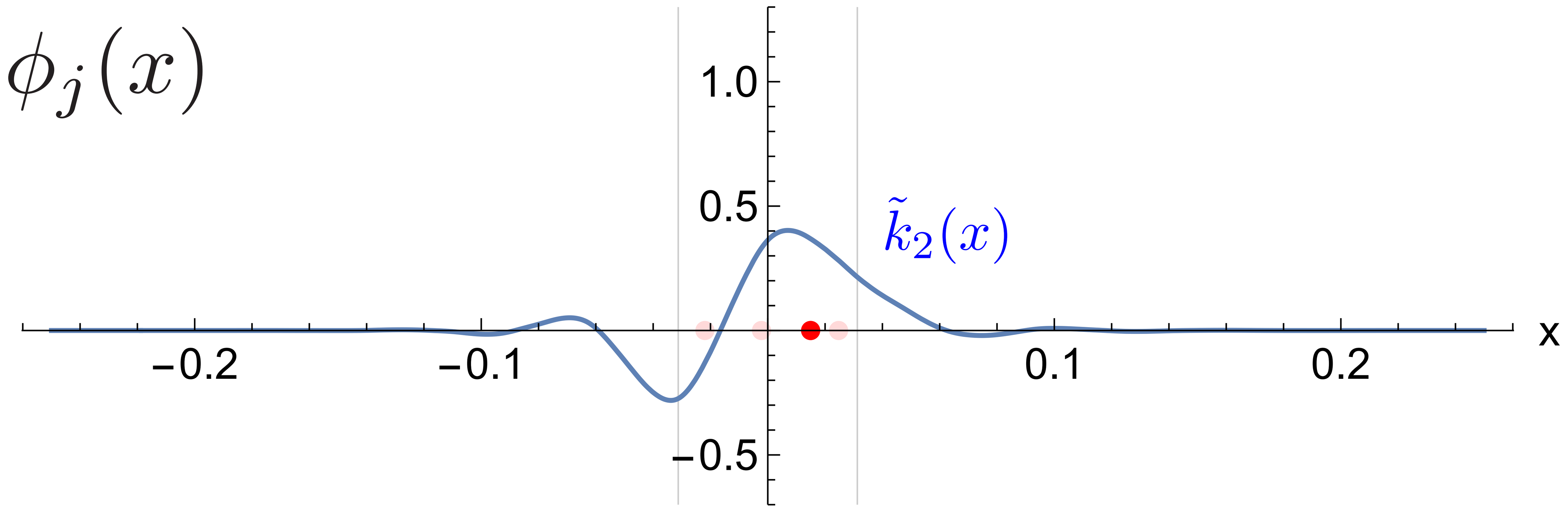
$$I_{ij} = \sum_{n=1}^{4} w_n \, \tilde{I}(x_n)$$

$$w_n = \int_{P_{ij}} \tilde{k}_n(x) \, dx$$

$\mathrm{sinc}_B(x - i)$



coiflet-$\phi_j(x)$

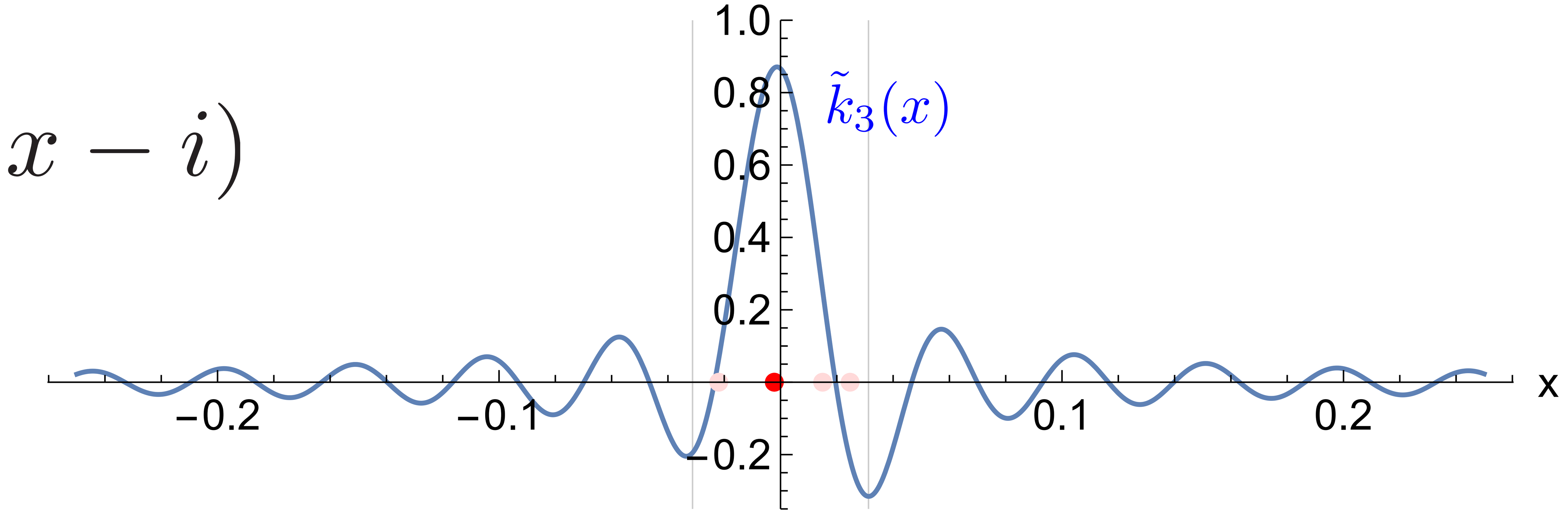


Quadrature weights

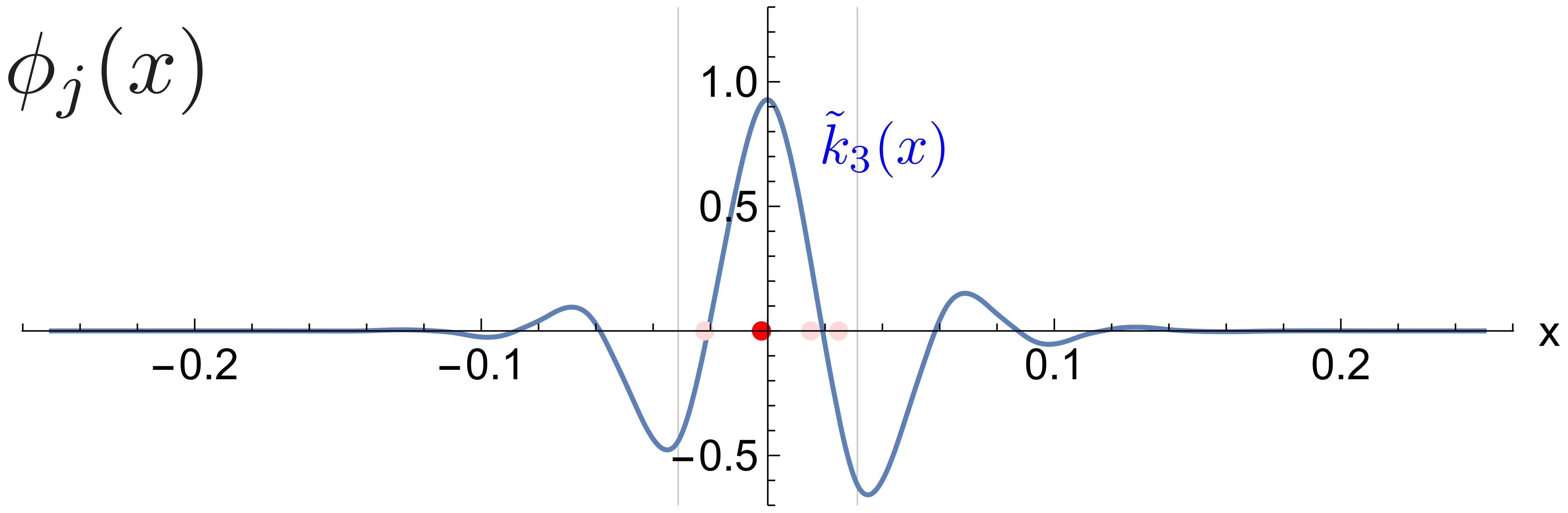
$$I_{ij} = \sum_{n=1}^{4} w_n \, \tilde{I}(x_n)$$

$$w_n = \int_{P_{ij}} \tilde{k}_n(x) \, dx$$

$\mathrm{sinc}_B(x - i)$



coiflet-$\phi_j(x)$

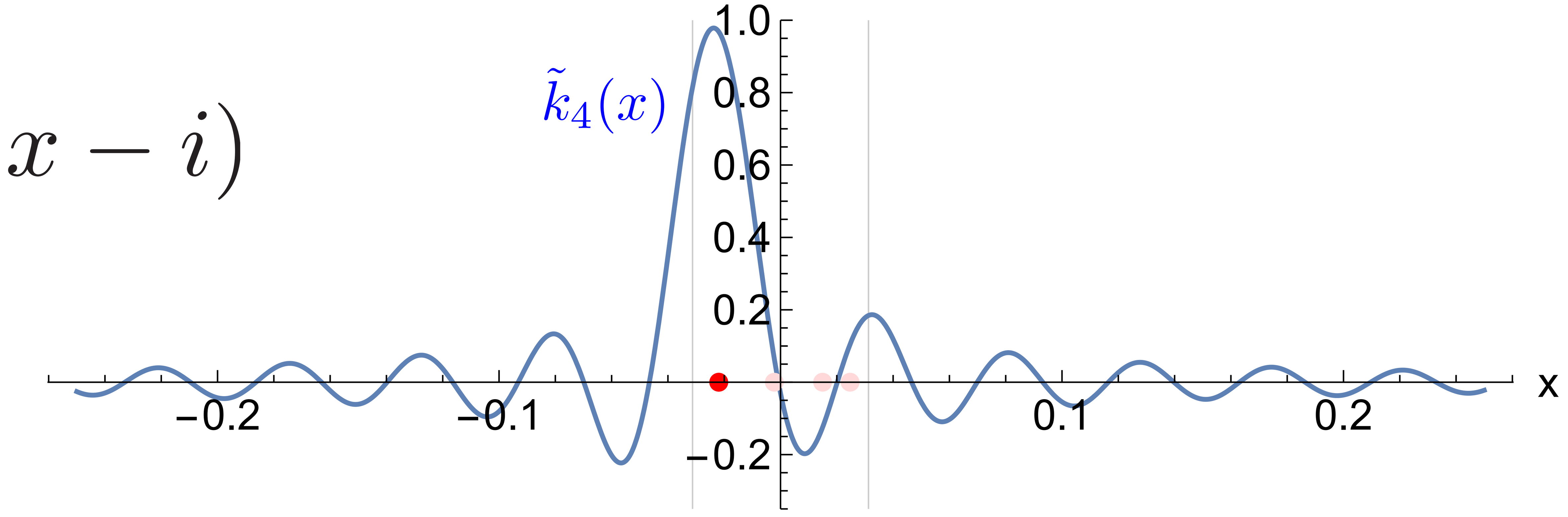


Quadrature weights

$$I_{ij} = \sum_{n=1}^{4} w_n \, \tilde{I}(x_n)$$

$$w_n = \int_{P_{ij}} \tilde{k}_n(x) \, dx$$

$\mathrm{sinc}_B(x - i)$



coiflet-$\phi_j(x)$

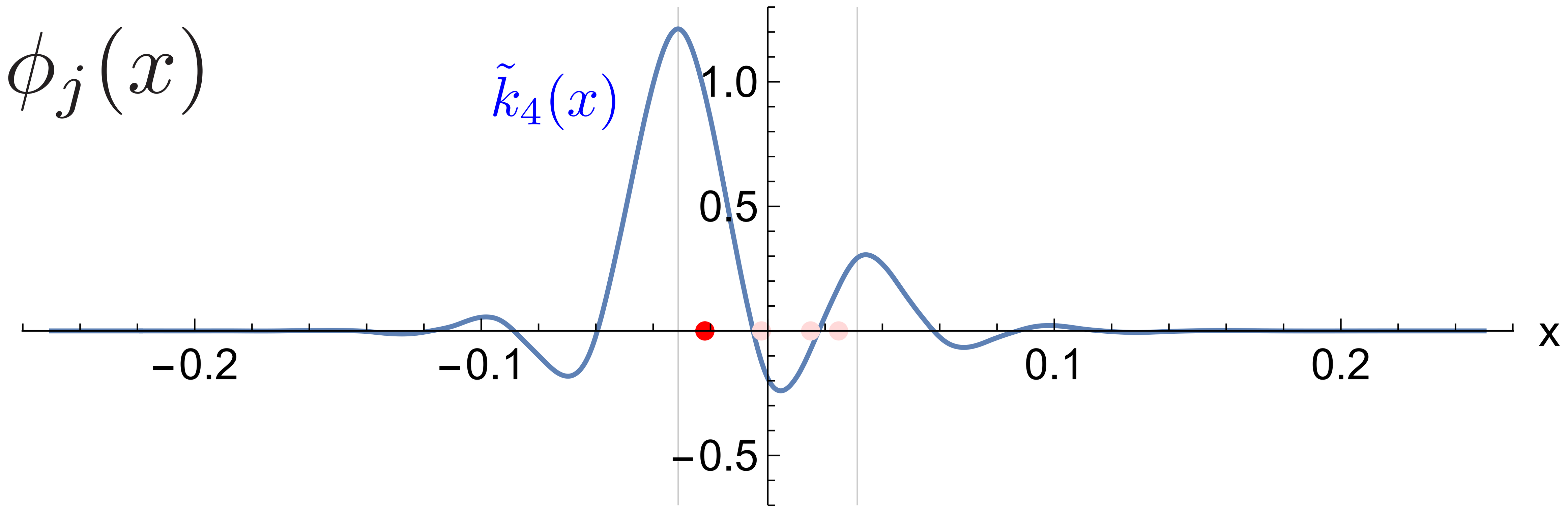


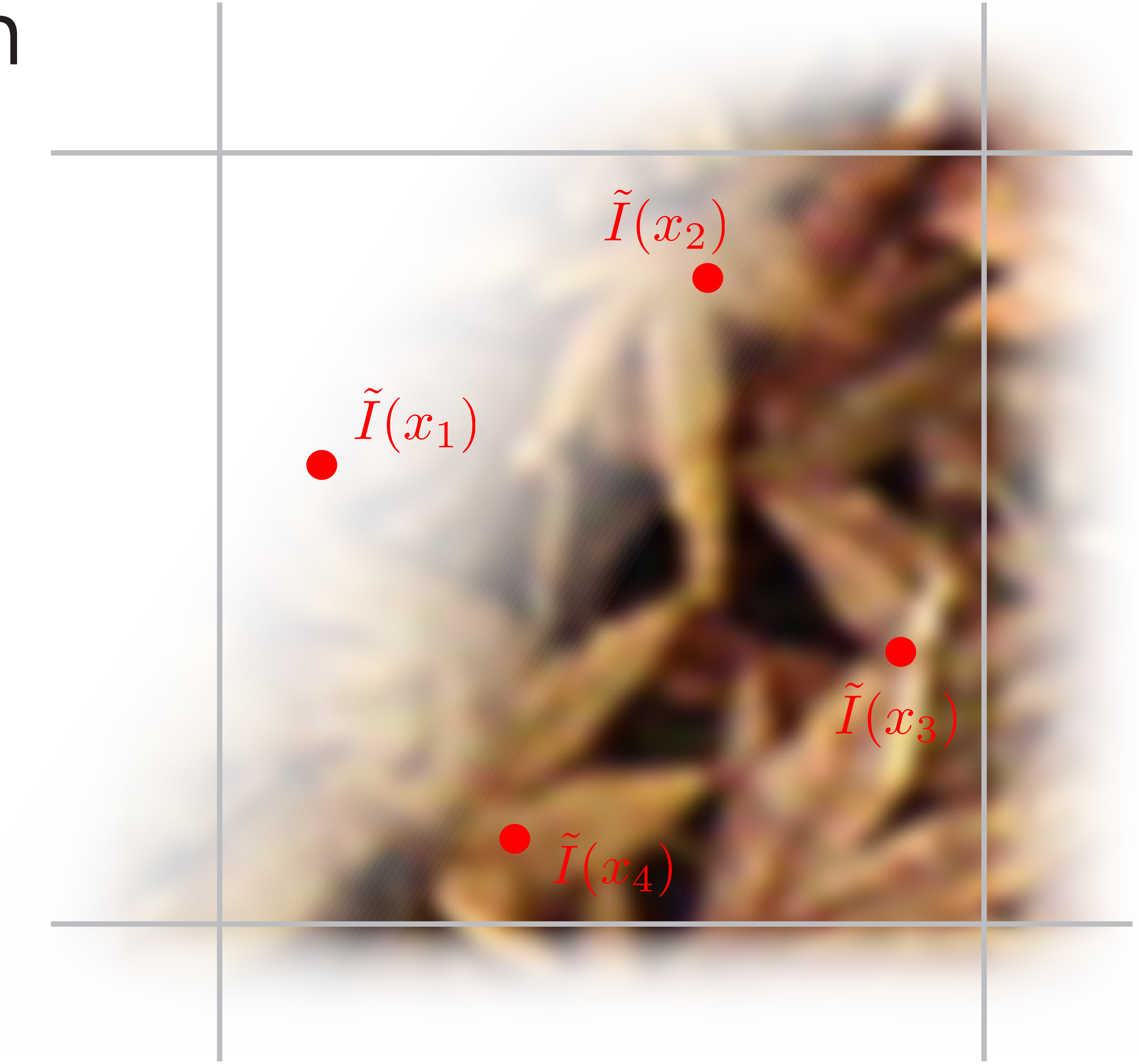
Image reconstruction

$$I_{ij} = \sum_{n=1}^{4} w_n \, \tilde{I}(x_n)$$

$$w_n = \int_{P_{ij}} \tilde{k}_n(x) \, dx$$

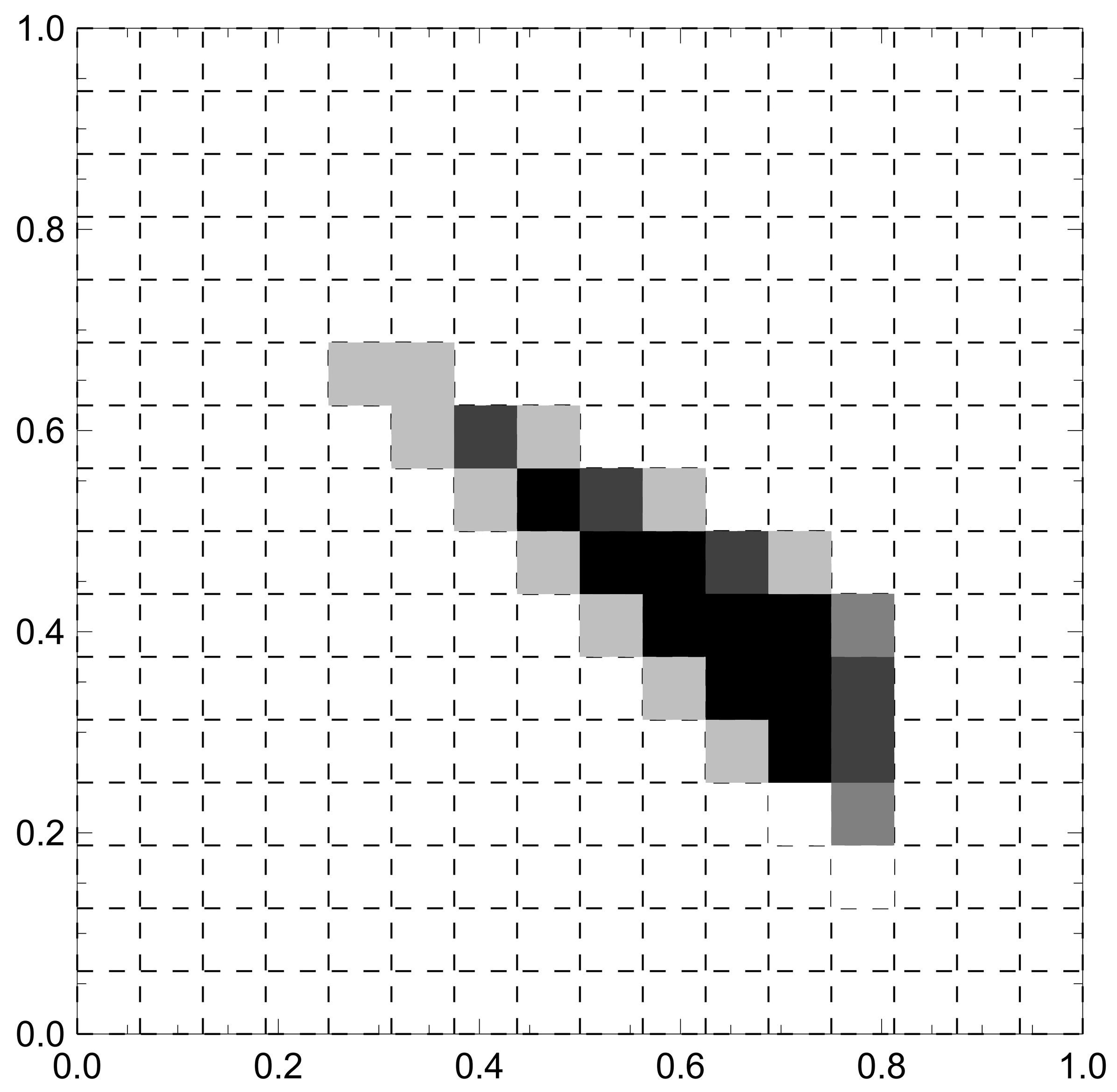
with

$$\tilde{I}(x_i) = \tilde{V}(x_i) \, t(x_i)$$

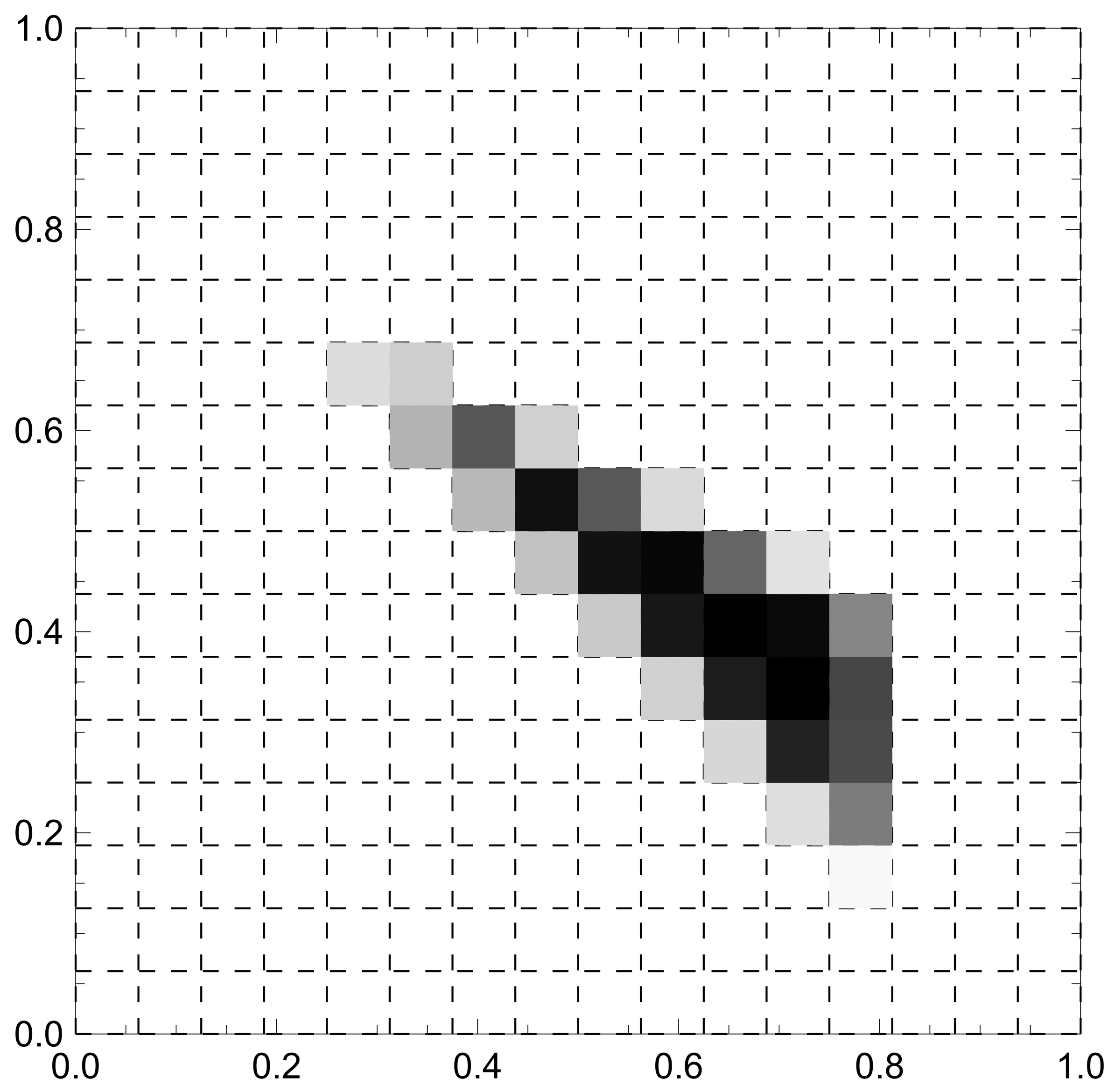


Results

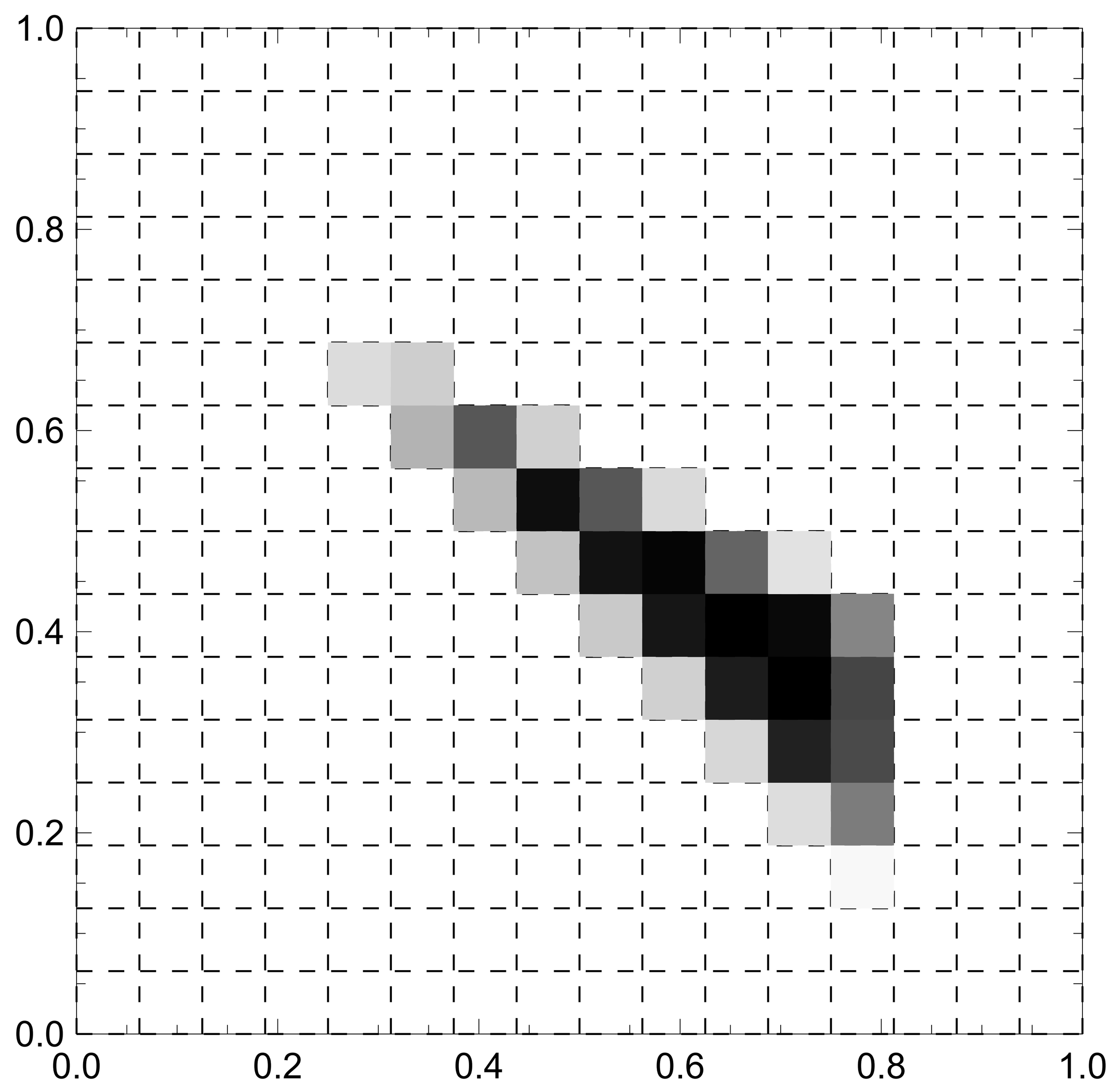
4 samples per pixel, monochrome triangles



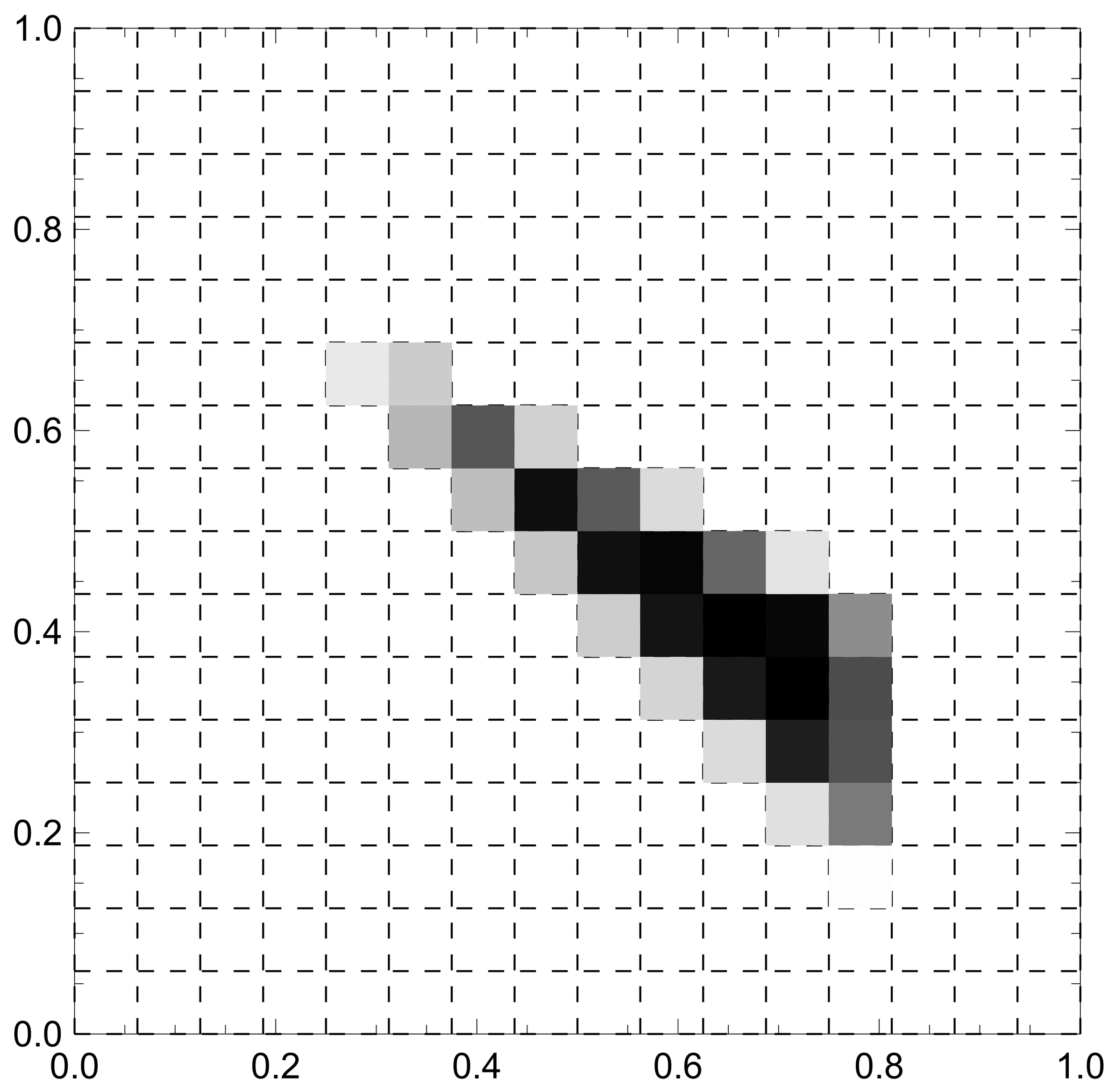
Classical



Sinc-based



Coiflet-based



Reference

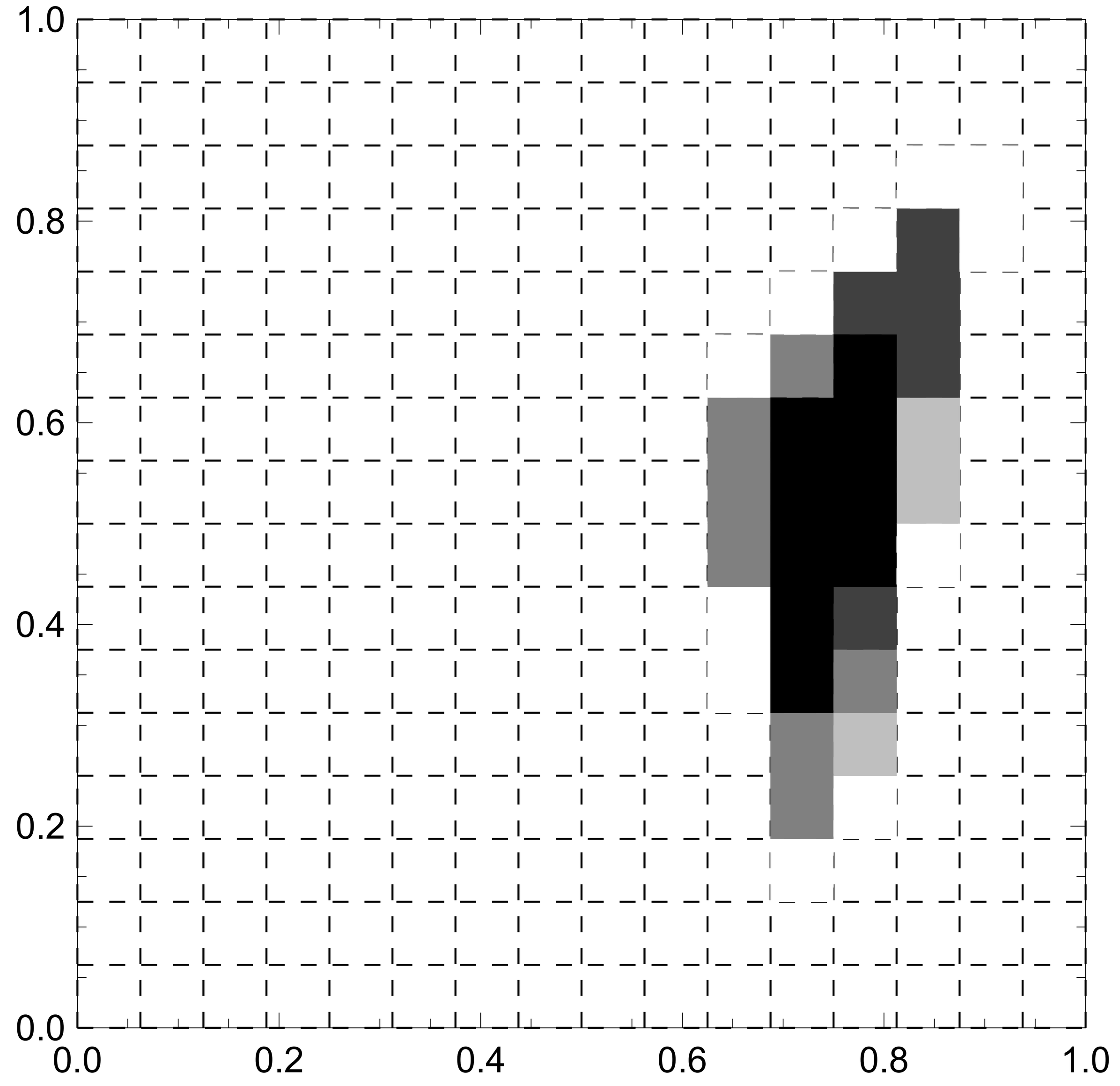
Results

4 samples per pixel, monochrome triangles

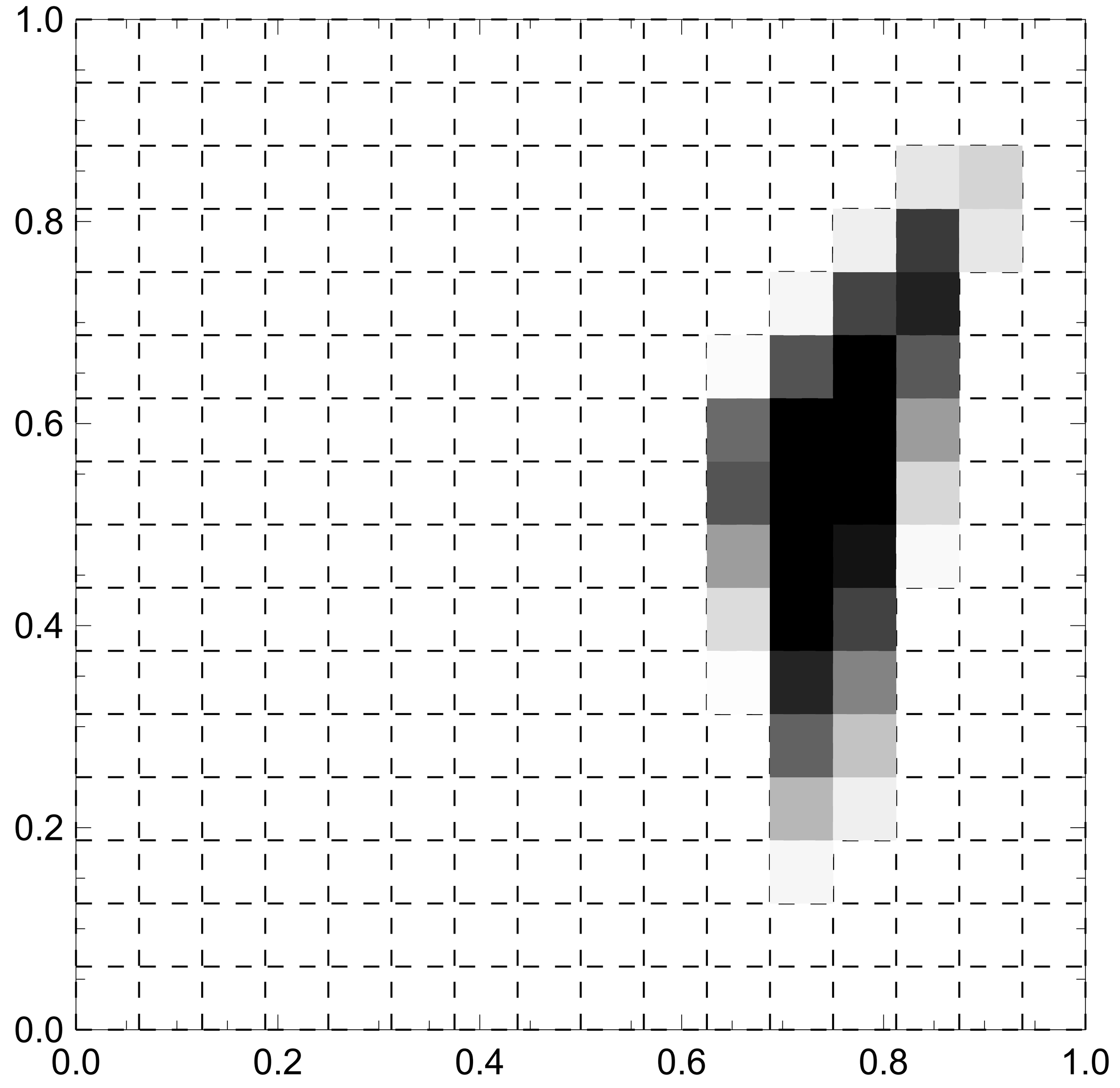
Classical Sinc-based Coiflet-based Reference

Results

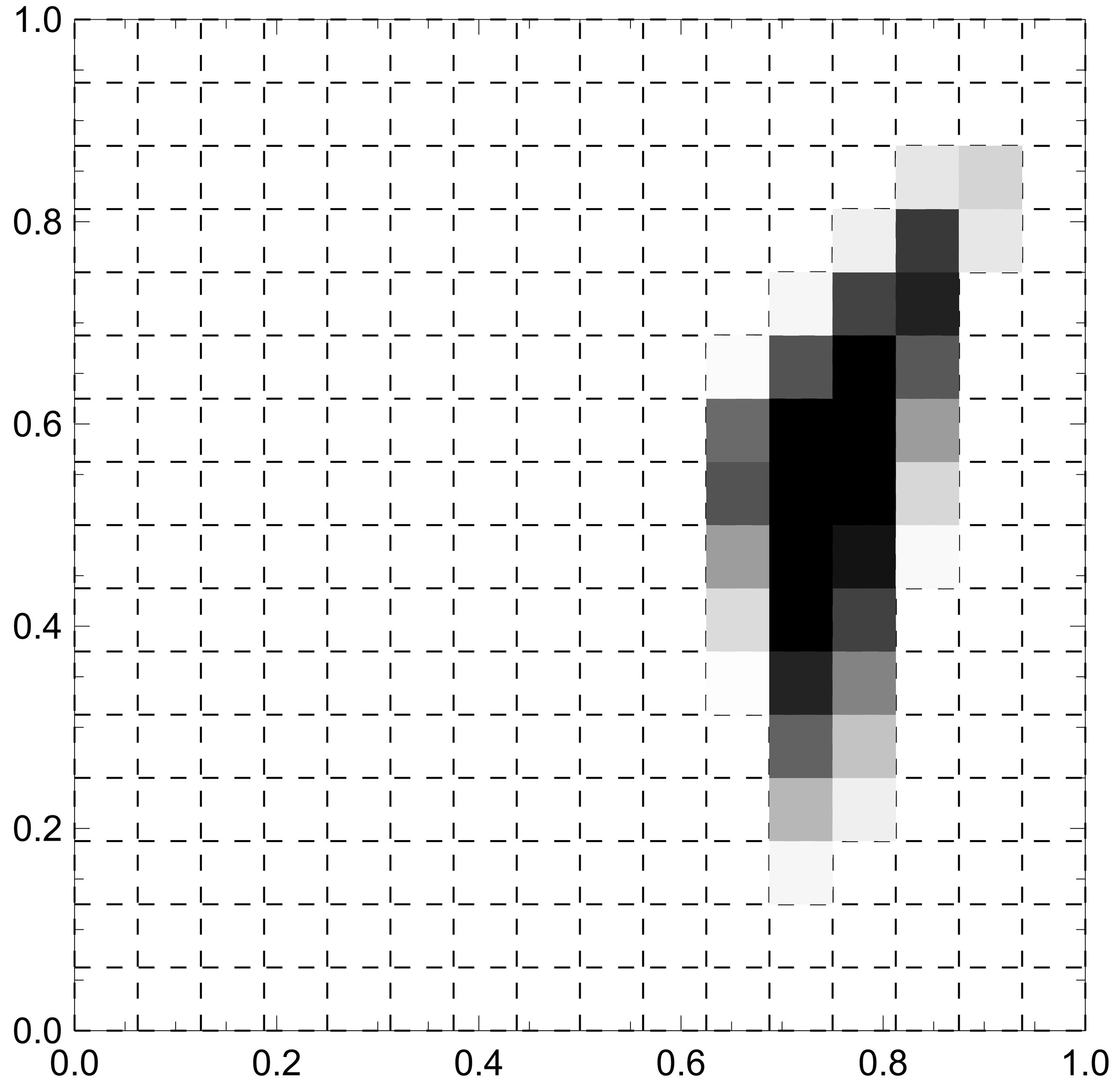
4 samples per pixel, monochrome triangles



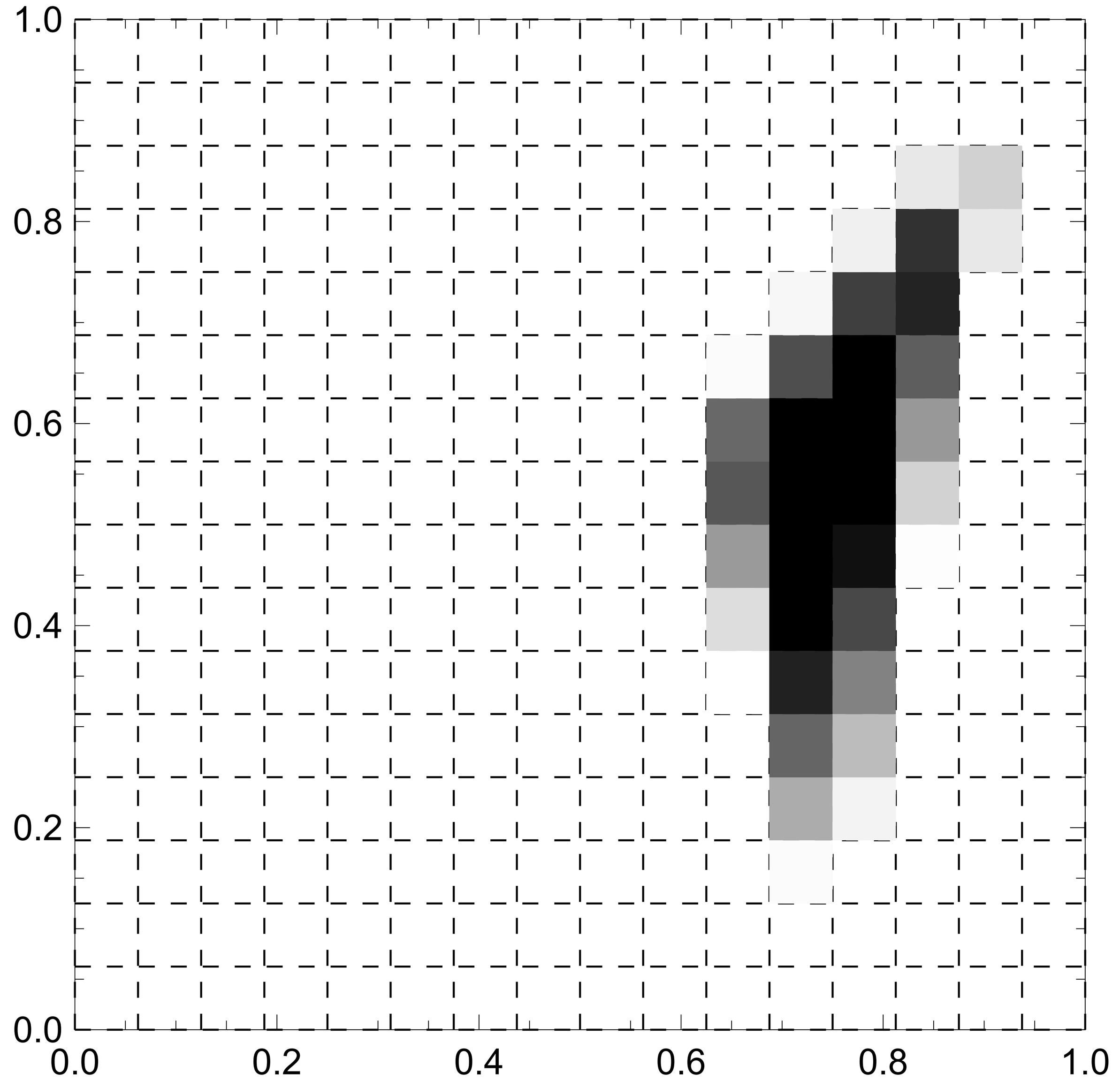
Classical



Sinc-based



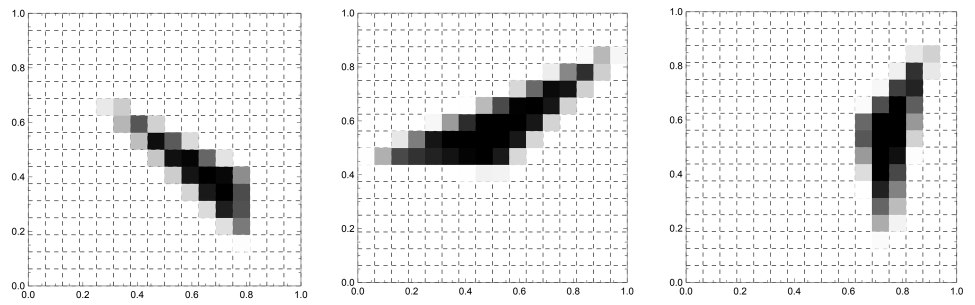
Coiflet-based

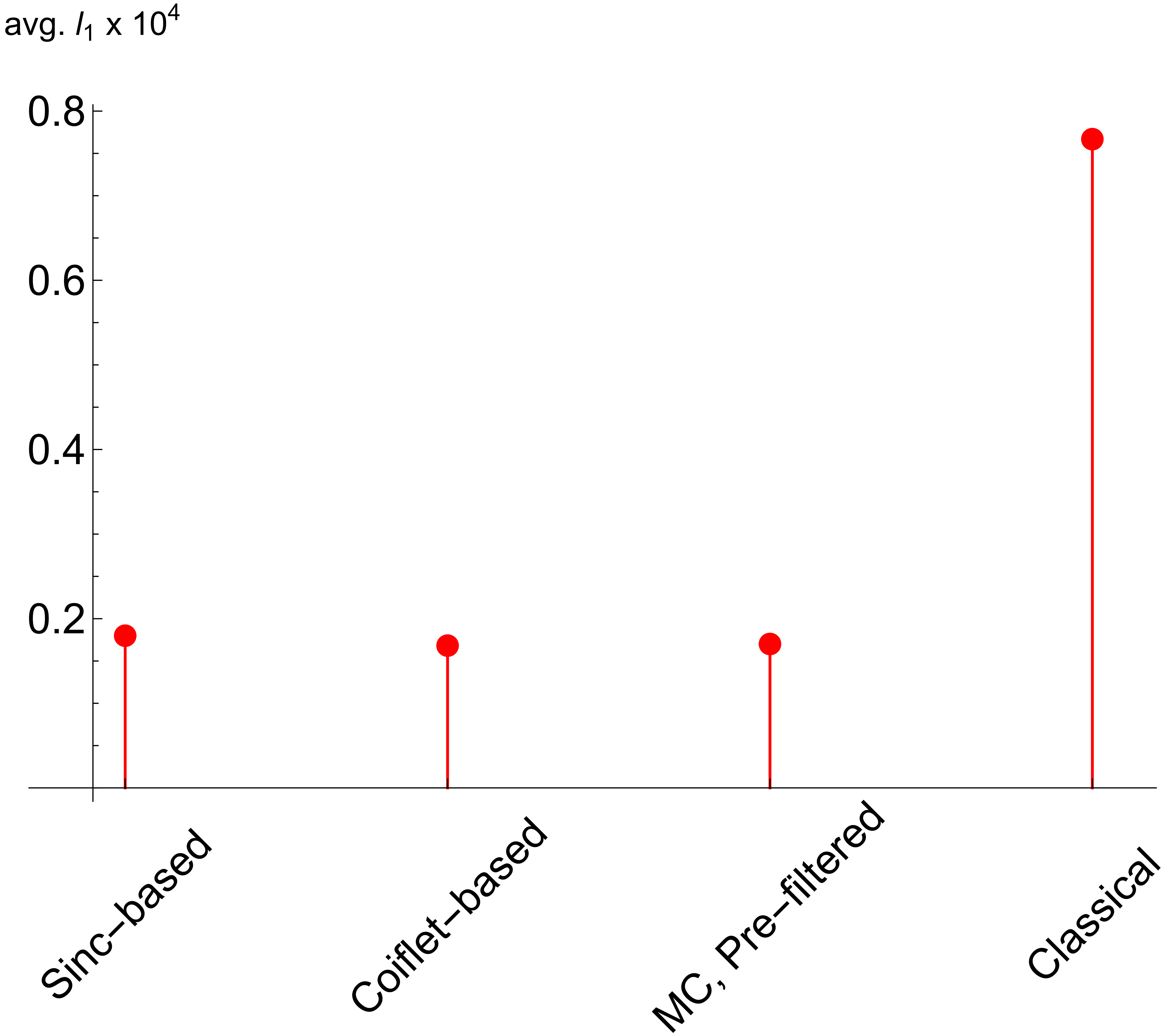


Reference

Results

- 4 samples per pixel
- monochrome triangles
- average error over random triangles





Results

- 4 samples per pixel
- monochrome triangles
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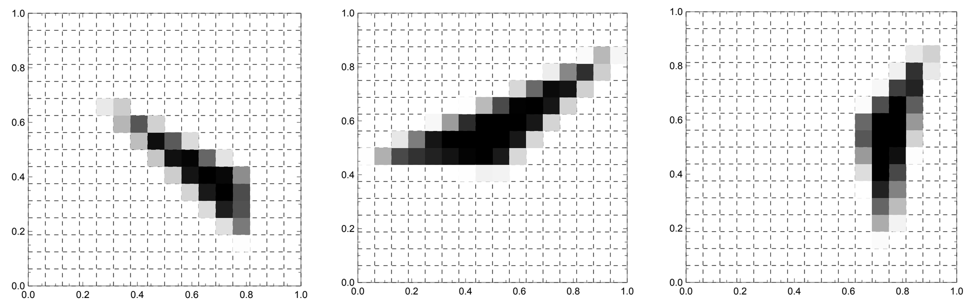
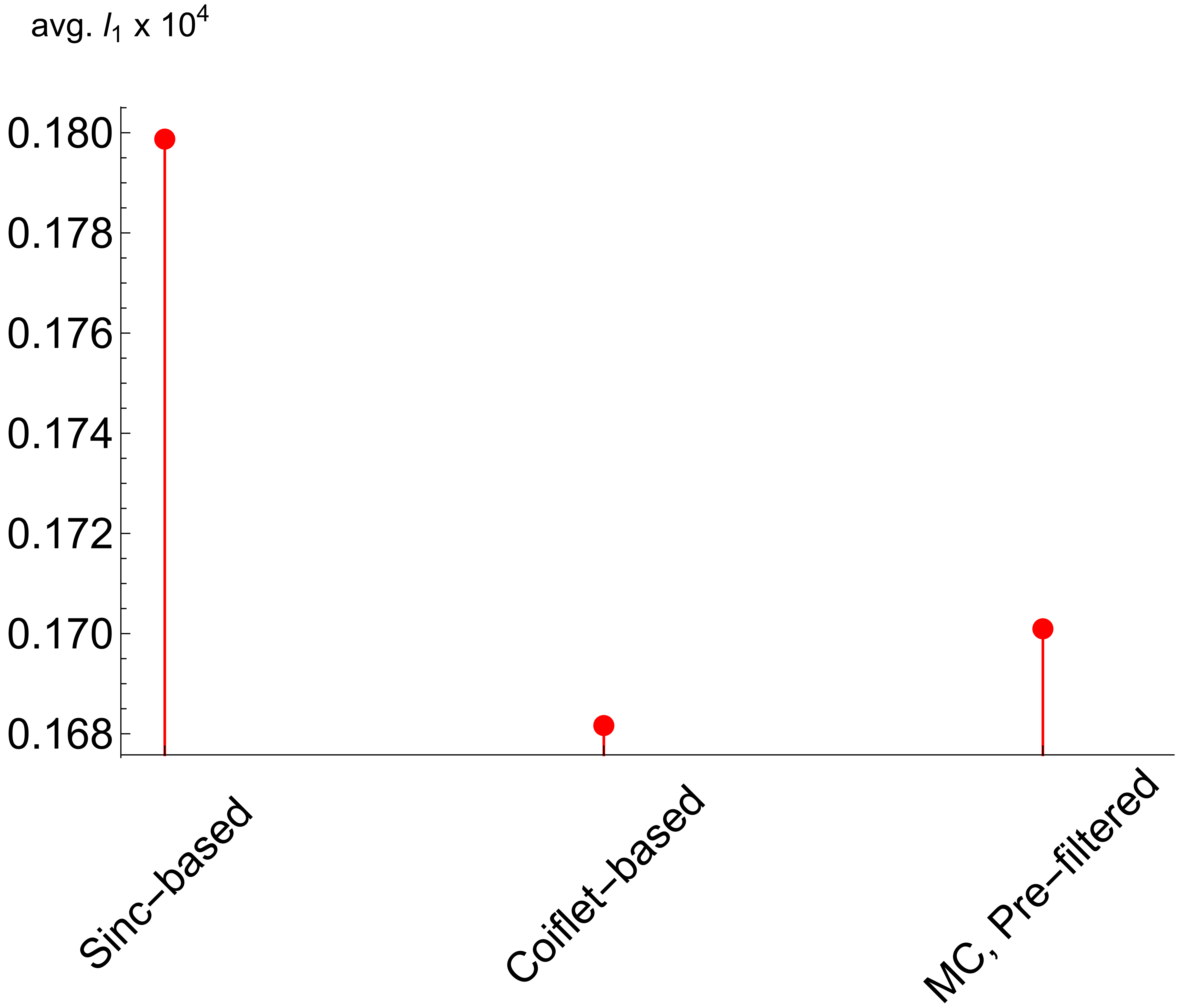
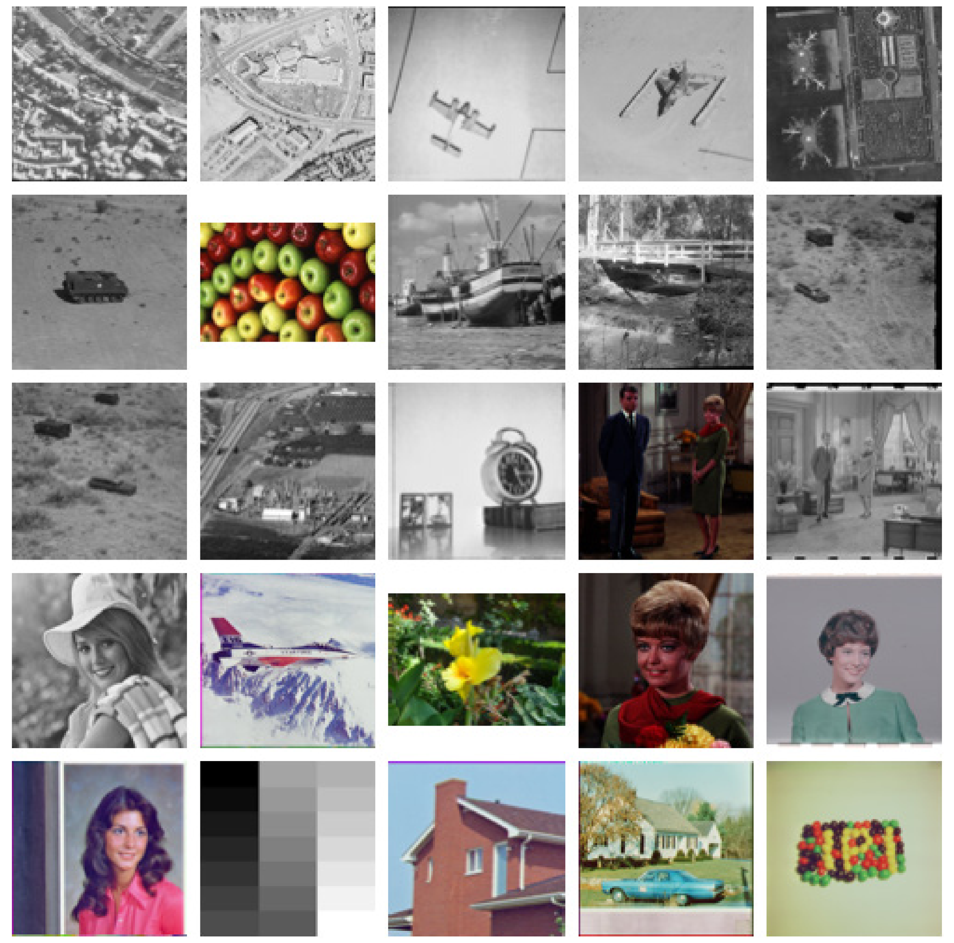
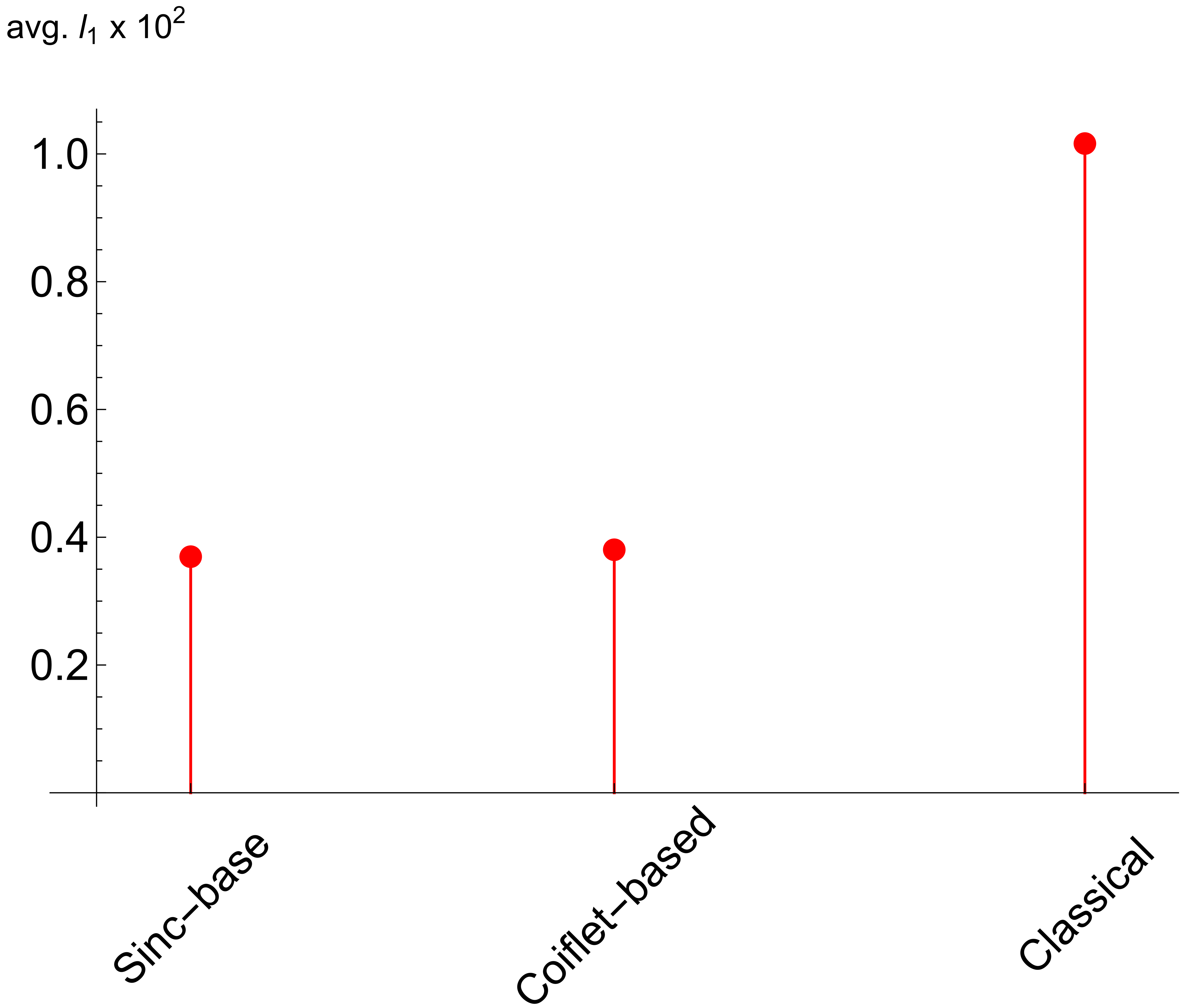




Results

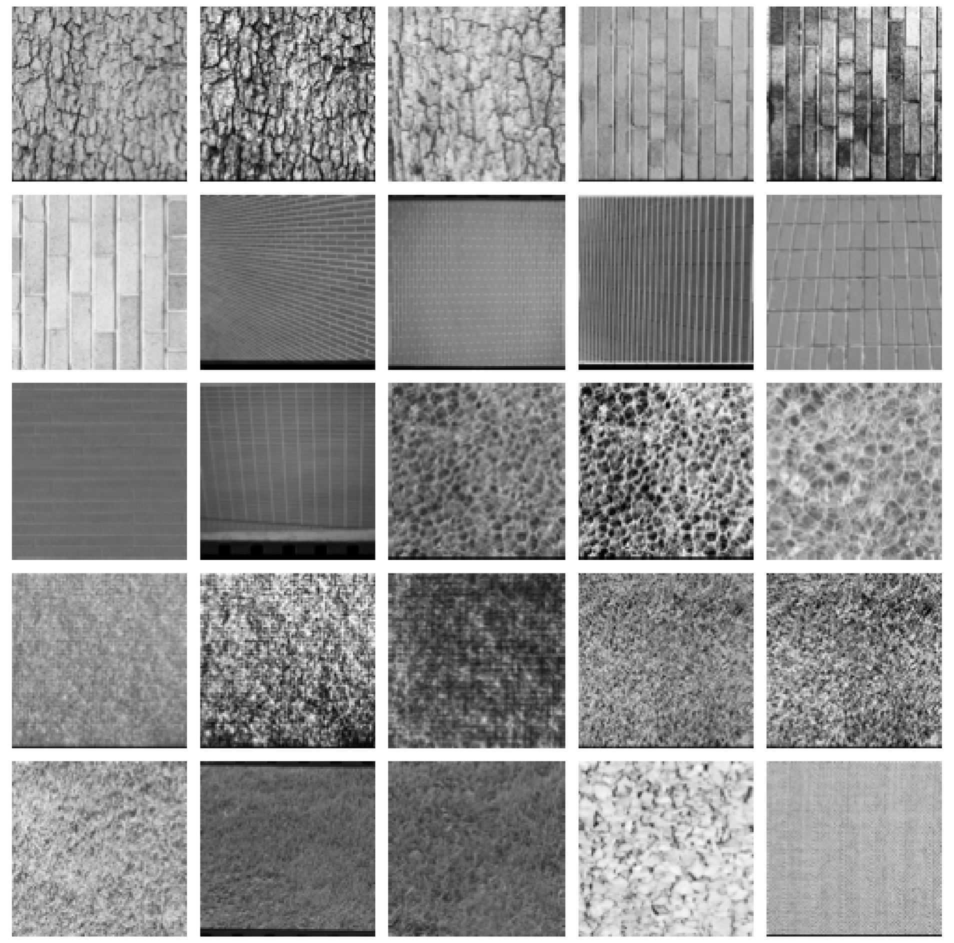
- 4 samples per pixel
- average error over random triangles and a random collection of natural images

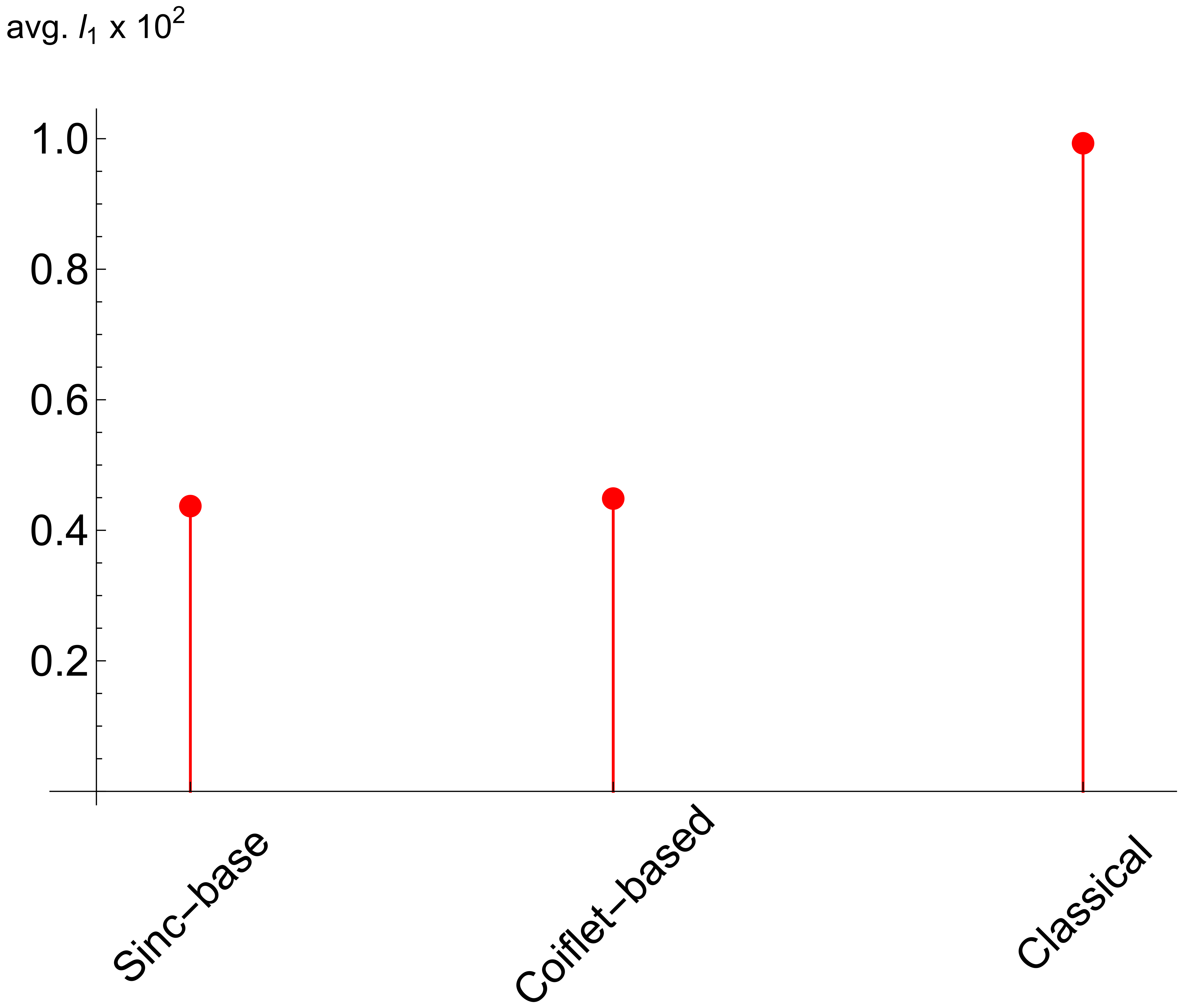




Results

- 4 samples per pixel
- average error over random triangles and a random collection of textures





More details in the paper on …

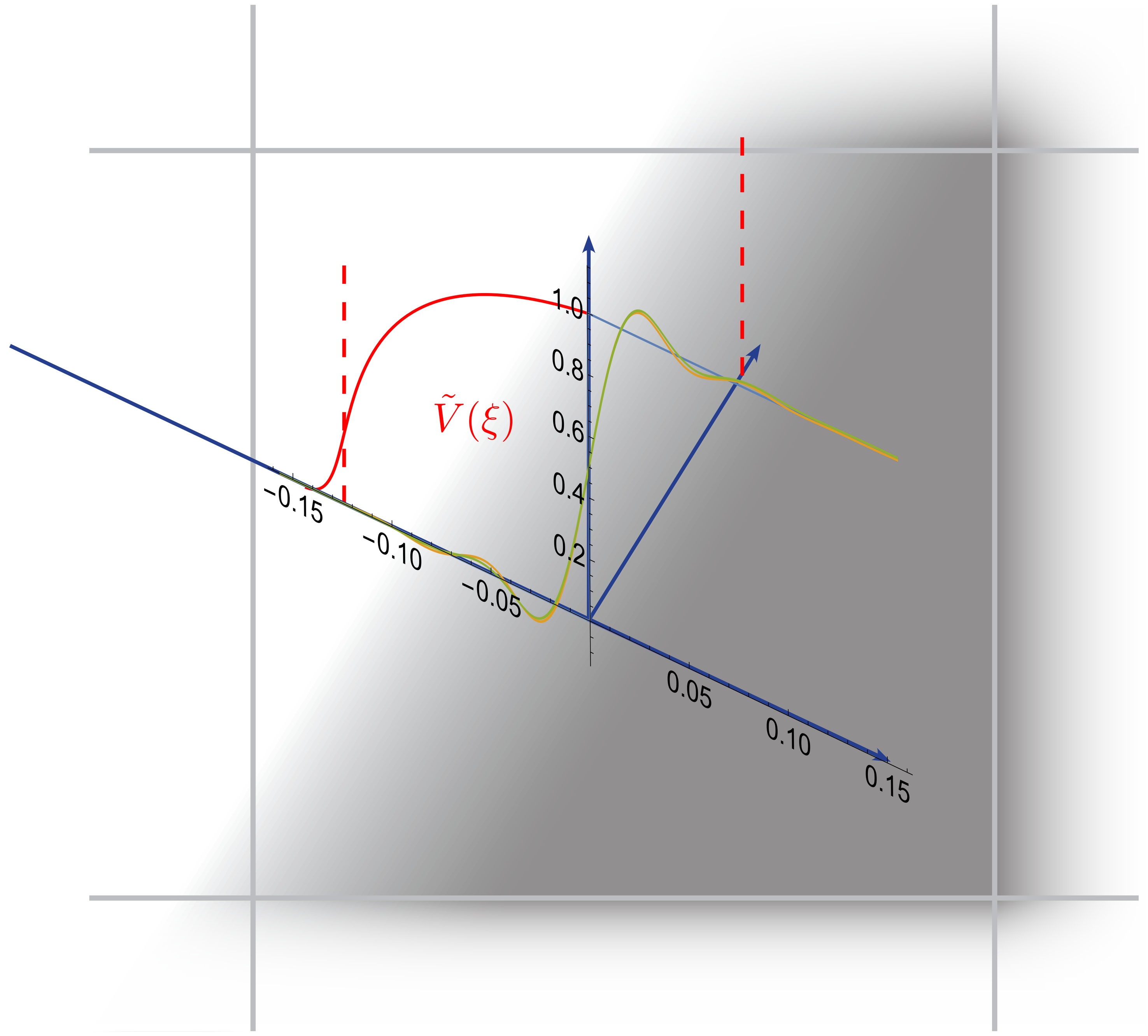
- Non-uniform sample patterns.
- Non-square number of samples (i.e. arbitrary density).
- How to construct pre-filters for wavelets.
- Results for 8-sample pattern.

Outlook

- What kernel $\phi(x - k)$ should we use?
- Translation- and rotation invariant pre-filtering.
- Extension to visibility sampling in the scene.
- More precise analytic treatment with predictive error bounds.

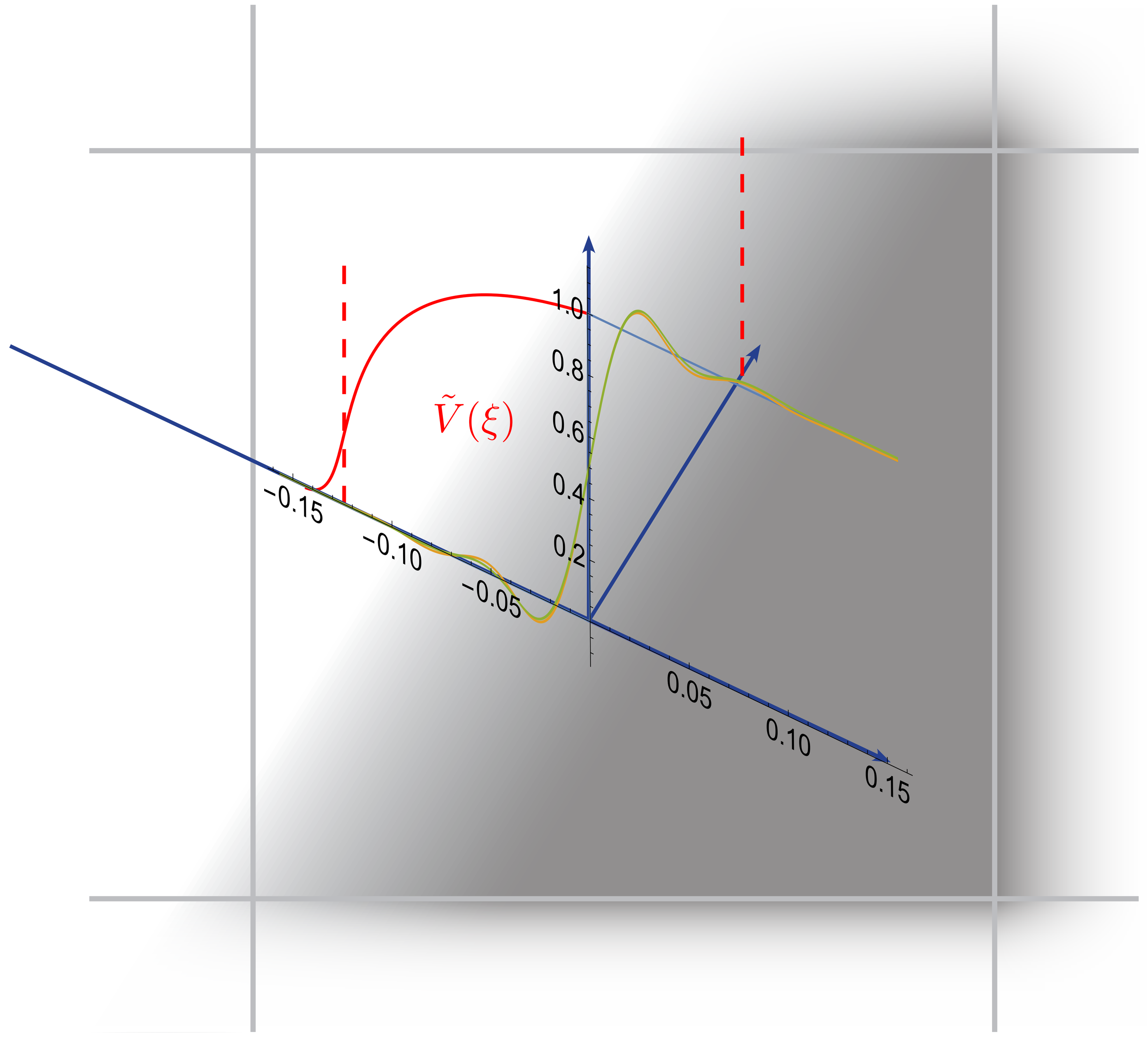
Summary

- Analytic "soft" pre-filtering of visibility discontinuity.

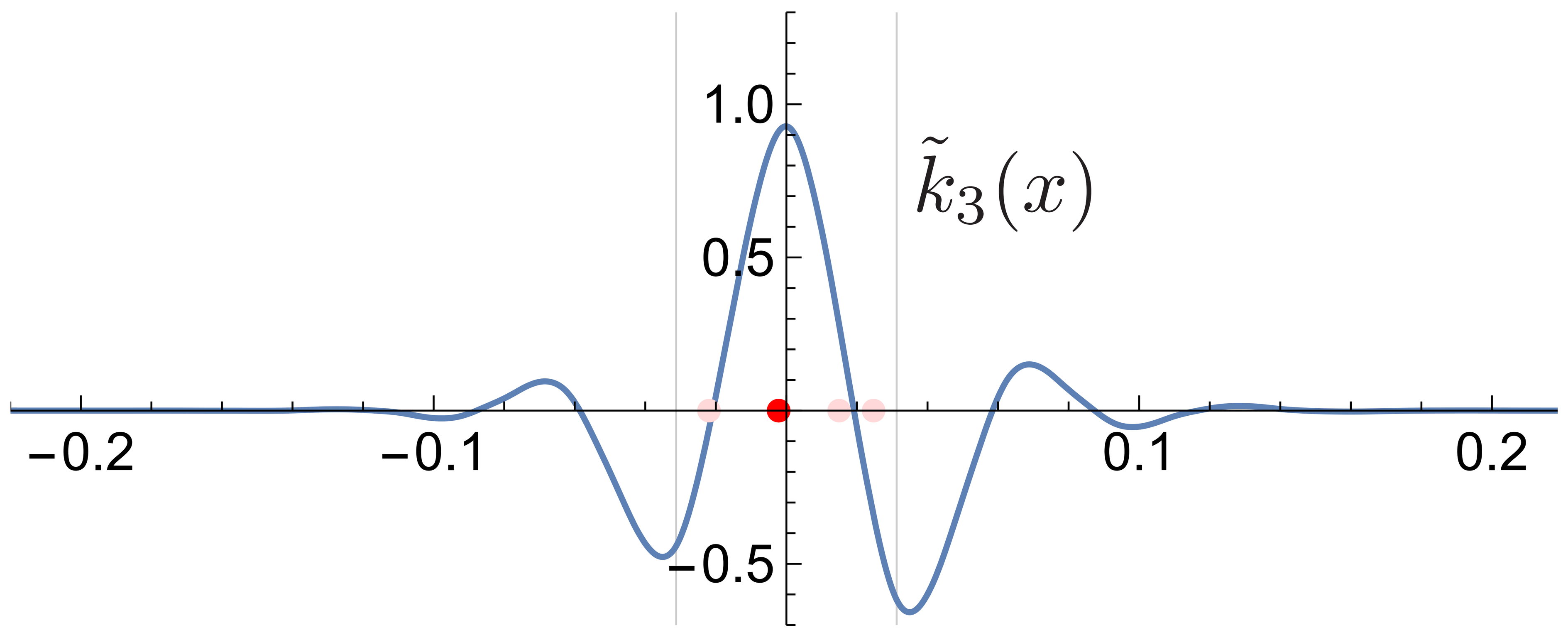


Summary

- Analytic “soft” pre-filtering of visibility discontinuity.

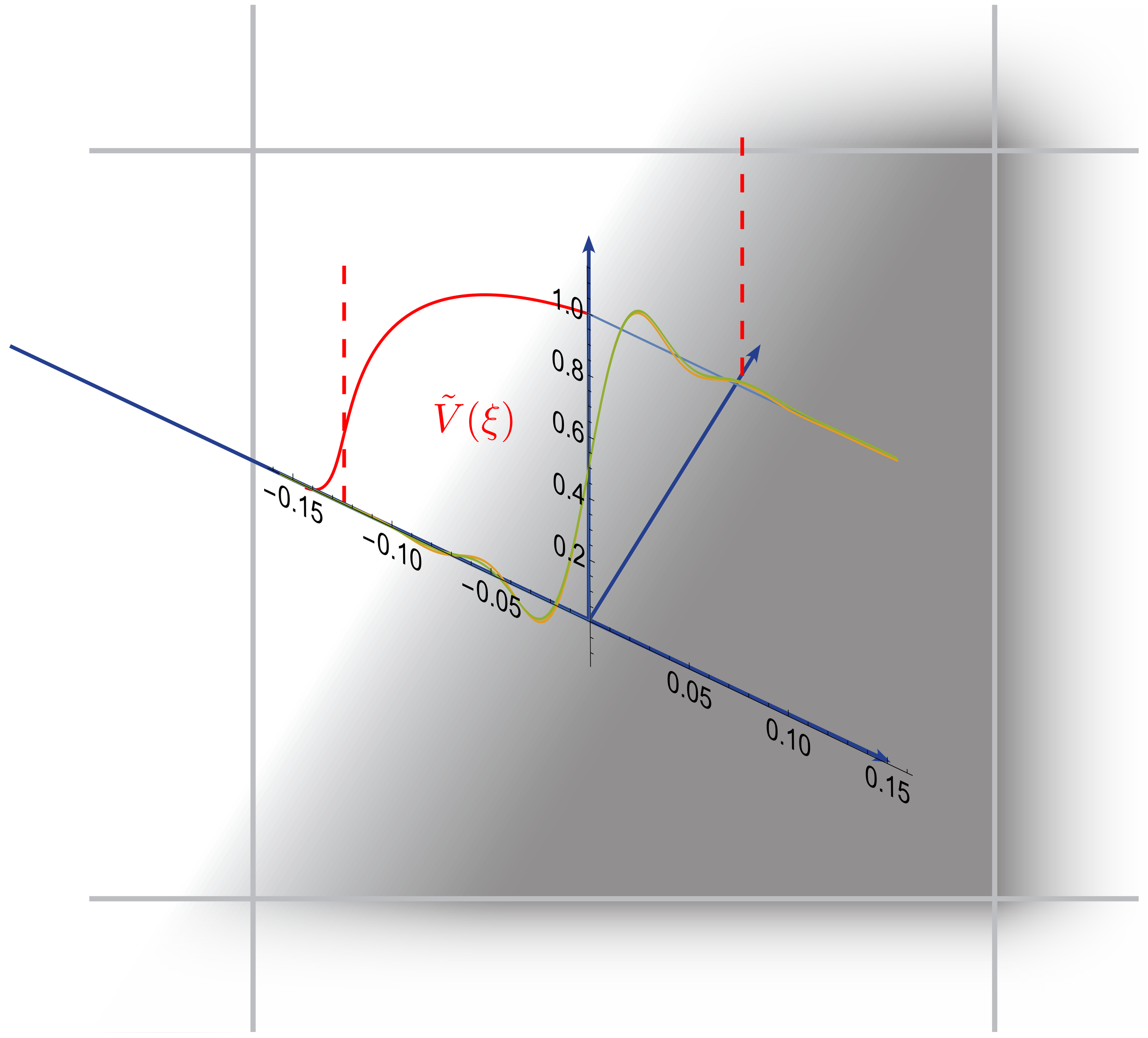


- Reconstruction using analytically determined quadrature weight.



Summary

- Analytic "soft" pre-filtering of visibility discontinuity.



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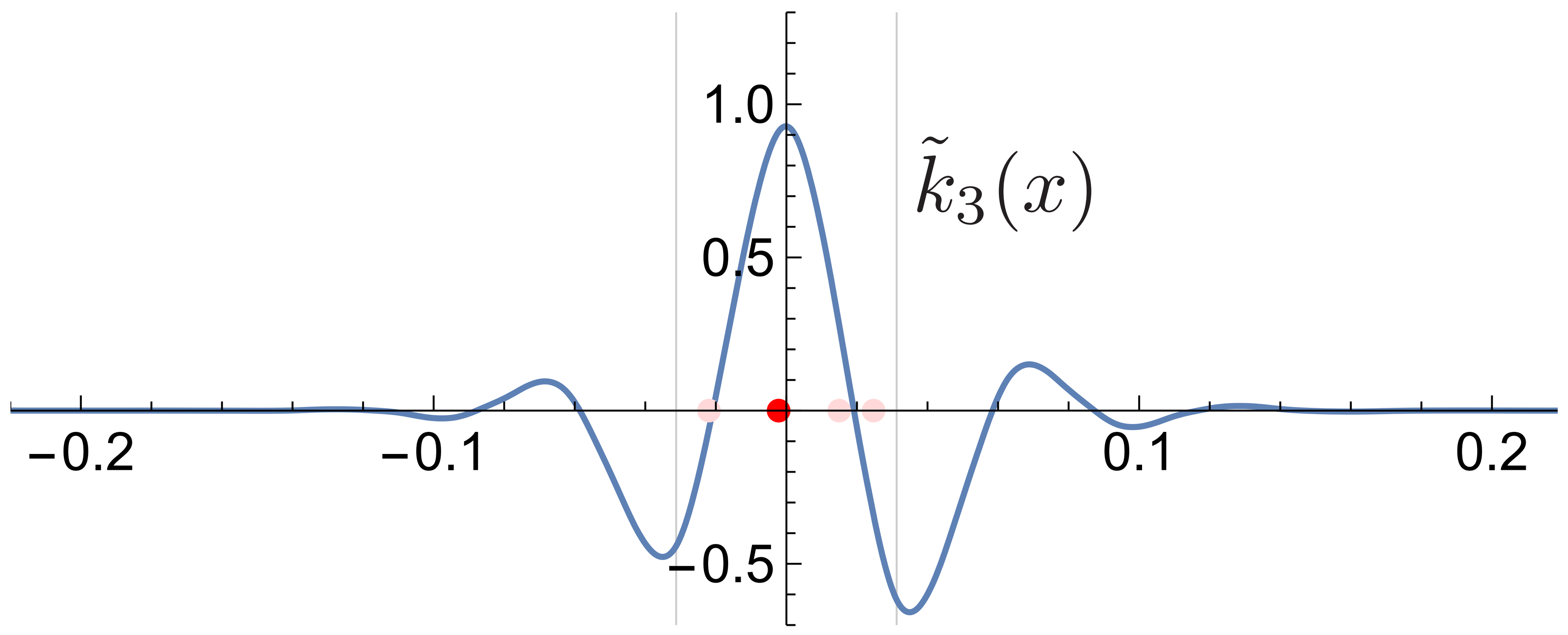

More details and source code: http://isgwww.cs.uni-magdeburg.de/graphics/projects/antialiasing/index.html